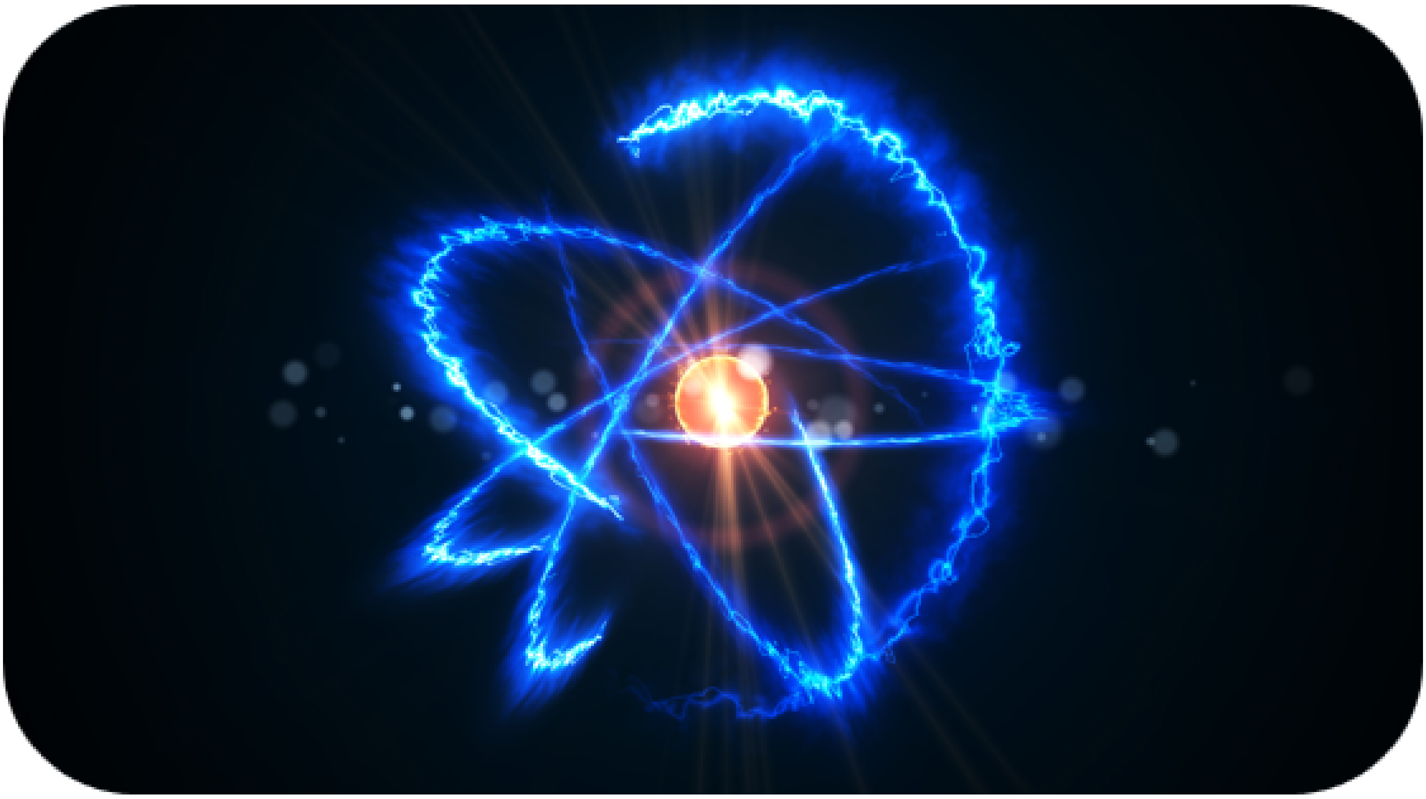
Physics

NEET Class 11th



01

UNITS AND MEASUREMENTS, & BASIC MATHEMATICS

Chapter 01

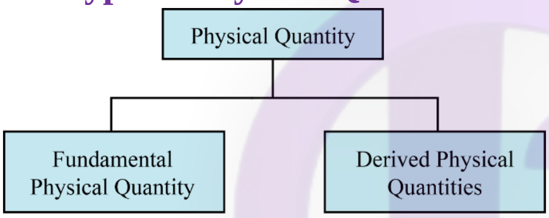
Units and Measurements

1. Fundamental and Derived Units

Physical quantity: Any quantity which can be measured is called a physical quantity.
Examples: length, weight, time etc.

Fig. 1.1

1.1 Type of Physical Quantities



1.2 Fundamental Physical Quantities

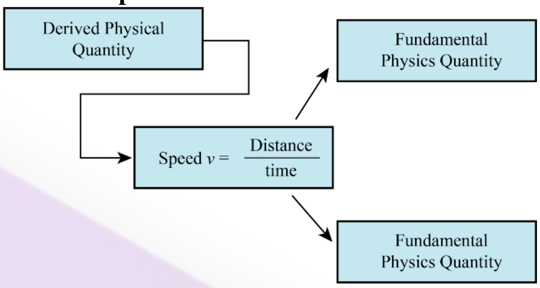
Physical quantities which are independent of other physical quantities are called fundamental physical quantity. These are the quantities we take as fundamental quantities.

Quantity
Length
Mass
Time
Electric Current
Temperature
Amount of Substance
Luminous Intensity

1.3 Derived Physical Quantities

Physical quantities which are dependent on other physical quantities are called derived physical quantity.

For Example:



1.4 Derived Physical Quantities

Examples:

- Acceleration = length/time^2
- Density = mass/ length^3
- Volume = length^3
- Force = mass (length)/time^2
- Momentum = mass. length/time
- Pressure = mass/length.time^2

1.5 How to Measure a Physical Quantity

- For measuring a physical quantity, we have to compare it with some reference, we call it a unit.
- A unit is a standard amount of a physical quantity.

Example: In old times people used to measure length by hand span or foot span.

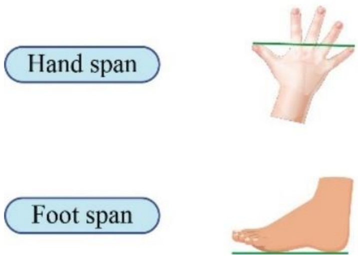


Fig. 1.2

Let's measure the length of a book using our hand span.
In this case the unit for measurement is the hand span.
But the length of hand span varies from person to person.
So, everyone will get a different result for measuring the same object.

So, there was a need of standardisation of units.

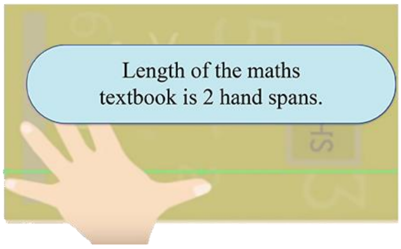


Fig. 1.3

1.6 Standard Units

Some of the standard units:
For measuring length: metre, centimetre, foot etc.
For measuring weight: kilogram, gram, pound etc.

Fig. 1.4

1.7 Expressing Measurement of Physical Quantity

Suppose we measure length of a rod and write
Length = 28
By this expression we didn't get any idea about the size of rod it can be anything like
28 m
28 mm
28 km
28 foot or 28 steps
So, we should always express a measurement with the unit of measurement.

Physical Quantity = Magnitude(n) x Unit(u) = n u

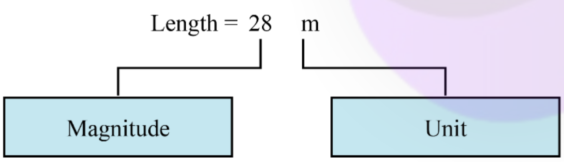


Fig. 1.5

NOTE:

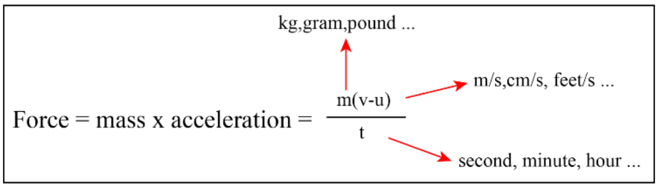
We always write a measurement of physical quantity as its magnitude multiplied by its unit.

If we measure a physical quantity in more than one unit then the multiplication of magnitude and unit is a constant.
If magnitude of a physical quantity is
= n_1 in the u_1 unit and n_2 in u_2 unit.
Physical Quantity = $n_1u_1 = n_2u_2$

1.8 Need of System of Units

What if everyone uses a unit of their choice for every measurement.

For Example:



If everyone decides to have his own way of measurement, then it will not be possible to come to the correct conclusion. Thus, a well-defined, universally accepted system must be developed.

1.9 System of Units

A system of units is a complete set of units which is used to measure all kinds of fundamental and derived quantities.
Let's see examples of some of the major system of units

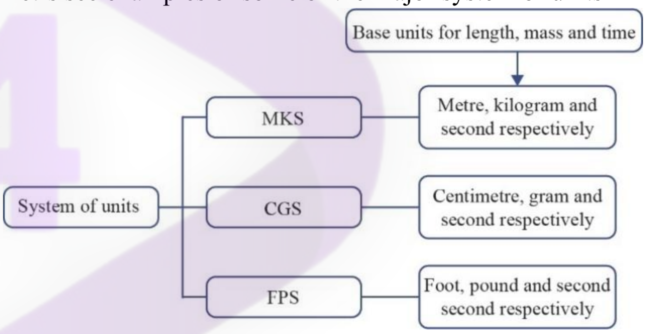


Fig. 1.6

1.10 The SI System of Units

Earlier different systems of units were used in different countries.
So, there was a need for an internationally accepted system of units.
Here comes the "International System of Units" or SI.
Currently it is the most popular system of units worldwide.
In the SI system there are 7 base units and 2 supplementary units.

1.11 Fundamental Units:

Quantity	Name of Units	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	Cd

1.12 Supplementary Units:

Quantity	Name of Units	Symbol
Plane angle	Radian	rad
Solid angle	Steradian	sr

1.13 Plane Angle

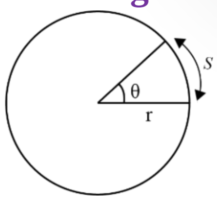


Fig. 1.7

$$\theta = \frac{s}{r} rad$$

1.14 Solid Angle

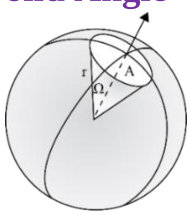


Fig. 1.8

$$\Omega = \frac{A}{r^2} sr$$

2. Dimensions

Dimensions of a physical quantity are the powers to which the fundamental units must be raised in order to get the unit of the derived quantity.

Fundamental quantity	Dimension
Mass	[M]
Length	[L]
Time	[T]
Current	[A]
Temperature	[K]
Amount of substance	[mol]
Luminous intensity	[cd]

2.1 Writing Dimensions of Physical Quantities

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} = \frac{\text{length}}{\text{time}}$$

$$\Rightarrow \text{dimension of velocity} = \left[L^1 T^{-1} \right]$$

$$\text{Acceleration}(a) = \frac{\text{change in velocity}}{\text{time}} = \frac{\text{length}}{(\text{time})^2}$$

$$\Rightarrow \text{Dimension of acceleration} = \left[L^1 T^{-2} \right]$$

$$\text{Force} = \text{Mass} \times \text{Acceleration} = \text{Mass} \times \frac{\text{length}}{(\text{time})^2}$$

$$\Rightarrow \text{Dimension of force} = \left[M^1 L^1 T^{-2} \right]$$

However, there are some quantities such as dimension of angle,

$$\text{Dimension of angle } = \frac{\text{arc length}}{\text{radius}} = \left[\frac{\text{L}}{\text{L}} \right]$$

i.e., $L = 0$

defining all fundamental quantities are zero.

2.2 How do Dimensions Behave in Mathematical Formulae?

Rule 1: All terms that are added or subtracted must have the same dimensions.

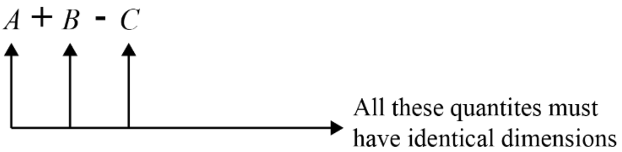


Fig. 1.9

Rule 2: Dimensions obey rules of multiplication and division.

$$D = \frac{AB}{C}$$

Given $A = \lfloor ML^0T^{-2} \rfloor, B = \lfloor M^0L^{-1}T^2 \rfloor, C = \lfloor ML^{-2}T^0 \rfloor$

$$[D] = \frac{\lfloor ML^0T^{-2} \rfloor \times \lfloor M^0L^{-1}T^2 \rfloor}{\left[ML^{-2}T^0\right]}$$

$$\Rightarrow [D] = \left[M^{1-1}L^{0-1+2}T^{-2+2}\right]$$

$$\Rightarrow [D] = \left[M^0L^1T^0\right]$$

2.3 Dimensional Analysis

Dimensional analysis is a tool to find or check relations among physical quantities by using their dimensions.
By using dimensional analysis, we can
1. Convert a physical quantity from one system of units to another.
2. Check the dimensional consistency of equations
3. Deduce relation among physical quantities.

2.4 Converting a Physical Quantity from One System of Unit to Another.

If u_1 and u_2 are the units of measurement of a physical quantity Q and n_1 and n_2 are there corresponding magnitude, then $Q = n_1u_1 = n_2u_2$

Let M_1, L_1 and T_1 be the fundamental units of mass, length and time in one system: and M_2, L_2, T_2 be corresponding units in another system. If the dimensional formula of quantity be $\left[M^aL^bT^c\right]$ then

$u_1 = \lfloor M_1^aL_1^bT_1^c \rfloor$ and $u_2 = \lfloor M_2^aL_2^bT_2^c \rfloor$

$$Q = n_1\lfloor M_1^aL_1^bT_1^c \rfloor = n_2\lfloor M_2^aL_2^bT_2^c \rfloor$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

This equation can be used to find the numerical value in the second or new system of units.
Let us convert one joule into erg.

Joule is SI unit of energy and erg is the CGS unit of energy.
Dimensional formula of energy is $[ML^2T^{-2}]$
a = 1, b = 2, c = - 2.

SI	CGS
$M_1 = 1kg = 1000g$	$M_2 = 1g$
$L_1 = 1m = 100\,cm$	$L_2 = 1cm$
$T_1 = 1s$	$T_2 = 1s$
$n_1 = 1(\text{Joule})$	$n_2 = ?(\text{erg})$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

This equation can be used to find the numerical value in the second or new system of units.

$$= 1\left[\frac{1000}{1}\right]^1\left[\frac{100}{1}\right]^2\left[\frac{1}{1}\right]^{-2}$$

$$= 1\times10^3\times10^4 = 10^7$$

$$\therefore 1\,\text{joule} = 10^7\,\text{erg.}$$

2.5 Checking the Dimensional Consistency of Equations

Principle of Homogeneity of Dimensions:
For an equation to be valid, the dimensions on the left side must match the dimensions on the right side, It is then dimensionally correct. Checking this is the basic way of performing dimensional analysis.
Let's check that the second equation of motion is correct or not.

$$s = ut + \frac{1}{2}at^2$$

$$s = \text{distance} = \text{length} = [L]$$

$$ut = \frac{\text{length}}{\text{time}}\times\text{time} = \text{length} = [L]$$

$$at^2 = \frac{\text{length}}{(\text{time})^2}\times(\text{time})^2 = \text{length} = [L]$$

$$\boxed{[L] = [L] + [L]}$$

If an equation is dimensionally incorrect, it must be wrong. On the other hand, dimensionally correct equations may or may not be correct.
Let's take an example to make it simple for you.
If I say the area of a circle = 2 x radius^2
- this is dimensionally correct (both sides have dimensions $[L^2]$)
- but it is wrong, as constant should be 'π' and not '2'

2.6 Deducing Relation among the Physical Quantities

The method of dimensions can sometimes be used to deduce relation among the physical quantities.
For this, we should know the dependence of the physical quantity on other quantities and consider it as a product type of the dependency.
Let's find the time period of a pendulum by using dimensional analysis. The period of oscillation of the simple pendulum depends on its length ($\boldsymbol{LL}$), mass of the bob ($\boldsymbol{mm}$)
and acceleration due to gravity ($\boldsymbol{gg}$).

Time period $T \propto m^a g^b L^c$

$\Rightarrow T = km^a g^b L^c$

Where k is dimensionless constant.
By considering dimensions on both sides,

$$\left[M^0L^0T^1\right] = \left[M^1\right]^a \cdot \left[LT^{-2}\right]^b \left[L^1\right]^c$$

$$\Rightarrow \left[M^0L^0T^1\right] = \left[M^a L^{b+c} T^{-2b}\right]$$

Comparing both sides

$a = 0$

$b = -\frac{1}{2}$

$c = \frac{1}{2}$

$$T = km^0 g^{-\frac{1}{2}} L^{\frac{1}{2}} = k\sqrt{\frac{L}{g}}$$

2.7 Limitations of Dimensional Analysis

1. Dimensionless quantities cannot be determined by this method.
2. Constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
3. This method is not applicable to trigonometric, logarithmic and exponential functions.
4. In some cases, the constant of proportionality also possesses dimensions. In such cases, we cannot use this system.
5. If one side of the equation contains addition or subtraction of physical quantities, we cannot use this method to derive the expression.

3. Significant Figures

The significant figures are normally those digits in a measured quantity which are known reliably plus one additional digit that is uncertain.
In this case a student takes reading 4.57 *mmmmmmmm*.
Here the digits 4 and 5 are certain and the digit 7 is an estimate.

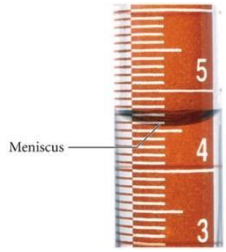


Fig. 1.10

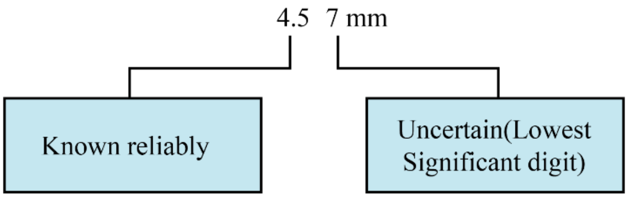


Fig. 1.11

3.1 Rules for Determining Significant Figures

Rule 1: Every non-zero digit in a reported measurement is assumed to be significant.
Example:
24.7 meters, no. of significant figures = 3
0.743 meters, no. of significant figures = 3
714 meters, no. of significant figures = 3

Rule 2: Zeros appearing between non-zero digits are significant.
Example:
70003 meters, no. of significant figures = 5
40.79 meters, no. of significant figures = 4
1.503 meters, no. of significant figures = 4

Rule 3: Left most zeros appearing in front of non-zero digits are not significant
Example:
0.0073 meters, no. of significant figures = 2
0.423 meters, no. of significant figures = 3
0.000099 meters, no. of significant figures = 2

NOTE:

Left most zeros act as place holders. By writing the measurements in scientific notation, we can eliminate such place holding zeros.

Left most zeros appearing in front of non-zero digits are not significant
0.0073 meter = 7.3×10^{-3} meter
0.423 meter = 4.23×10^{-1} meter
0.000099 meter = 9.9×10^{-5} meter

As the power of ten does not contribute to significant figures, thus even by changing units the number of significant digits will remain the same.

Rule 4: Zeros to the right of the last non-zero digit (trailing zeros) in a number with the decimal point are significant if they are within the measurement or reporting resolution.
Example: 1.200 has four significant figures (1, 2, 0, and 0) if they are allowed by the measurement resolution.

Rule 5: The trailing zeros in a number without decimal point are not significant example, 010100 has 3 SF. But if the number comes from some actual measurement, then the trailing zeros become significant example: m = 100 kg has 3 SF

3.2 Significant Figures in Calculations

Rules for arithmetical operations with significant figures
Rule I: In addition, or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places.
e.g., 12.587-12.5 = 0.087 = 0.1 (∵ second term contain lesser i.e., one decimal place)
Rule II: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g., $2.4 \times 3.65 = 8.8$
So, let's read about rounding off.

3.3 Rounding Off

Rule 1: If the last significant digit(d) < 5 then drop it.
Example:
Round off 12.3 to 2 significant figures.
Last significant digit is 3 < 5
So, the answer is 12.

Rule 2: If the last significant digit(d) > 5, then increase the preceding digit by 1 and drop 'd'.

Example:
Round off 14.56 to 3 significant figures.
Last significant digit is 6 > 5
So, the answer is 14.6.

Rule 3: If the last significant digit(d) = 5, then look at the preceding digit.
(i) If the preceding digit is even, drop 'd'.
(ii) If the preceding digit is odd then increase the preceding digit by 1 and drop 'd'.

Example:
Round off 1.45 to 2 significant figures.
Last significant digit is 5 and preceding digit is 4 which is even. So, the answer is 1.4

Example:
Round off 147.5 to 3 significant figures.
Last significant digit is 5 and preceding digit is 7 which is odd. So, the answer is 148

4. Errors

What is an error?
An error is a mistake of some kind causing an error in your results, so the result is not accurate.

4.1 Types of Errors

Errors can be divided into two main classes
- Random errors
- Systematic errors

4.2 Random Errors

Random error has no pattern. One minute your readings might be too small. Next, they might be too large. You can't predict random error and these errors are usually unavoidable.
- Random errors cannot be rectified but can be minimized.
- Random errors can be reduced by taking a lot of readings, and then calculating the average (mean).
  Two main causes of random error are
- Human errors
- Faulty technique

4.3 Causes of Random Errors

1.H uman Error
Example:

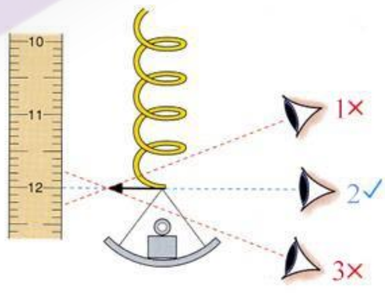


Fig. 1.12

Way of taking reading 2 is best, 1 and 3 give the wrong readings.
This is called a parallax error.

2. Faulty Technique
Using the instrument wrongly

Fig. 1.13

4.4 Systematic Errors

Systematic error is a consistent, repeatable error associated with faulty equipment or a flawed experiment design. These errors are usually caused by measuring instruments that are incorrectly calibrated.

- These errors cause readings to be shifted one way (or the other) from the true reading.

4.5 Causes of Systematic Errors

Example:

1.Z ero error

- There is no weight, and the weighing machine is not showing zero.

Fig. 1.14

2. **Faulty Instrument**
   Example:

- If a ruler is wrongly calibrated, or if it expands, then all the readings will be too low

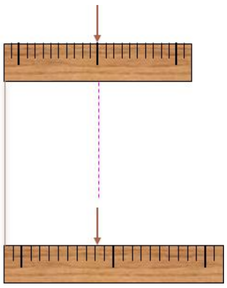


Fig. 1.15

3. **Personal Error**
   Example:

- If someone have a habit of taking measurements always from above the reading, then due to parallax you will get a systematic error and all the readings will be deviated from actual reading.

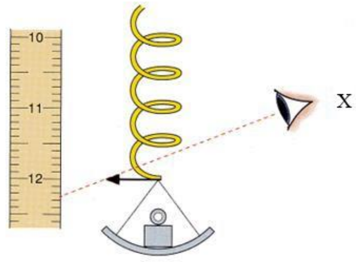


Fig. 1.16

Now, Let's learn about some common terms used during, measurements and error analysis

4.6 Accuracy and Precision

Accuracy is an indication of how close a measurement is to the accepted value.

- An accurate experiment has a low systematic error. Precision is an indication of the agreement among a number of measurements.
- A precise experiment has a low random error.

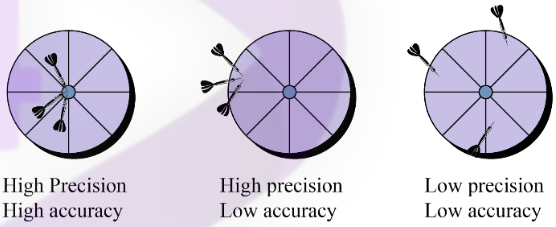


Fig. 1.17

4.7 Calculation of Errors

For calculation purpose we divide the errors in three types

- Absolute error
- Relative error
- Percentage error

4.8 Absolute Errors

The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement.

Absolute error is denoted by $|\Delta a|$ and it is always taken positive.

For Example:
Let's say, values obtained in several measurements are a_1, a_2, a_3, …,a_n
If true value is not available, we can consider arithmetic mean as true value.

$$a_{mean} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}$$

Absolute Errors in measurements =

$$|\Delta a_1| = |a_1 - a_{mean}|$$
$$|\Delta a_2| = |a_2 - a_{mean}|$$
… … …
… … …
$$|\Delta a_n| = |a_n - a_{mean}|$$

Mean Absolute Error

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|}{n}$$

So, we show the measurement by $a_{mean} \pm \Delta a_{mean}$

4.9 Relative Errors

The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

Relative error $= \frac{\Delta a_{mean}}{a_{mean}}$

When the relative error is expressed in percent, it is called the percentage error (δ).

Percentage error $\delta = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$

4.10 Range of Uncertainty

Range of uncertainty is reported as a nominal value plus or minus an amount called the tolerance or percent tolerance.

Reported value 120 mm ± 2% = 117.6 mm to 122.4 mm

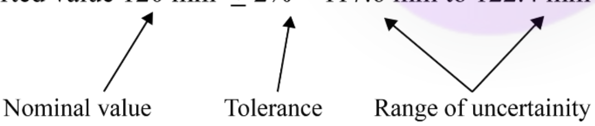


Fig. 1.18

NOTE:

2% of 120 = 2.4
120 - 2.4 = 117.6
120 + 2.4 = 122.4

4.11 Limit of Reading or Least Count

The limit of reading of a measurement is equal to the smallest graduation of the scale of an instrument.

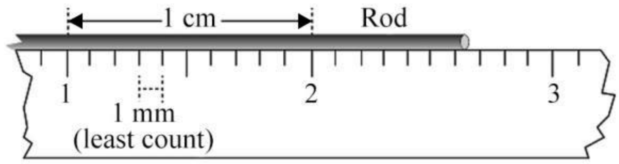


Fig. 1.19

Least count of this scale is 1 mm

4.12 Least Count Error

When a measurement falls between two divisions, then error due to approximate measurement made by the observer is called least count error.

4.13 Propagation of Errors

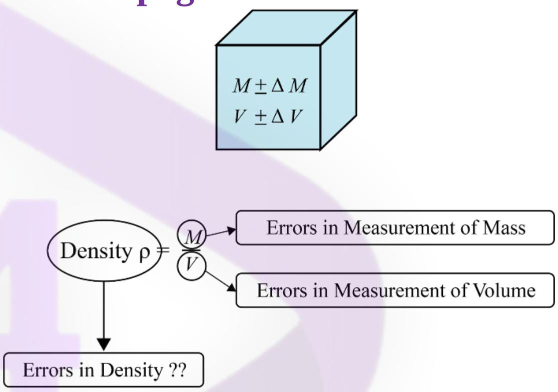


Fig. 1.20

4.14 Errors of a Sum or a Difference

When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.
Measured value of physical quantity AA and BB is respectively $AA \pm \Delta AA$ and $BB \pm \Delta BB$
If a Physical Quantity $ZZ = AA + BB$ or
$ZZ = AA - BB$
Then Maximum possible Error in Z
$\Delta ZZ = \Delta AA + \Delta\Delta BB$

4.15 Errors of a Multiplication or Division

Measured value of physical quantity AA and BB is respectively $AA \pm \Delta AA$ and $BB \pm \Delta BB$
If a Physical Quantity $ZZ = AA \times BB$ or
$ZZ = AA/BB$
Then maximum relative error in ZZ,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

4.16 Error of a Measured Quantity Raised to a Power

The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.
Measured value of physical quantity AA and BB is respectively
$AA \pm \Delta AA$ and $BB \pm \Delta BB$
If a Physical Quantity $ZZ = AA^2$
Then maximum relative error in ZZ,
$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta A}{A} = 2\frac{\Delta A}{A}$$
In general, if $Z = A^p B^q C^r$
Then maximum relative error in Z,
$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Basic Mathematics

5. Quadratic Equation

A quadratic equation is an equation of second degree, meaning it contains at least one term that is squared.
The standard form of quadratic equation is
$ax^2 + bx + c = 0$ where $a \neq 0$
The solution of the above quadratic equation is the values of variable 'x' which will satisfy it. It basically has two solutions $(x_1$ and $x_2)$
If we try to calculate time when football is at height H, then we will observe that we will get two answers

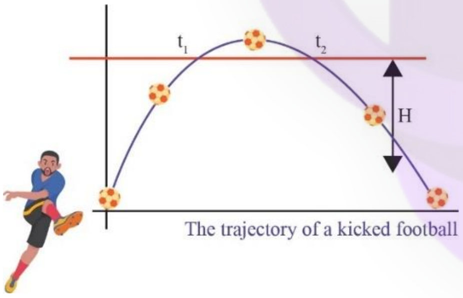


Fig. 1.21

t_1 - While going up
t_2 - While coming down
What if we take a height which is greater than maximum height covered by ball, and we are trying to find the time?

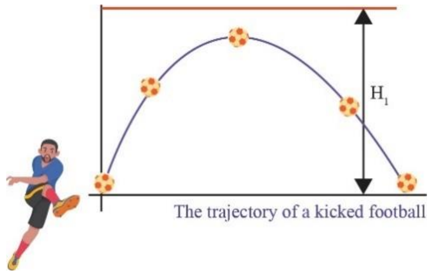


Fig. 1.21

By this diagram we can easily say that at no real value of time, the ball is at height H_1. We might will not have a diagram every time though.
For finding out if a quadratic equation has a real solution or not, we shall use the **'DISCRIMINANT'**.

5.1 Discriminant of a Quadratic Equation

Discriminant of a quadratic $ax^2 + bx + c = 0$ equation is represented by D.
$D = b^2 - 4ac$

The roots are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- If D < 0, No real roots for given equation.

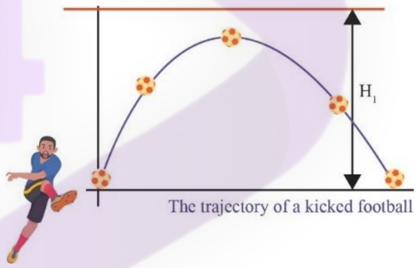


Fig. 1.22

- If D > 0, Two distinct real roots

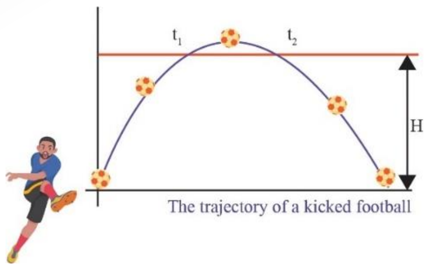


Fig. 1.23

- The roots are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- If D = 0, Equal and real roots. Then we will get only one root

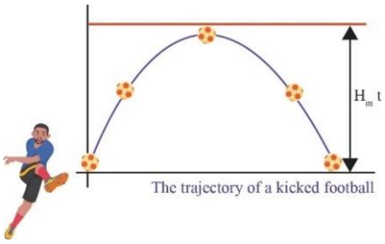


Fig. 1.24

- The roots are given by $-\frac{b}{2a}$

i. Sum of roots $= x_1 + x_2 = -\frac{b}{a}$

ii. Product of roots $= x_1 x_2 = \frac{c}{a}$

iii. Difference of the roots $= x_1 - x_2 = \frac{\sqrt{D}}{a}$

6. Basic Graph

(i) Straight line graph

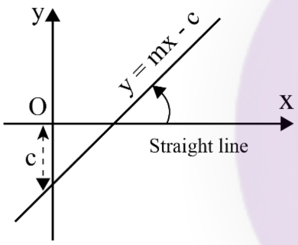


Fig. 1.25

Equation of graph: $y = mx - c$

(ii) Straight line graph

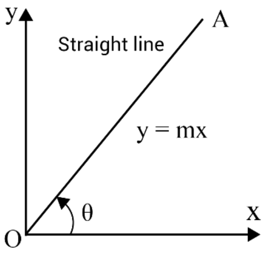


Fig. 1.26

Equation of graph: $y = mx$

(iii) Straight line graph

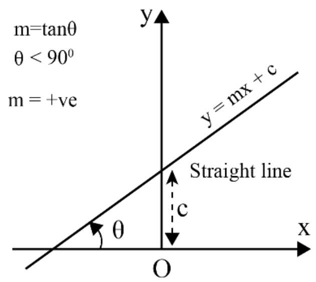


Fig. 1.27

Equation of graph: $y = mx + c$

$m = \tan\theta$

$\theta < 90°$

$m = +ve$

(iv) Straight line graph

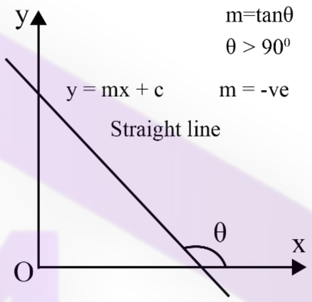


Fig. 1.28

Equation of graph: $y = mx + c$

$m = \tan\theta$

$\theta > 90°$

(v) Parabola graph

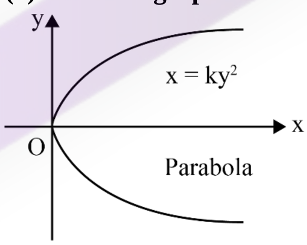


Fig. 1.29

Equation of graph: $x = ky^2$

(vi) Parabola graph

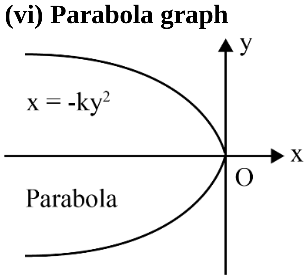


Fig. 1.30

Equation of graph: $x = -ky^2$

(vii) Parabola graph

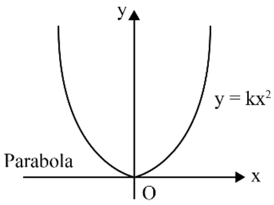


Fig. 1.31

Equation of graph: $y = kx^2$

(viii) Parabola graph

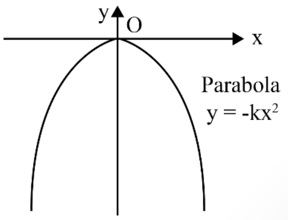


Fig. 1.32

Equation of graph: $y = -kx^2$

(ix) Rectangular Hyperbola graph

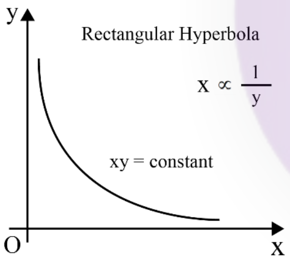


Fig. 1.33

Equation of graph: $xy = \text{cons tan t}$

$x \propto \frac{1}{y}$

(x) Circle graph

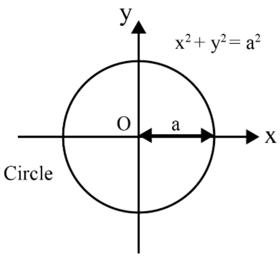


Fig. 1.34

Equation of graph: $x^2 + y^2 = a^2$

(xi) Ellipse graph

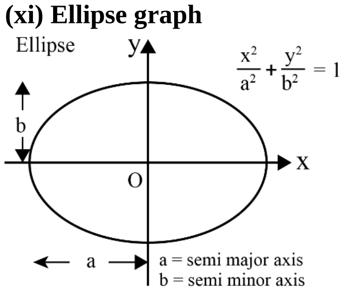


Fig. 1.35

Equation of graph: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(xii) Exponential Decay graph

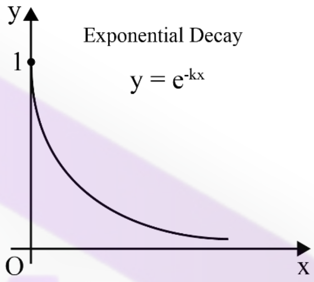


Fig. 1.36

Equation of graph: $y = e^{-kx}$

(xiii) sin graph:

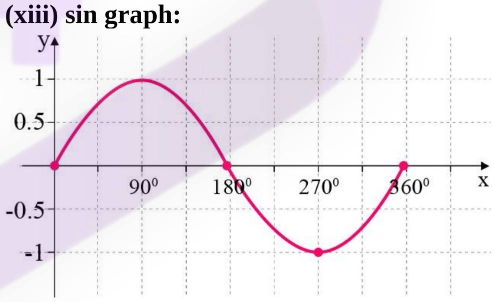


Fig. 1.37

Max value of Graph	Min value of the graph
1 at 90°, 450° etc.	−1 at 270°, 630° etc

- y = sin x
- The roots or zeros of y = sin x is at the multiples of 180°
- The sin graph passes the x-axis as sin x = 0.
- Period of the sine function is 360°

(xiv) cos graph:

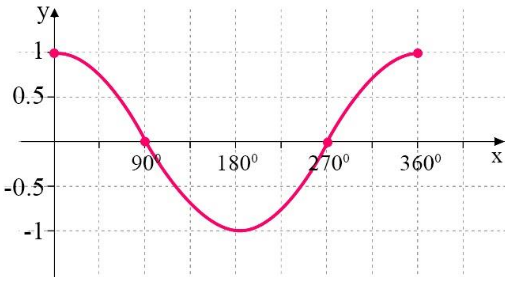


Fig. 1.38

Max value of Graph	Min value of the graph
1 at 0, 360°, 720°	−1 at 180°, 540°, 900°

- y = cos x
- sin (x + 90°) = cos x
- The y = cos x graph is obtained by shifting the y = sin x, 90° units to the left
- Period of the cosine function is 360°

There are a few similarities between the sine and cosine graphs they are:

- Both have the same curve which is shifted along the x-axis.
- Both have an amplitude of 1
- Have a period of 360°

The combined graph of sine and cosine function can be represented as follows:

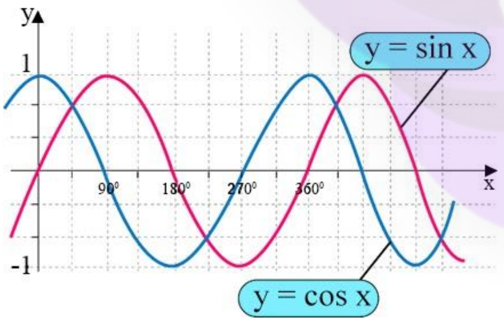


Fig. 1.39

(xv) tan graph:
The tan function is completely from sin and cos function. The function here goes between negative and positive infinity, crossing through y = 0 over a period of 180°

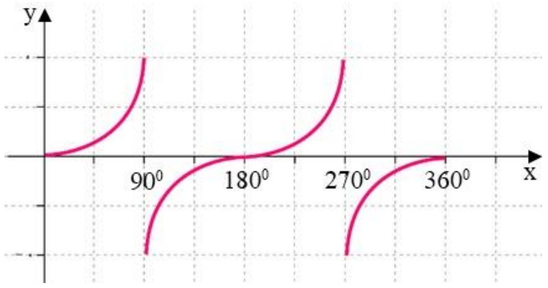


Fig. 1.40

- y = tan x
- The tangent graph has an undefined amplitude as the curve tends to infinity
  It also has a period of 180°.

7. Binomial Expansion

An algebraic expression containing two terms is called a binomial expression.

For example, $(a+b)$, $(a+b)^3$, $(2x-3y)^{-1}$, $\left(x+\dfrac{1}{y}\right)$ etc. are binomial expressions.

Binomial Theorem

$$(a+b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2\times 1}a^{n-2}b^2 + \ldots\ldots,$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\times 1}x^2 + \ldots\ldots$$

Binomial Approximation
If x is very small, compared to 1, then terms containing higher powers of x can be neglected so $(1+x)^n \approx 1+nx$

8. Componendo Dividendo Method

If $\dfrac{p}{q} = \dfrac{a}{b}$ then $\dfrac{p+q}{p-q} = \dfrac{a+b}{a-b}$

9. Logarithmic and Exponential Function

Common formulae:

- $\log mn = \log m + \log n$
- $\log \dfrac{m}{n} = \log m - \log n$
- $\log m^n = n \log m$
- $\log_e m = 2.303 \log_{10} m$

10. Trigonometry and Geometry

10.1 Angle

Consider a revolving line OP.

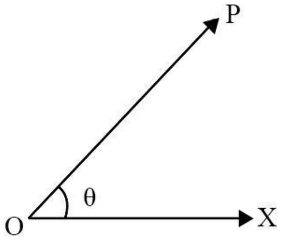


Fig. 1.41

Suppose that it revolves in anticlockwise direction starting from its initial position OX. The angle is defined as the amount of revolution that the revolving line makes with its initial position.

From fig. the angle covered by the revolving line OP is $\theta = \angle POX$

The angle is taken positive if it is traced by the revolving line in anticlockwise direction and is taken negative if it is covered in clockwise direction.

1° = 60' (minute)

1' = 60" (second)

1 right angle = 90° (degrees) also 1 right angle $= \frac{\pi}{2}$ rad

One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle. $1 \text{ rad} = \frac{180°}{\pi} \approx 57.3°$

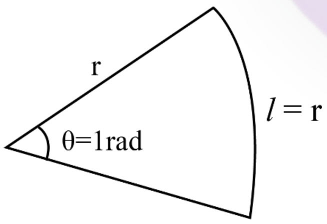


Fig. 1.42

To convert an angle from degree to radian multiply it by $\frac{\pi}{180}$

To convert an angle from radian to degree multiply it by $\frac{180°}{\pi}$

10.2 Trigonometrical Ratios (or T ratios)

Let two fixed lines XOX' and YOY' intersect at right angles to each other at point O. Then,

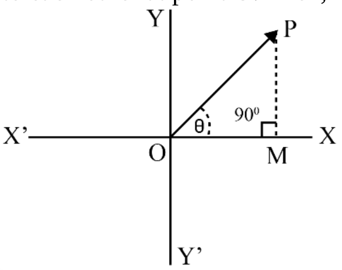


Fig. 1.43

(i) Point O is called origin.

(ii) XOX' is known as X-axis and YOY' as Y-axis.

(iii) Portions XOY, YOX', X'OY' and Y'OX are called I, II, III and IV quadrants respectively.

Consider that the revolving line OP has traced out angle θ (in I quadrant) in anticlockwise direction. From P, draw perpendicular PM on OX. Then, side OP (in front of right angle) is called hypotenuse, side MP (in front of angle θ) is called opposite side or perpendicular and side OM (making angle θ with hypotenuse) is called adjacent side or base.

The three sides of a right-angled triangle are connected to each other through six different ratios, called trigonometric ratios or simply T-ratios:

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP} \qquad \cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$$

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM} \qquad \cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM} \qquad \operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$$

It can be easily proved that:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

Table: The T-ratios of a few standard angles ranging from 0° to 360°

Angle (in Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
Angle (in Radians)	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	–1	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	–1	∞	1

10.3 Four Quadrants and ASTC Rule

In first quadrant, all trigonometric ratios are positive.
In second quadrant, only sinθ and cosecθ are positive.
In third quadrant, only tanθ and cotθ are positive.
In fourth quadrant, only cosθ and secθ are positive.

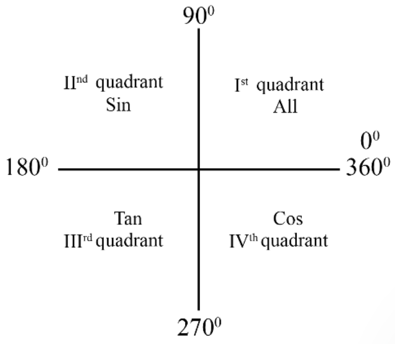


Fig. 1.44

NOTE:

> Remember as Add Sugar To Coffee or After School To College.

10.4 Trigonometrical Ratios of General Angles (Reduction Formulae)

(i) Trigonometric function of an angle $(2n\pi+\theta)$ where n = 0, 1, 2, 3, …. will be remain same.

$\sin(2n\pi+\theta)=\sin\theta$

$\cos(2n\pi+\theta)=\cos\theta$

$\tan(2n\pi+\theta)=\tan\theta$

(ii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will remain same if n is even and sign of trigonometric function will be according to value of that function in quadrant.

$\sin(\pi-\theta)=+\sin\theta$

$\cos(\pi-\theta)=-\cos\theta$

$\tan(\pi-\theta)=-\tan\theta$

$\sin(\pi+\theta)=-\sin\theta$

$\sin(2\pi-\theta)=-\sin\theta$

$\tan(\pi+\theta)=+\tan\theta$

$\cos(\pi+\theta)=-\cos\theta$

$\cos(2\pi-\theta)=+\cos\theta$

$\tan(2\pi-\theta)=-\tan\theta$

(iii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will be changed into co-function if n is odd and sign of trigonometric function will be according to value of that function in quadrant.

$\sin\left(\frac{\pi}{2}+\theta\right)=+\cos\theta$

$\sin\left(\frac{\pi}{2}-\theta\right)=+\cos\theta$

$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

$\cos\left(\frac{\pi}{2}-\theta\right)=+\sin\theta$

$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$

$\tan\left(\frac{\pi}{2}-\theta\right)=+\cot\theta$

(iv) Trigonometric function of an angle –θ (negative angles)

$\sin(-\theta)=-\sin\theta$

$\cos(-\theta)=+\cos\theta$

$\tan(-\theta)=-\tan\theta$

10.5 A few important trigonometric formulae

$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

$\sin 2A=2\sin A\cos A$

$\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$

$\cos(A+B)=\cos A\cos B-\sin A\sin B$

$\cos(A-B)=\cos A\cos B+\sin A\sin B$

$\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$\cos 2A=\cos^2 A-\sin^2 A$

$\cos 2A=2\cos^2 A-1=1-2\sin^2 A$

$1+\cos A=2\cos^2\frac{A}{2},\ 1-\cos A=2\sin^2\frac{A}{2}$

10.6 Range of Trigonometric Functions

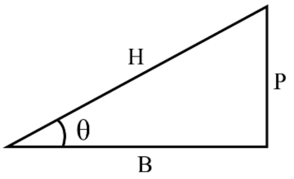


Fig. 1.45

As $\sin\theta = \frac{P}{H}$ and $P \le H$ so $-1 \le \sin\theta \le 1$

As $\cos\theta = \frac{B}{H}$ and $B \le H$ so $-1 \le \cos\theta \le 1$

As $\tan\theta = \frac{P}{B}$ so $-\infty < \tan\theta < \infty$

Remember: $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

10.7 Small Angle Approximation

If θ is small, then $\sin\theta \approx \theta$, $\cos\theta \approx 1$ & $\tan\theta \approx \theta$. Here θ must be in radians.

11. Basic Geometry

11.1 Formulae for Determination of Area:

1. Area of a square = $(\text{side})^2$
2. Area of rectangle = length × breadth
3. Area of a triangle $= \frac{1}{2}(\text{base} \times \text{height})$
4. Area of trapezoid $= \frac{1}{2}$ (distance between parallel sides) × (sum of parallel sides)
5. Area enclosed by a circle = πr^2 (r = radius)
6. Surface area of a sphere = $4\pi r^2$ (r = radius)
7. Area of a parallelogram = base × height
8. Area of curved surface of cylinder $= 2\pi r\ell \quad (r = \text{radius and } \ell = \text{length})$
9. Area of ellipse = πab (a and b are semi major and semi minor axes respectively)
10. Surface area of a cube = 6 $(\text{side})^2$
11. Total surface area of cone $= \pi r^2 + \pi r\ell$ where $\pi r\ell = \pi r\sqrt{r^2+h^2}$ = lateral area

11.2 Formulae for Determination of Volume:

1.V olume of a rectangular slab = length × breadth × height = abt

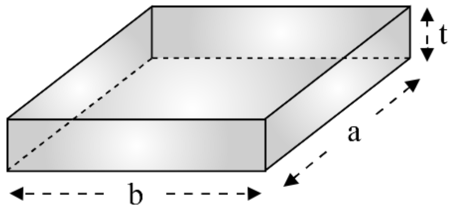


Fig. 1.46

2. Volume of a cube = $(\text{side})^3$
3. Volume of a sphere $= \frac{4}{3}\pi r^3$ (r = radius)
4. Volume of a cylinder = $\pi r^2 \ell$ (r = radius and ℓ is length)
5. Volume of a cone $= \frac{1}{3}\pi r^2 h$ (r = radius and h, is height)

Note: $\pi = \frac{22}{7} = 3.14$; $\pi^2 = 9.8776 \approx 10$ and $\frac{1}{\pi} = 0.3182 \approx 0.3$.

12. Scalars and Vectors

12.1 What is a Scalar?

A scalar is a quantity that is fully described by a magnitude only. It is described by just a number.

Examples:
Speed, volume, mass, temperature, power, energy, time, etc.

12.2 What is a Vector?

Vector is a physical quantity which has magnitude as well as direction and follows the rule of vector addition.
Vector quantities are important in the study of physics.

Examples:
Force, velocity, acceleration, displacement, momentum, etc.

12.3 Representation of Vectors

- A vector is drawn as an arrow with a head and a tail.
- The magnitude of the vector is often described by the length of the arrow.
- The arrow points in the direction of the vector.

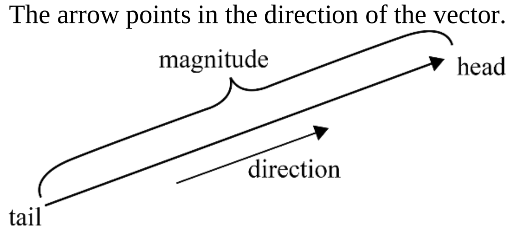


Fig. 1.47

- Vectors can be defined in two dimensional or three-dimensional space

  How to write a vector?

  Vectors are generally written with an arrow over the top of the letter. (Ex: $\vec{a}$)

  $\overrightarrow{AB} = \vec{a}$

Magnitude:

$|\overrightarrow{AB}| = a$

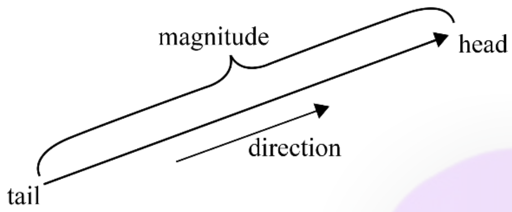


Fig. 1.48

12.4 Properties of Vectors

Vectors are mathematical objects, and we will now study some of their mathematical properties.

(1). Equality of vectors

Two vectors are equal if they have the same magnitude and the same direction.

(2). Negative Vector

A negative vector is a vector that has the opposite direction to the reference positive direction but same magnitude.

12.5 Types of Vectors

1. Zero Vector
2. Unit Vector
3. Position Vector
4. Co-initial Vector
5. Like and Unlike Vectors
6. Coplanar Vector
7. Collinear Vector
8. Displacement Vector

12.6 Zero Vector:

- A zero vector is a vector when the magnitude of the vector is zero and the starting point of the vector coincides with the terminal point.
- In other words, a vector $\overrightarrow{AB}$ coordinates of the point A are the same as that of the point B then the vector is said to be a zero vector and is denoted by $\vec{O}$.

12.7 Unit Vector:

A vector which has a magnitude of unit length is called a unit vector.

Suppose if $\vec{x}$ is a vector having a magnitude $|\vec{x}|$ then the unit vector is denoted by $\hat{x}$ in the direction of the vector x and has the magnitude equal to 1.

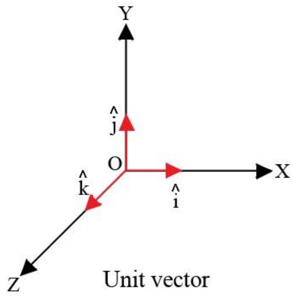


Fig. 1.49

$$\therefore \hat{x} = \frac{\vec{x}}{|\vec{x}|}$$

It must be carefully noted that any two-unit vectors must not be considered as equal, because they might have the same magnitude, but the direction in which the vectors are taken might be different

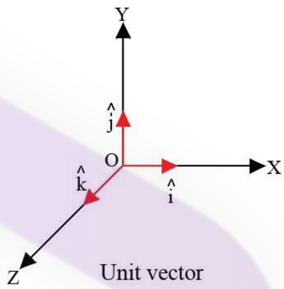


Fig. 1.50

- A unit vector is a vector that has a magnitude of 1.
- Any vector can become a unit $\vec{a}$ vector on dividing it by the vector's magnitude.
- Unit vector in the direction of is $\hat{a}$.

  $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

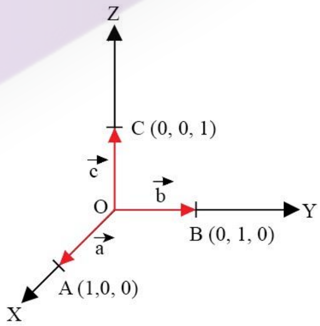


Fig. 1.51

$\vec{a}$ $|\vec{a}| = 1$ $\times$ $\boxed{a} \rightarrow \boxed{\hat{i}}$

$\vec{b}$ $|\vec{b}| = 1$ $\times$ $\boxed{\hat{b}} \rightarrow \boxed{j}$

$\vec{c}$ $|\vec{c}| = 1$ $\times$ $\boxed{\hat{c}} \rightarrow \boxed{k}$

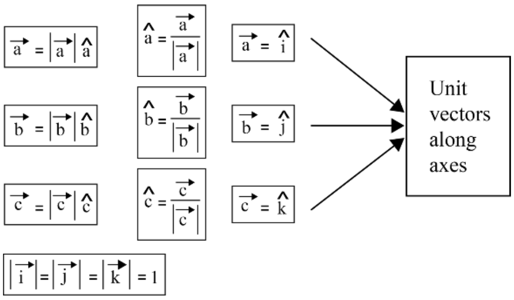


Fig. 1.52

12.8 Position Vector:

If O is taken as reference origin and P is an arbitrary point in space, then the vector OP is called as the position vector of the point.
Position vector simply denotes the position or location of a point in the three-dimensional Cartesian system with respect to a reference origin.

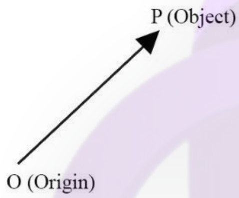


Fig. 1.53

12.9 Co-initial Vector:

The vectors which have the same starting point are called co-initial vectors.

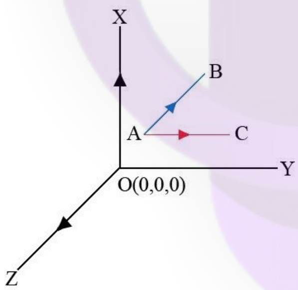


Fig. 1.54

The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called co-initial vectors as they have same starting point.

12.10 Like and Unlike Vectors:

The vectors having the same direction are known as like vectors. On the contrary, the vectors having the opposite direction with respect to each other are termed to be unlike vectors.

12.11 Coplanar Vectors:

Three or more vectors lying in the same plane or parallel to the same plane are known as coplanar vectors.

12.12 Collinear Vectors:

Vectors which lie along the same line are known to be collinear vectors.

12.13 Displacement Vector:

If a point is displaced from position A to B, then the displacement AB represents a vector $\overrightarrow{AB}$ which is known as the displacement vector.

12.14 Multiplication of Vectors with Scalar

- When a vector is multiplied by a scalar quantity, then the magnitude of the vector changes in accordance with the magnitude of the scalar and the direction of the vector depends on whether scalar quantity is positive or negative.
- Suppose we have a vector $\vec{a}$, then if this vector is multiplied by a scalar quantity k, then we get a new vector with magnitude as $|k\vec{a}|$ and the direction depends on whether k is positive or negative.

12.15 Multiplication of Vectors with Real Numbers

Fig. 1.55

NOTE:

Multiplying the vector with a negative number inverts the direction of vector.

Now let us understand visually the scalar multiplication of the vector.
Let us take the values of ‘k’ to be = 2, 3, -3, -1/2 and so on.

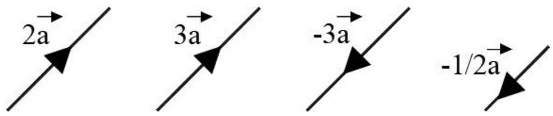


Fig. 1.56

12.16 Position Vector

A vector representing the straight-line distance and the direction of any point or object with respect to the origin, is called position vector.

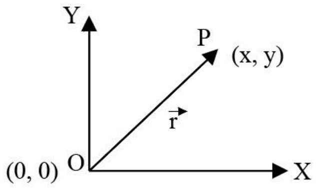


Fig. 1.57

$\overrightarrow{OP} = x\hat{i} + y\hat{j}$

$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2} = |\vec{r}| = r$

12.17 Displacement Vector

A vector representing the straight-line distance and the direction of any point or object with respect to another point is called displacement vector.

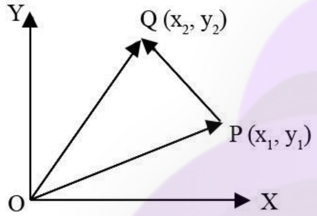


Fig. 1.58

$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j}$

$\overrightarrow{OQ} = x_2\hat{i} + y_2\hat{j}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$\left|\overrightarrow{PQ}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

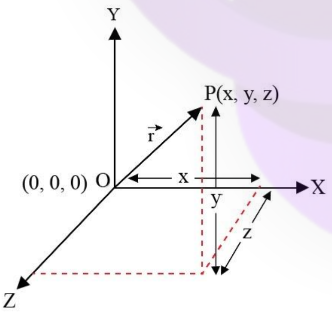


Fig. 1.59

$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2} = |\vec{r}| = r$

12.18 Components of a Vector

In physics, when you break a vector into its parts, those parts are called its components.

Typically, a physics problem gives you an angle and a magnitude to define a vector

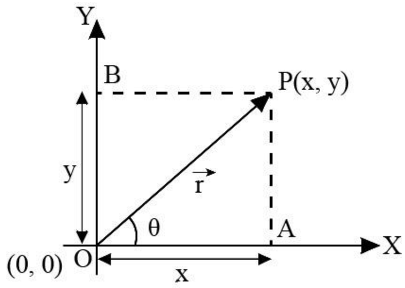


Fig. 1.60

$\overrightarrow{OP} = x\hat{i} + y\hat{j}$

$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2} = |\vec{r}| = r$

$\overrightarrow{OA} = x\hat{i} \Rightarrow r\cos\theta = \left|\overrightarrow{OA}\right|$

$\left|\overrightarrow{OB}\right| = y\hat{i} \Rightarrow r\sin\theta = \left|\overrightarrow{OB}\right|$

$\tan\theta = \dfrac{\left|\overrightarrow{OB}\right|}{\left|\overrightarrow{OA}\right|}$

$\overrightarrow{OA} = x\hat{i}$

$\overrightarrow{OB} = y\hat{i} = \overrightarrow{AD}$

$\overrightarrow{OC} = z\hat{k} = \overrightarrow{DP}$

In ΔODP

$\overrightarrow{OP} = \overrightarrow{OD} + \overrightarrow{DP} = x\hat{i} + y\hat{j} + z\hat{k}$

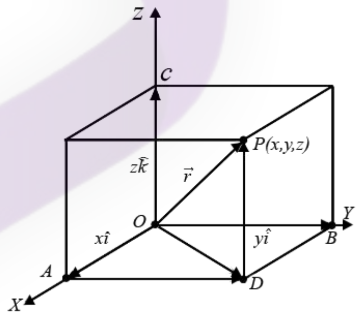


Fig. 1.61

$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$

$\boxed{|\vec{r}| = \sqrt{x^2 + y^2 + z^2}}$

$\boxed{\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}} \rightarrow$ unit vector along $\vec{r}$

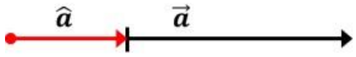
$\overrightarrow{OA} = x\hat{i}$, is the component of vector $\vec{r}$ in X-axis

$\overrightarrow{OB} = y\hat{j}$, is the component of vector $\vec{r}$ in Y-axis

$\overrightarrow{OC} = z\hat{k}$, is the component of vector $\vec{r}$ in Z-axis

12.19 Finding a Unit Vector (2D/3D)

- We have already studied about it in previous classes. Just to recall:
- Unit vector in the direction of $\vec{a}$ is $\hat{a}$



$\hat{\alpha} = \frac{\vec{\alpha}}{|\alpha|}$ $|\hat{\alpha}| = 1$

Fig. 1.62

It will be more clear by solving some problems pertaining 2D/3D cases.

13. Rules of Vector Algebra

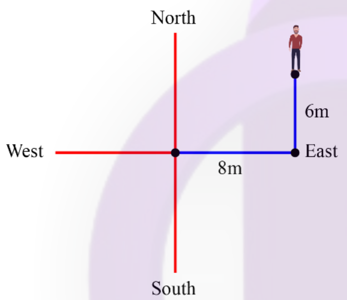


Fig. 1.63

Can we add these vectors directly as (8 m + 6 m) = 14 m ?
(a) Yes
(b) No
Sol: We add vectors considering their directions.
So, now we will learn about the addition of vectors.

13.1 Triangle Law of Vector Addition

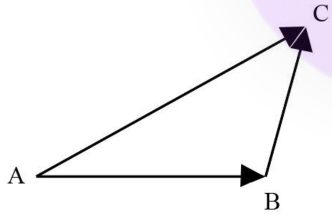


Fig. 1.64

$\vec{AC} = \vec{AB} + \vec{BC}$ Triangle law of Addition

$0 = \vec{AB} + \vec{BC} - \vec{AC}$ $\vec{AB} + \vec{BC} + \vec{CA} = 0$

13.2 Both Addition and Subtraction can be shown as:

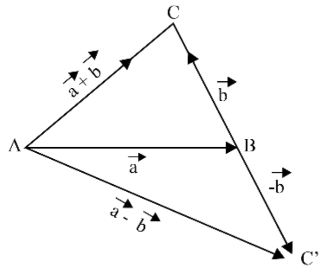


Fig. 1.65

13.3 Polygon Law of Vector Addition

It states that if number of vectors acting on a particle at a time are represented in magnitude and direction by the various sides of an open polygon taken in same order, their resultant vector R is represented in magnitude and direction by the closing side of polygon taken in opposite order. In fact, polygon law of vectors is the outcome of triangle law of vectors.

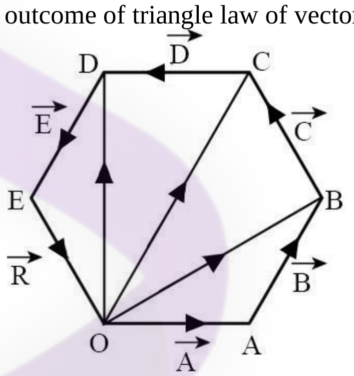


Fig. 1.66

$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$

$\vec{OA} + \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} = \vec{OE}$

13.4 Polygon Law of Vector Addition

- Resultant of two unequal vectors cannot be zero.
- Resultant of three coplanar vectors may or may not be zero.
- Resultant of three non-coplanar vectors cannot be zero, minimum number of non-coplanar vectors whose sum can be zero is four.
- Polygon law should be used only for diagram purpose for calculation of resultant vector (For addition of more than 2 vectors), we use components of vector.
- Minimum no. of coplanar vector for zero resultant is 2 (for equal magnitude) & 3 (for unequal magnitude).

13.5 Addition of Vectors

Adding Vectors Analytically

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$

$\vec{r} = \vec{r}_1 + \vec{r}_2$

$= \left(x_1\hat{i} + y_1\hat{j}\right) + \left(x_2\hat{i} + y_2\hat{j}\right)$

$= x_1\hat{i} + y_1\hat{j} + x_2\hat{i} + y_2\hat{j} = x_1\hat{i} + x_2\hat{i} + y_1\hat{j} + y_2\hat{j}$

$= (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j}$

13.6 Addition of Vectors: Components

Step 1: Identify the x-and y-axes that will be used in the problem.

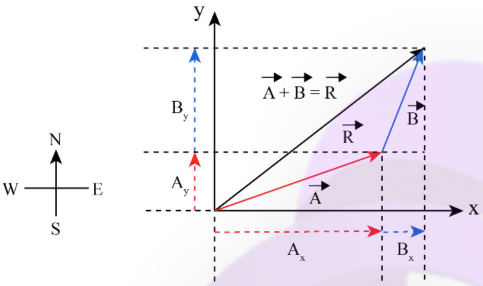


Fig. 1.67

Then, find the components of each vector to be added along the chosen perpendicular axes. Use the equations $A_x = A\cos\theta$, $A_y = A\sin\theta$ to find the components. In figure, these components are A_x, A_y, B_x and B_y.

The angles that vectors A and B make with the x-axis are θ_A and θ_B. respectively.

Step 2: Find the components of the resultant along each axis by adding the components of the individual vectors along that axis. That is, as shown in figure,

$R_x = A_x + B_x$

and $R_y = A_y + B_y$

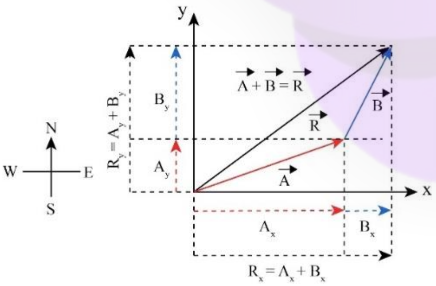


Fig. 1.68

ALWAYS REMEMBER:

A Vector can be changed either by changing its magnitude or direction or by changing both of them.

components along the same axis, say the x-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the y-axis. So, resolving vectors into components along common axes makes it easier to add them. Now that the components of R are known, its magnitude and direction can be found.

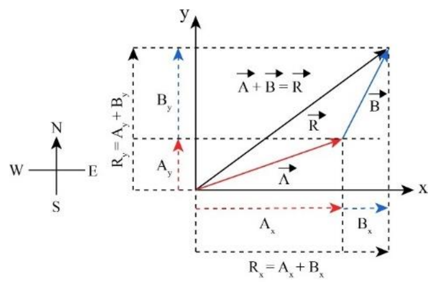


Fig. 1.69

Step 3: To get the magnitude R of the resultant, use the Pythagorean theorem.

$R = \sqrt{R_x^2 + R_y^2}$

Step 4: To get the direction of the resultant.

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

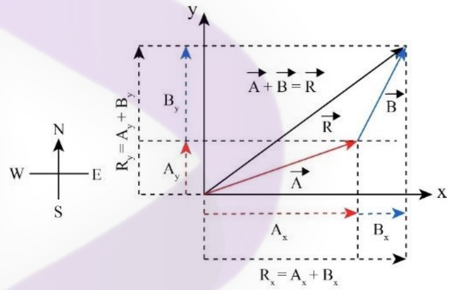


Fig. 1.70

13.7 Parallelogram Law of Vector Addition

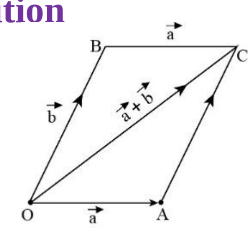


Fig. 1.71

Suppose the magnitude of $\vec{a} = a$ and that of $\vec{b} = b$.

What is the magnitude of $\vec{a}+\vec{b}$ and what its direction?

Suppose the angle between $\vec{a}$ and $\vec{b}$ is θ.

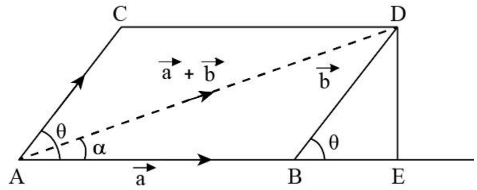


Fig. 1.72

It is easy to say from fig. that

$$AD^2 = (AB + BE)^2 + (DE)^2$$

$$= (a + b\cos\theta)^2 + (b\sin\theta)^2$$

$$= a^2 + 2ab\cos\theta + b^2$$

Thus, the magnitude of is

$$\sqrt{a^2 + b^2 + 2ab\cos\theta}$$

Its angle with $\vec{a}$ is α where,

$$\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{a + b\cos\theta}$$

13.8 Some Properties of Vector Addition

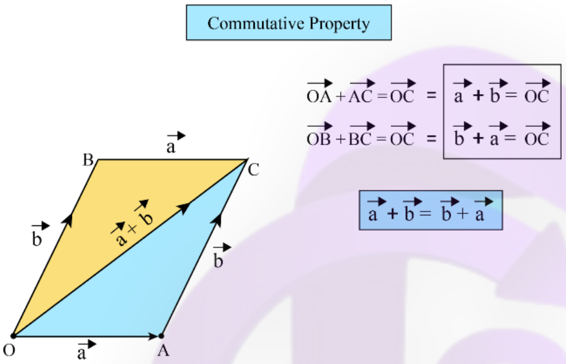


Fig. 1.73

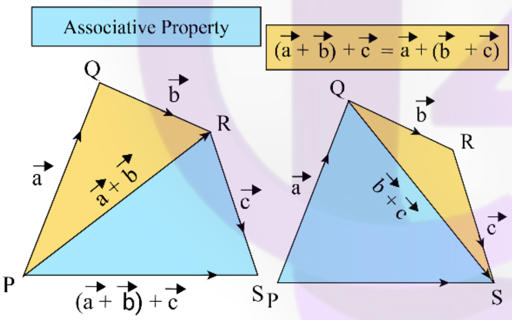


Fig. 1.74

13.9 Subtraction of Vectors

- Subtracting vectors algebraically

$$\vec{a} = a_x\hat{i} + a_y j$$

$$\vec{b} = b_x\hat{i} + b_y j$$

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = a_x\hat{i} + a_y j + (-b_x\hat{i} - b_y j)$$

$$= (a_x - b_x)\hat{i} + (a_y - b_y) j$$

- Subtracting vectors geometrically

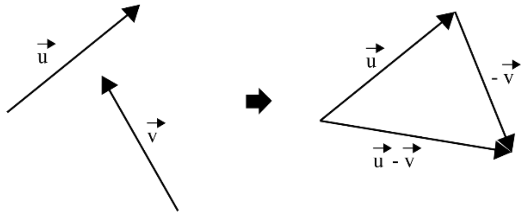


Fig. 1.75

13.10 Change in Vectors

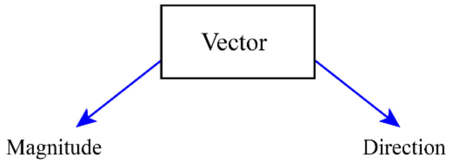


Fig. 1.76

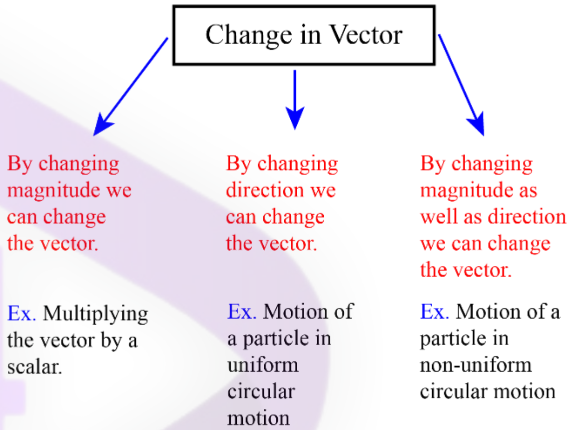


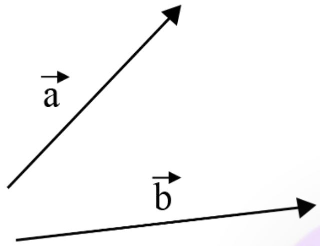
Fig. 1.77

14.Product of Two Vectors

- A vector can be multiplied by another but may not be divided by another vector.
- There are two kinds of products of vectors used broadly in physics and engineering.
- One kind of multiplication is a scalar multiplication of two vectors. Taking a scalar product of two vectors results in a number (a scalar), as its name indicates.
- Scalar products are used to define work and energy relations.
- For example, the work that a force (a vector) performs on an object while causing its displacement (a vector) is defined as a scalar product of the force vector with the displacement vector.
- A quite different kind of multiplication is a vector multiplication of vectors. Taking a vector product of two vectors returns a vector, as its name suggests.
- Vector products are used to define other derived vector quantities.

- For example, in describing rotations, a vector quantity called torque is defined as a vector product of an applied force (a vector) and its distance from pivot to force (a vector).
- It is important to distinguish between these two kinds of vector multiplication because the scalar product is a scalar quantity and a vector product is a vector quantity.

14.1 Scalar Product or Dot Product



$\vec{a}$ Fig. 1.78

Dot product of vector and $\vec{b}$ is denoted by $\vec{a}\cdot\vec{b}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$0 \le \theta \le \pi$

- Dot product gives us a scalar quantity.
- Angle between vectors, $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

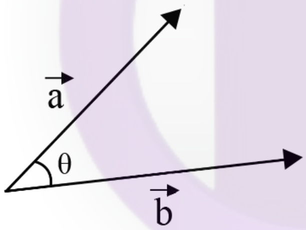


Fig. 1.79

- When $\theta = 0°$,

  $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|$

  $\vec{a}\cdot\vec{b}$ is maximum

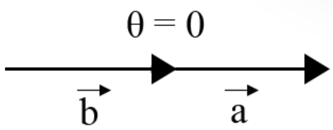


Fig. 1.80

- When $\theta = \pi$,

  $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\pi = -|\vec{a}||\vec{b}|$

  $\vec{a}\cdot\vec{b}$ is minimum.

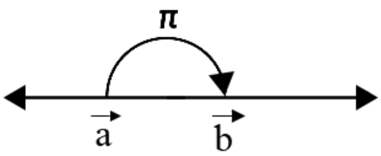


Fig. 1.81

- $\theta = \dfrac{\pi}{2}$, $\vec{a}\cdot\vec{b} = 0$

14.2 Properties of Dot Product

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\vec{b}\cdot\vec{a} = |\vec{b}||\vec{a}|\cos\theta$

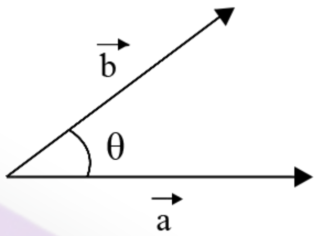


Fig. 1.82

- Dot product is commutative.

  $\vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b}$
- Dot product is distributive over addition or subtraction.

  $\vec{a}\cdot\left(\vec{b}\pm\vec{c}\right) = \vec{a}\cdot\vec{b}\pm\vec{a}\cdot\vec{c}$
- When vectors are given in component form,

  $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

  $\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j}\right)\cdot\left(B_x\hat{i} + B_y\hat{j}\right)$
- We know that, $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = 0$

  $\Rightarrow \vec{A}\cdot\vec{B} = A_xB_x + Ay + By$
- Thus, for 3D, when

  $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

  $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

14.3 Application of Dot Product in Physics

Work done (W): It is defined as the scalar product of the force $\left(\vec{F}\right)$, acting on the body and the Displacement $\left(\vec{s}\right)$ produced.

Thus $W = \vec{F}\cdot\vec{s}$

Instantaneous power (P): It is defined as the scalar product of force $\left(\vec{F}\right)$ and the instantaneous velocity $\left(\vec{v}\right)$ of the body.

Thus $P = \vec{F}\cdot\vec{v}$

Magnetic flux (ϕ): The magnetic flux linked with a surface is defined as the scalar product of magnetic intensity $\left(\vec{B}\right)$ and the area $\left(\vec{A}\right)$ vector. Thus $\phi = \vec{B}\cdot\vec{A}$

Note: As the scalar product of two vectors is a scalar quantity, so work, power and magnetic flux are all scalar quantities.

14.4 Cross Product of Two Vectors

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n} \qquad 0°\le\theta\le180°$

$$n = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$\hat{n}$ is the unit vector in direction normal to the a and b

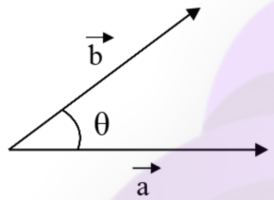


Fig. 1.83

It is also called Vector Product.

- Vector product is distributive over addition i.e., $\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

14.5 Direction of Cross Product

Right Hand Thumb Rule: Curl the fingers of the right hand in such a way that they point in the direction of rotation from vector $\vec{a}$ *to* $\vec{b}$ through the smaller angle, then the stretched thumb points in the direction of $\vec{a}\times\vec{b}$

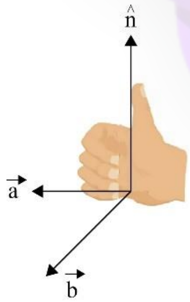


Fig. 1.84

Direction of $\vec{a}\times\vec{b}$

Direction of $\vec{b}\times\vec{a}$

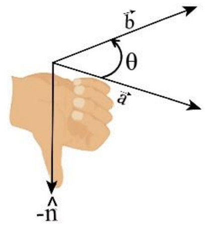


Fig. 1.85

$$\boxed{\vec{b}\times\vec{a} = |\vec{b}||\vec{a}|\sin\theta\left(-n\right)}$$

14.6 Properties of Vector Product

- Vector product is anti - commutative i.e., $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- Vector of two parallel or antiparallel vectors is a null vector. Thus
  $\vec{A}\times\vec{B} = AB\sin\left(0° \text{ or } 180°\right)\hat{n} = \vec{0}$
- Vector product of a vector with itself is a null vector.
  $\vec{A}\times\vec{A} = AA\sin 0°\, n = \vec{0}$
  $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = k\times k = \vec{0}$
- The magnitude of the vector product of two mutually perpendicular vectors is equal to the product of their magnitudes.
  $|\vec{A}\times\vec{B}| = AB\sin 90° = AB$
- Sine of the angle between two vectors. If θ is the angle between two vectors $\vec{A}$ and $\vec{B}$, then
  $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$
  $$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$$
- If $\hat{n}$ is a unit vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$, then $n = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$
- Vector product of orthogonal unit vectors
  $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$
  $\hat{j}\times\hat{i} = -\hat{k}, \hat{k}\times\hat{j} = -\hat{i}, \hat{i}\times\hat{k} = -\hat{j}$

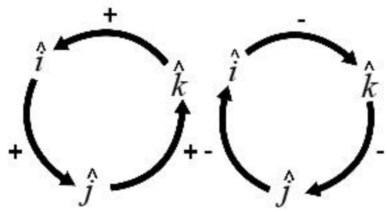


Fig. 1.86

Summary

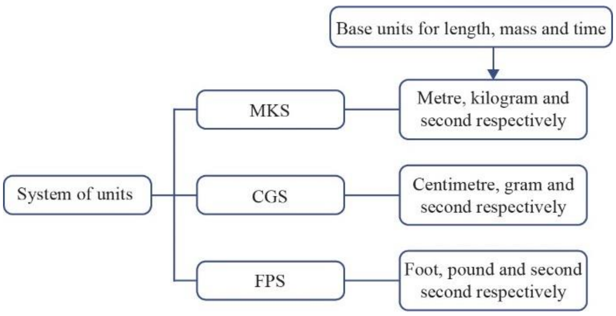
- Any quantity which can be measured is called a physical quantity.
- **Fundamental Unit:**

Quantity	Name of Units	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	Cd

- **Supplementary Units:**

Quantity	Name of Units	Symbol
Plane angle	Radian	rad
Solid angle	Steradian	sr

- **System of Units:**



- **Dimension**
  Dimensions of a physical quantity are the powers to which the fundamental units must be raised in order to get the unit of derived quantity.

- Dimensional analysis is a tool to find or check relations among physical quantities by using their dimensions.
  By using dimensional analysis, we can
  1. Convert a physical quantity from one system of unit to another.
  2. Check the dimensional consistency of equations
  3. Deduce relation among physical quantities.

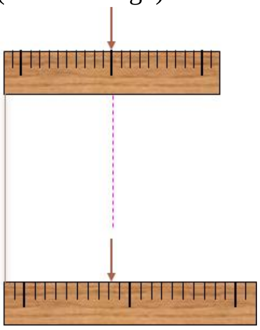
- **Limitations of Dimensional Analysis**
- In some cases, the constant of proportionality also possesses dimensions. In such cases, we cannot use this system.
- If one side of the equation contains addition or subtraction of physical quantities, we cannot use this method to derive the expression.

- **Systematic Errors**
  Systematic error is a consistent, repeatable error associated with faulty equipment or a flawed experiment design. These errors are usually caused by measuring instruments that are incorrectly calibrated.
- These errors cause readings to be shifted one way (or the other) from the true reading.

- **Causes of Systematic Errors**
  1. Zero Error
  Example:
  - There is not any weight, and the weighing machines are not showing zero.

2. **Faulty Instrument**
   Example:
   - If a ruler is wrongly calibrated, or if it expands, then all the readings will be too low

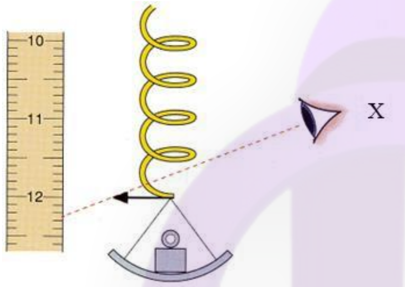
(or all too high).



3. Personal Error

Example:

If someone have a habit of taking measurements always from above the reading, then due to parallax you will get a systematic error and all the readings will be too high.



Now, let's learn about some common terms used during, measurements and error analysis.

- **Accuracy and Precision**
  - Accuracy is an indication of how close a measurement is to the accepted value.
  - An accurate experiment has a low systematic error. Precision is an indication of the agreement among a number of measurements.
  - A precise experiment has a low random error

- **Quadratic Equation**
  A quadratic equation is an equation of second degree, meaning it contains at least one term that is squared. The standard form of quadratic equation is
  $ax^2 + bx + c = 0$ where $a \neq 0$

- **Discriminant of a Quadratic Equation:**
  Discriminant of a quadratic $ax^2 + bx + c = 0$ equation is represented by D.
  $D = b^2 - 4ac$

  The roots are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- **Binomial Expansion**
  A binomial is a polynomial with two terms.
  There are a few similarities between the sine and cosine graphs. They are:
  - Both have the same curve which is shifted along the x-axis.
  - Both have an amplitude of 1.
  - Have a period of 360^o or $2\pi\pi$ radians.

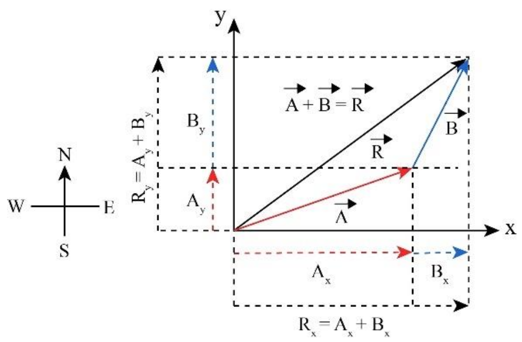
- **Vectors**
  - Scalar and Vector
  - Representation and Properties of Vectors
  - Types of Vectors

- **Negative Vector:**
  A negative vector is a vector that has the opposite direction to the reference positive direction.

- **Types of Vectors**
  (i) Zero Vector
  (ii) Unit Vector
  (iii) Position Vector
  (iv) Co-initial Vector
  (v) Like and Unlike Vectors
  (vi) Coplanar Vector
  (vii) Collinear Vector
  (viii) Displacement Vector
- A unit vector is a vector that has a magnitude of 1.
- Any vector can become a unit vector on dividing it by the vector's magnitude.
- A vector representing the straight-line distance and the direction of any point or object with respect to the origin, is called position vector.

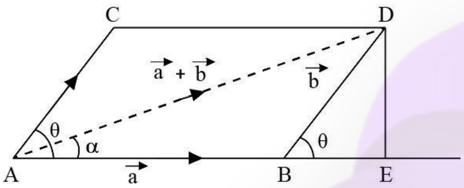
- **Polygon Law:** It states that if number of vectors acting on a particle at a time are represented in magnitude and direction by the various sides of an open polygon taken in same order, their resultant vector R is represented in magnitude and direction by the closing side of polygon taken in opposite order.

- **Addition of Vectors Components:**
  To get the magnitude R of the resultant, use the Pythagorean theorem:
  $R = \sqrt{R_x^2 + R_y^2}$
  To get the direction of the resultant.
  $\theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$



- **Addition of vectors:** Law of Parallelogram of vector addition. Thus, the magnitude of

  $\vec{a}+\vec{b}$ is $\sqrt{a^2+b^2+2ab\cos\theta}$



Its angle with $\vec{a}$ is α where $\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{a+b\cos\theta}$

- **Vector Subtraction:**

  $\vec{a} = a_x\hat{i}+a_y\hat{j}$

  $\vec{b} = b_x\hat{i}+b_y\hat{j}$

  $\vec{a}-\vec{b} = \vec{a}+\left(-\vec{b}\right) = a_x\hat{i}+a_y\hat{j}+\left(-b_x\hat{i}-b_y\hat{j}\right)$

  $=\left(a_x-b_x\right)\hat{i}+\left(a_y-b_y\right)\hat{j}$

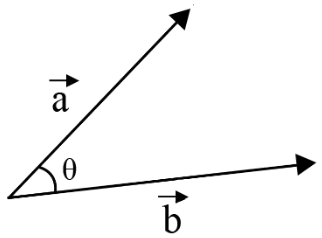
- Scalar Product or Dot Product

  $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

  $0 \le \theta \le \pi$

- Dot product gives us a scalar quantity.
- Angle between vectors,

  $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$



- Dot product is commutative.

  $\vec{b}\cdot\vec{a} = \vec{a}\cdot\vec{b}$

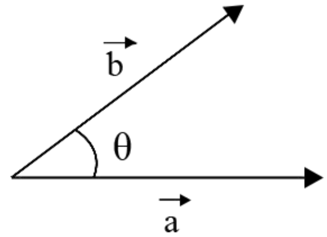
- Dot product is distributive over addition or subtraction.

  $\vec{a}\cdot\left(\vec{b}\pm\vec{c}\right) = \vec{a}\cdot\vec{b}\pm\vec{a}\cdot\vec{c}$

- Cross Product:

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n} \qquad 0^\circ \le \theta \le 180^\circ$

$\hat{n}$ is the unit vector in direction normal to the a and b



- Properties of Cross Product:
- Vector product is anti - commutative i.e.,

  $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

- Vector product is distributive over addition i.e.,

  $\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

- Vector of two parallel or antiparallel vectors is a null vector. Thus

  $\vec{A}\times\vec{B} = AB\sin\left(0^\circ \text{ or } 180^\circ\right)\hat{n} = \vec{0}$

- Vector product of a vector with itself is a null vector.

  $\vec{A}\times\vec{A} = AA\sin 0^\circ n = \vec{0}$

- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = k\times k = 0$
- $\left(\vec{a}\times\vec{b}\right) = -\left(\vec{b}\times\vec{a}\right)$
- $\vec{a}\times\left(\vec{b}+\vec{c}\right) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$
- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = k\times k = \vec{0}$
- $\hat{i}\times\hat{j} = k \qquad \hat{j}\times\hat{i} = -k$
- $\hat{j}\times k = \hat{i} \qquad k\times\hat{j} = -\hat{i}$
- $k\times\hat{i} = \hat{j} \qquad \hat{i}\times k = -\hat{j}$
- If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3k$ and $= b_1\hat{i}+b_2\hat{j}+b_3k$, then

  $$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

  $=\left(a_2b_3-a_3b_2\right)\hat{i}-\left(a_1b_3-a_3b_1\right)\hat{j}+\left(a_1b_2-a_2b_1\right)k$

Dimensional Formulae of Physical Quantities

S. No.	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
1.	Area	Length × breadth	$[L^2]$	$[M^0L^2 T^0]$
2.	Volume	Length × breadth × height	$[L^3]$	$[M^0L^3 T^0]$
3.	Mass density	Mass/volume	$[M]/[L^3]$ or $[ML^{-3}]$	$[ML^{-3}T^0]$
4.	Frequency	1/time period	$1/[T]$	$[M^0L^0T^{-1}]$
5.	Velocity, speed	Displacement/time	$[L]/[T]$	$[M^0LT^{-1}]$
6.	Acceleration	Velocity/time	$[LT^{-1}]/[T]$	$[M^0LT^{-2}]$
7.	Force	Mass × acceleration	$[M][LT^{-2}]$	$[MLT^{-2}]$
8.	Impulse	Force × time	$[MLT^{-2}][T]$	$[MLT^{-1}]$
9.	Work, Energy	Force × distance	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$
10.	Power	Work/time	$[ML^2T^{-2}]/[T]$	$[ML^2T^{-3}]$
11.	Momentum	Mass × velocity	$[M][LT^{-1}]$	$[MLT^{-1}]$
12.	Pressure, stress	Force/area	$[MLT^{-2}]/[L^2]$	$[ML^{-1}T^{-2}]$
13.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[L]/[L]$	$[M^0L^0T^0]$
14.	Modulus of elasticity	Stress/strain	$\frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]}$	$[ML^{-1}T^{-2}]$
15.	Surface tension	Force/length	$[MLT^{-2}]/[L]$	$[ML^0T^{-2}]$
16.	Surface energy	Energy/area	$[ML^2T^2]/[L^2]$	$[ML^0T^{-2}]$

17.	Velocity gradient	Velocity/distance	$[LT^{-1}]/[L]$	$[M^0L^0T^{-1}]$
18.	Pressure gradient	Pressure/distance	$[ML^{-1}T^{-2}]/[L]$	$[M^1L^{-2}T^{-2}]$
19.	Pressure energy	Pressure × volume	$[ML^{-1}T^{-2}][L^3]$	$[ML^2T^{-2}]$
20.	Coefficient of viscosity	Force/area × velocity gradient	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[ML^{-1}T^{-1}]$
21.	Angle, Angular displacement	Arc/radius	$[L]/[L]$	$[M^0L^0T^0]$
22.	Trigonometric ratio $(\sin\theta, \cos\theta, \tan\theta, \text{etc.})$	Length/length	$[L]/[L]$	$[M^0L^0T^0]$
23.	Angular velocity	Angle/time	$[L^0]/[T]$	$[M^0L^0T^{-1}]$

Physics

Class 11th NEET

02

MOTION IN A STRAIGHT LINE

Motion In a Straight Line

Important Terms

1. Mechanics

It is the branch of Physics, which deals with the study of motion of physical objects. Mechanics can be broadly classified into following branches

1.1 Statics

It is the branch of mechanics, which deals with the study of physical objects at **rest**.

1.2 Kinematics

It is the branch of mechanics, which deals with study of motion of physical bodies without taking into account the factors, which causes motion.

1.3 Dynamics

It is the branch of mechanics, which deals with the study of motion of physical bodies taking into account the factors which causes motion.

2. States of Objects

2.1 Rest

- An object is said to be at rest if it does not change its position with respect to the surrounding.
- The white board in the classroom is at rest with respect to the classroom

2.2 Motion

- An object is said to be in motion if it changes its position with respect to the surrounding.
- When we walk, run or ride a bike we are in motion with respect to the ground.

NOTE:

Rest and Motion are relative

Rest and motion depend upon the **observer**. The object in one situation may be at rest whereas the same object in another situation may be in motion. For Example, the driver of a moving car is in motion with respect to an observer standing on the ground whereas, the same driver is at rest with respect to the man (observer) in the passenger's seat.

3. While Studying this Chapter

- We will treat the object as **Point mass** object
- An object can be considered as a point mass object if during the course of motion, it covers distances much greater than its own size.
- We shall confine ourselves to the study of rectilinear motion
- Rectilinear motion is the study of motion of objects along a straight line.

4. Position, Distance & Displacement

4.1 Position

- Position of an object is always defined with respect to some reference point which we generally refer to as origin.

4.2 Distance

- It is the actual path traversed by the body during the course of motion
- SI unit is 'm'.
- Dimension is $[M^0L^1T^0]$

4.3 Displacement

- It is the shortest path joining initial and final position of the object.
- SI unit is 'm'
- Dimension is $[M^0L^1T^0]$
- It is a vector quality.

Table 2.1: Difference between Distance & Displacement

Distance	Displacement
It is the actual path traversed by the object during the course of motion.	It is the difference between the initial and the final positions $\Delta x = x_2 - x_1$ where,

	x_2 and x_1 are final and initial position respectively.
It is a **scalar** quantity.	It is a **vector** quantity.
The distance travelled by an object during the course of motion is never negative or zero and is always positive	The displacement of an object may be positive, negative or, zero during the course of motion.

NOTE:

The distance travelled is never less than displacement (in magnitude).
$|\text{Displacement}| \le \text{Distance}$.

NOTE:

If the motion of an object is along a straight line and in the same direction, the magnitude of displacement is equal to the total path length. In that case, the magnitude of average velocity is equal to the average speed. This is not always the case.
The average velocity tells us how fast an object has been moving over a given interval but does not tell us how fast it moves at different instants of time during that interval.

5. Scalar and Vector Quantities

5.1 Scalar Quantities

The physical quantities which have only magnitude but no direction, are called scalar quantities.
Example: - mass, length, time, distance, speed, work, temperature.

5.2 Vector Quantities

The physical quantities which have magnitude as well as direction, are called vector quantities.
Example: - displacement, velocity, acceleration, force, momentum, torque

6. Average velocity and Average Speed

6.1 Average Velocity

- It is defined as the change in position or displacement (Δx) divided by the time intervals (Δt) in which the displacement occurs.
- SI unit of velocity is m/s, although km/hr is used in many everyday applications
- Dimension is $[M^0L^1T^{-1}]$

6.2 Average Speed

- It is defined as the total path length travelled divided by the total time interval during which the motion has taken place.
- SI unit is m/s.
- Dimension $[M^0L^1T^{-1}]$

7. Instantaneous Velocity and Instantaneous Speed

7.1 Instantaneous Velocity

- It is velocity at an instant of time t. The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.
- Instantaneous velocity = $\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ = dx/dt
- The quantity on the right-hand side of equation is the differential coefficient of x with respect to t and is denoted by dx/dt.
- It is the rate of change of position with respect to time at that instant.
- SI unit is m/s
- Dimension is $[M^0L^1T^{-1}]$

7.2 Instantaneous Speed

- Instantaneous speed or speed is the magnitude of velocity at any particular instant of time.
- SI unit is m/s
- Dimension is $[M^0L^1T^{-1}]$

Table 2.2: Difference between Speed & Velocity

Speed	Velocity
It is defined as the total path length travelled divided by the total time interval during which the motion has taken place.	It is defined as the change in position or displacement divided by the time intervals, in which displacement occurs of.
It is a **scalar** quantity.	It is a **vector** quantity.
It is always positive during the course of the motion.	It may be positive, negative or zero during the course of the motion.
It is greater than or equal to the magnitude of velocity.	It is less than or equal to the speed.

8. Acceleration

8.1 Average Acceleration

- The average acceleration over a time interval is defined as the change of velocity divided by the time interval: $a = \frac{(v_2 - v_1)}{(t_2 - t_1)}$ where, v_2 and v_1 are velocities at time t_2 & t_1.
- It is the average change of velocity per unit time.
- SI unit is m/s^2.
- Dimension is $[M^0L^1T^{-2}]$.

8.2 Instantaneous Acceleration

- Instantaneous acceleration is defined in the same way as the instantaneous velocity:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$
$$= dv/dt$$

(for very small interval of time dt)

- SI unit is m/s^2
- Dimension is $[M^0L^1T^{-2}]$
- When the acceleration is uniform, obviously, instantaneous acceleration equals the average acceleration over that period.
- Since velocity is a quantity having both magnitude and direction, a change in the velocity may involve either or both of these factors.
- Acceleration, therefore, may result from a change in the speed (magnitude), a change in direction or changes in both.
- Like velocity, acceleration can also be positive, negative or zero.

NOTE: -

We will restrict ourselves to the study of constant acceleration for this chapter. In this case average acceleration equals the constant value of acceleration during the interval.

9. Kinematics Equations:

9.1 Equations of Uniformly Accelerated Motion

If a body starts with velocity (u) and after time t its velocity changes to v, if the uniform acceleration is a and the distance travelled in time t in s, then the following relations are obtained, which are called equations of uniformly accelerated motion.

(i) $v = u + at$

(ii) $S = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

(iv) Distance travelled in nth second

$$S_n = u + \frac{a}{2}(2n - 1)$$

If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then the velocity at the mid point of its path $\sqrt{\frac{(u^2 + v^2)}{2}}$.

10. Vertical Motion Under Gravity

If an object is falling freely (u = 0) under gravity, then equations of motion

(i) $v = u + gt$

(ii) $h = ut + \frac{1}{2}gt^2$

(iii) $v^2 = u^2 + 2gh$

NOTE:

If an object is thrown upward then g is replaced by – g in above three equations. It thus follows that

(i) Time taken to reach maximum height

$$T = u/g$$

(ii) Maximum height reached by the body

$$h_{max} = u^2 / 2g$$

(iii) A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively, then

$$t = \sqrt{t_1 t_2}$$

(iv) When a body is dropped freely from the top of the tower and another body is projected horizontally from the same point, both will reach the ground at the same time.

11. Calculus

11.1 Differentiation of a Function

If we say y as a function of x then we write

y = f (x)

x = Independent variable

y = Dependent variable.

In physics we study variation of a quantity y with respect to quantity x and we also study rate at which y changes when x changes.

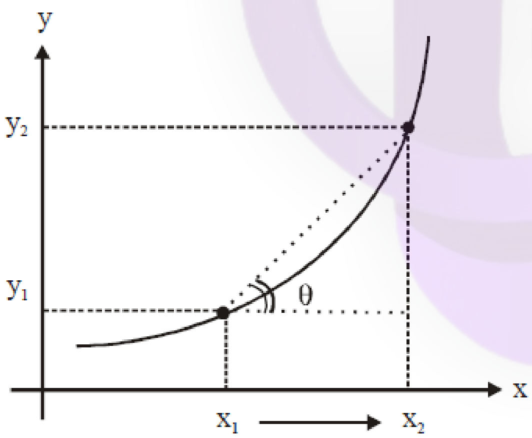


Fig. 2.1

One simple way to see variation of y with x

Rate of change of y with change in $x = \frac{\Delta y}{\Delta x}$

Graphically one can see that $\frac{\Delta y}{\Delta x} = \tan\theta$

Now let's say x_1 approaches to x_2 then $\Delta x \to 0$ or it will be very-very small, we write it dx. In that way, y approaches to y_2 and $\Delta y \to 0$, written as dy. So,

Rate of change of y with respect to $x = \frac{dy}{dx}$ (at one point)

Graphically it is tangent of curve on given point.

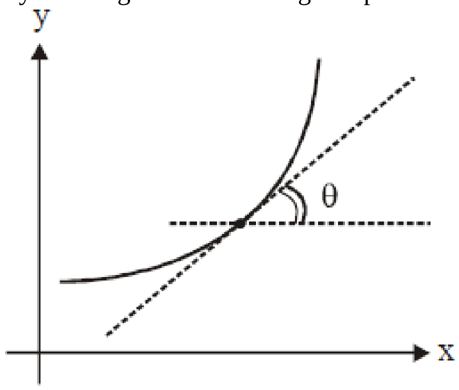


Fig. 2.2

$\frac{dy}{dx} =$ slope of tangent on curve between y and x at one point. Mathematically that is called differentiation of y with respect to $x = \frac{dy}{dx}$

$\frac{dy}{dx}$ →
1. Rate of change of y with respect to x
2. Slope of tangent on curve between y & x at one point
3. Differentiation of y with respect to x

In physics, first we will study the linear motion of an object where position of object is represented by x which changes with time t, then

$\frac{dx}{dt}$ →
- Rate of change of x with respect to t
- Slope of tangent on curve between x & t at one point
- Differentiation of x with respect to t

and that is equal to velocity (magnitude of velocity)

$$v = \frac{dx}{dt}$$

$$|v| = \left|\frac{dx}{dt}\right|$$

Above expression will give speed.

Similarly, when velocity changes with time then we say

$\frac{dv}{dt} = a \Rightarrow$ acceleration

so, we can define

Velocity $\Rightarrow$ that is rate of change of position with respect to t.

Acceleration $\Rightarrow$ that is rate of change of velocity with respect to t.

Tips:

If $x = f(t)$

$$v = \frac{dx}{dt} = \dot{x} = f'(t)$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = \ddot{x} = f''(t)$$

11.2 Standard Rules and Formulae of Differentiation:

1. $y = x^n$	$\dot{y} = nx^{n-1}$
2. $y = c \Rightarrow y = cx^0$	$\dot{y} = 0$
3. $y = \sin x$	$\dot{y} = \cos x$
4. $y = \cos x$	$\dot{y} = -\sin x$
5. $y = \tan x$	$\dot{y} = \sec^2 x$
6. $y = \cot x$	$\dot{y} = -\text{cosec}^2 x$
7. $y = \ell n(x)$	$\dot{y} = \frac{1}{x}$
8. $y = e^x$	$\dot{y} = e^x$
9. $y = a^x$	$\dot{y} = a^x \ell n(x)$

Rules

1. $y = f_1(x) + f_2(x)$	$\dot{y} = f_1'(x) + f_2'(x)$
2. $y = cf(x)$	$\dot{y} = cf'(x)$
3. $y = f_1(x) f_2(x)$	$\dot{y} = f_1(x) f_2'(x) + f_1'(x) f_2(x)$
4. $y = \frac{f_1(x)}{f_2(x)}$	$\dot{y} = \frac{f_2(x) f_1'(x) - f_1(x) f_2'(x)}{[f_2(x)]^2}$

11.3 Chain Rule

If y = f(z) and z = f(x), then differentiation of y with respect to x can be given by:

$$\frac{dy}{dx} = f'(z) \cdot f'(x) = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

12. Increasing and Decreasing Function

12.1 Increasing Function

Suppose y = f (x) and if x is increasing, y also increases, then the function is increasing function. There are two types of possible graphical variations.

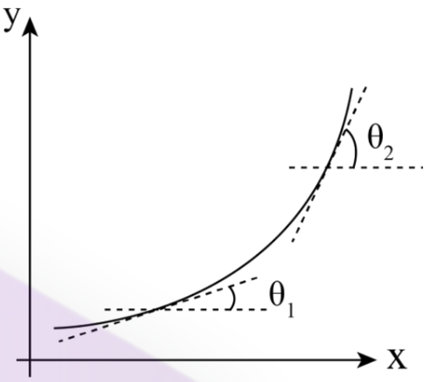


Fig 2.3

$\theta_2 > \theta_1$

$\Rightarrow \tan\theta_2 > \tan\theta_1$

$\Rightarrow \left.\frac{dy}{dx}\right|_2 > \left.\frac{dy}{dx}\right|_1$

If slope is increasing that mean first derivative is also increasing, so second derivative should be +ve, $\frac{d^2y}{dx^2} > 0$

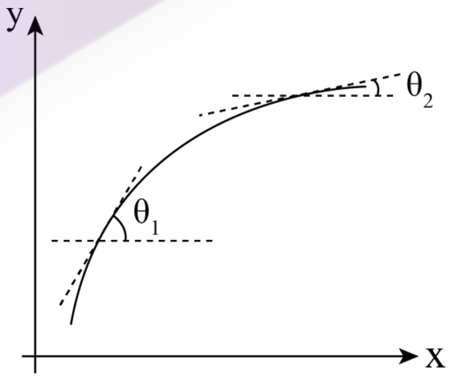


Fig. 2.4

$\theta_2 < \theta_1$

$\Rightarrow \tan\theta_2 < \tan\theta_1$

$\Rightarrow \left.\frac{dy}{dx}\right|_2 < \left.\frac{dy}{dx}\right|_1$

If slope decreasing that means first derivative is decreasing that means second derivative will be negative.

$$\frac{d^2y}{dx^2} < 0$$

NOTE:

(i) Increasing graph with decreasing slope will be Concave downward.
(ii) Increasing graph, with increasing slope will be Concave upward.

13. Decreasing Graphs

On increasing value of x, y decreases.
There also two types of graphs.

1.

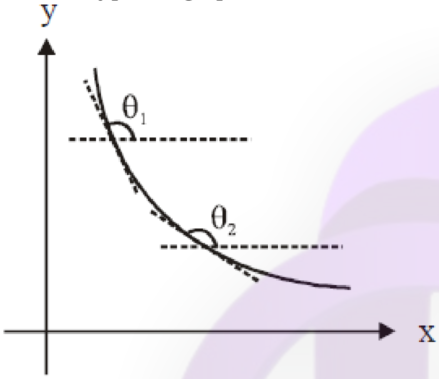


Fig. 2.5

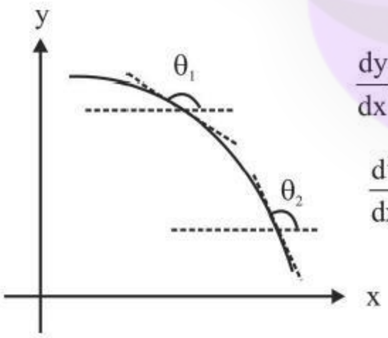
$$\theta_1 > \theta_2 > \frac{\pi}{2}$$

$$\Rightarrow \tan\theta_2 > \tan\theta_1$$

Slope is negative but increasing so $\frac{d^2y}{dx^2} > 0$

$$\frac{dy}{dx} < 0, \frac{d^2y}{dx^2} > 0$$

2.



$$\frac{dy}{dx} < 0$$

$$\frac{d^2y}{dx^2} < 0$$

Fig. 2.6

$\theta_1 > \theta_2$

$\tan\theta_1 > \tan\theta_2$

Slope decreasing and negative so second derivative is negative.

13.1 Applications in Physics:

1.

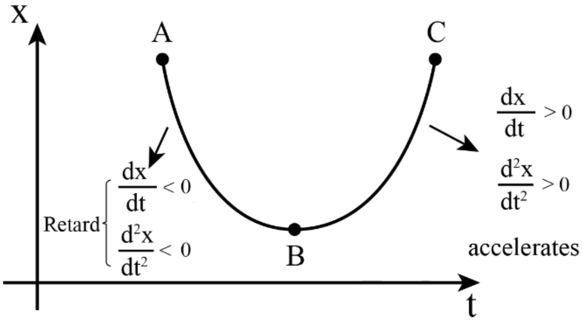


Fig. 2.7

2.

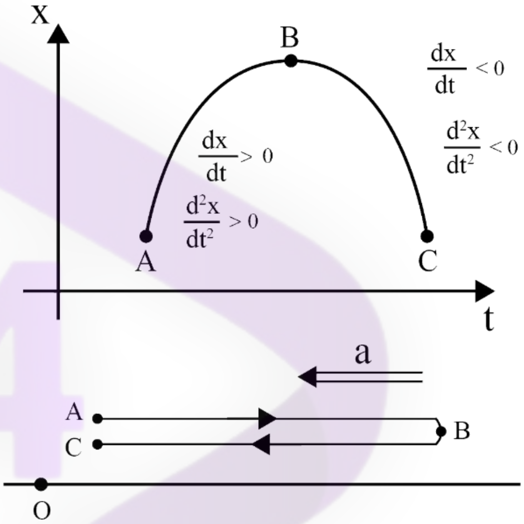


Fig. 2.8

NOTE:

If the graph is parabola, then second derivative will be constant

$x = at^2 + bt + c$ if $a > 0$

$\dot{x} = 2at + b$ $\quad \ddot{x} > 0$ and constant

$\ddot{x} = 2a$ $\quad$ if $a < 0$

$\ddot{x} < 0$ and constant

So if acceleration is constant then x and t graph will be parabola.
(ii) Increasing graph, with increasing slope will be Concave upward.

14. Maxima Minima of a Function

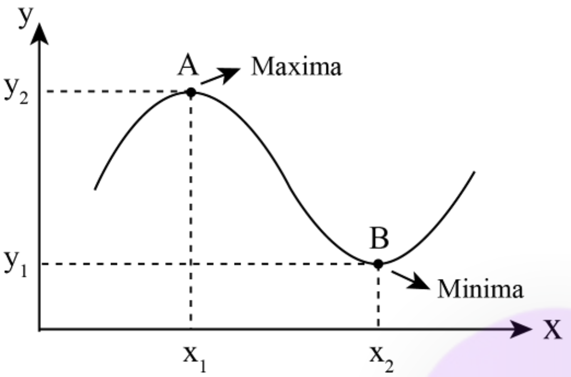


Fig. 2.9

Maxima

(Condition to locate and check point of maxima)

$$\frac{dy}{dx} = 0$$

$$\left.\frac{d^2y}{dx^2}\right|_{x_1} < 0$$

where x_1 is the point of maxima.

Minima

(Condition to locate and check point to minima)

$$\frac{dy}{dx} = 0$$

$$\left.\frac{d^2y}{dx^2}\right|_{x_2} > 0$$

where x_1 is the point of maxima.

15. Point of Inflexion

Concavity change at A is known as the point of inflexion.

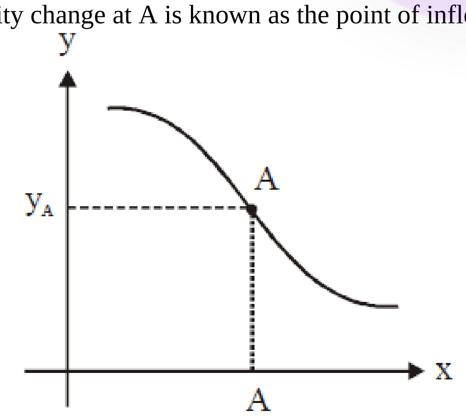


Fig. 2.10

$$\frac{dy}{dx} \neq 0$$

$$\frac{d^2y}{dx^2} = 0$$

16. Integration

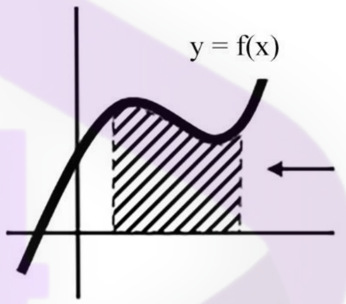
Integration of a function. Let y = f (x) Area of shaded region of curve is dA = ydx. Total area bounded by curve y = f (x)

$$A = \sum_{x=x_1}^{x=x_2} dA = \int_{x_1}^{x_2} dA$$

That is called area of graph with integration from x_1 to x_2.

x_1 = lower limit of integration

x_2 = upper limit of integration



$$A = \int_{x_1}^{x_2} f(x)\,dx$$

Fig 2.11

$\int_{x_1}^{x_2} ydx =$ Definite Integral

$\int ydx =$ Indefinite Integral (without limit)

Integration is reverse process of differentiation in which we find a function for which the given function is the derivative of function.

16.1 Formulae

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
2. $\int \frac{1}{x} dx = \ell n(x) + c$
3. $\int \sin x dx = -\cos x + c$
4. $\int \cos x dx = \sin x + c$
5. $\int \tan x dx = \ell n(\sec x) + c$
6. $\int e^x dx = e^x + c$

16.2 Rule of Integration

1. $\int dx = x + c$
2. $\int cf(x)dx = c\int f(x)dx$
3. $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$

NOTE:

Like differentiation, rules of substitution are also applicable to integration as well in a similar way.

16.3 Applications in Physics

$$v = \frac{dx}{dt}$$

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} vdt$$

$$x_2 - x_1 = \int_{t_1}^{t_2} vdt = \text{Change in position or displacement.}$$

(Area under the curve of v and t graph is displacement)

$$a = \frac{dv}{dt}$$

$$\int_{u}^{v} dv = \int_{t_1}^{t_2} adt$$

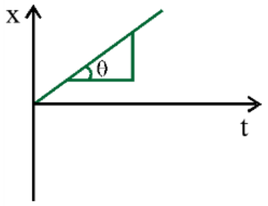
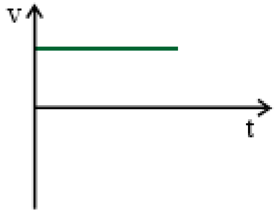
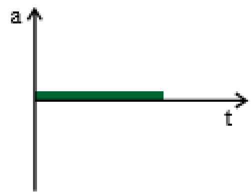
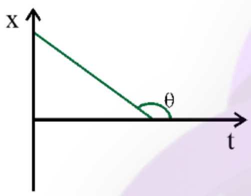
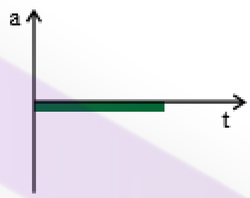
$$v - u = \int_{t_1}^{t_2} adt = \text{Change in velocity.}$$

(Area under the curve of a and t graph is change in velocity)

17. Graphs

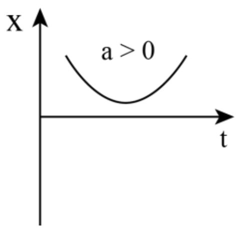
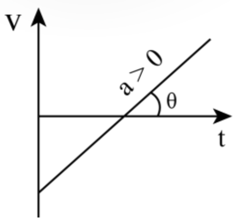
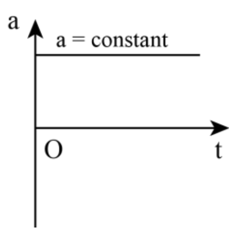
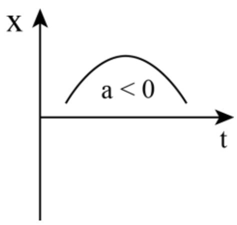
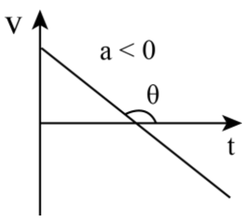
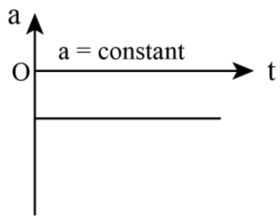
17.1 Uniform Motion

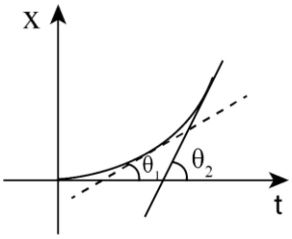
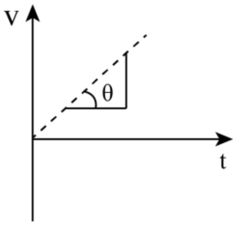
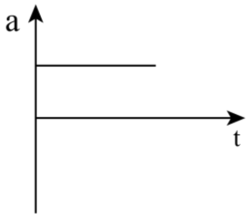
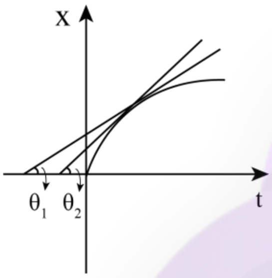
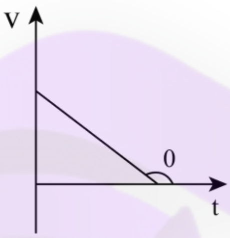
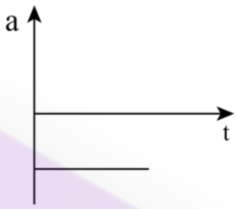
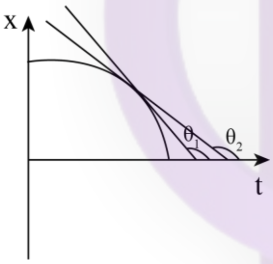
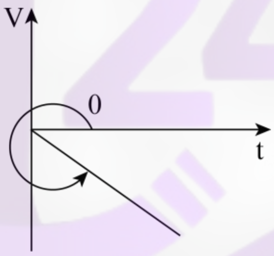
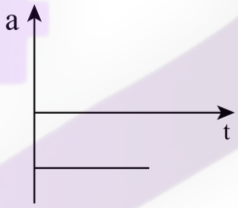
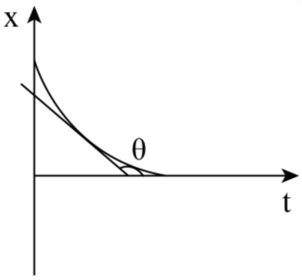
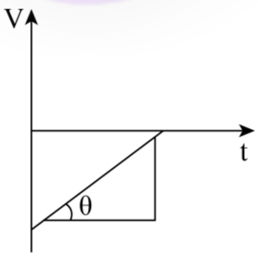
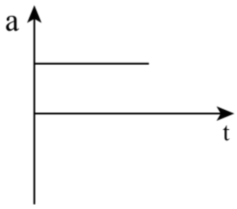
- In a uniform motion a body covers equal distance in equal intervals of time.
- Velocity is constant during the course of motion.
- Acceleration is zero during the course of motion.
  If we try to represent the same on the number line with x, v, a on the Y-axis and t on the X-axis then we will have

S.No	Displacement – Time graph	Velocity – Time graph velocity = Slope of x – t graph	Acceleration – Time graph Acceleration = Slope of v – t graph
(i)	 nature of slope: positive magnitude of slope: constant	 nature of slope: zero magnitude of slope: constant	 nature of slope of a – t: zero magnitude of slope: constant
(ii)	 nature of slope: negative magnitude of slope: constant	 nature of slope: zero magnitude of slope: constant	 nature of slope: zero magnitude of slope: Constant

17.2 Non-Uniform Motion

- In a non-uniform motion, a body covers unequal distances in equal intervals of time.
- Uniformly accelerated motion
- Accelerated motion
- Magnitude of velocity increases or decreases with time

S. No	Displacement – Time graph	Velocity – Time graph velocity = Slope of x – t graph	Acceleration – Time graph Acceleration = Slope of v – t graph
(i)			
(ii)			

(iii)	 nature of slope: positive magnitude of slope: Increasing	 nature of slope: positive magnitude of slope: constant	 Slope of a - t graph gives jerk, i.e., $\vec{J} = \frac{d\vec{a}}{dt} = 0$
(iv)	 nature of slope: positive magnitude of slope: decreasing	 nature of slope: negative magnitude of slope: constant	 Slope of a - t graph gives jerk, i.e., $\vec{J} = \frac{d\vec{a}}{dt} = 0$
(v)	 Nature of slope: negative magnitude of slope: decreasing	 Nature of slope: negative Magnitude of slope: constant	 Nature of slope: negative Magnitude of slope: constant
(vi)	 nature of slope: negative magnitude of slope: decreasing	 nature of slope: positive magnitude of slope: constant	 nature of slope: positive magnitude of slope: constant

NCERT Corner

(Some important Points to Remember)

1. An object is said to be in motion if its position changes with time. The position of the object can be specified with reference to a conveniently chosen origin. For motion in a straight fine. position to the right of the origin is taken as positive and to the left as negative.

2. Path length is defined as the total length of the path traversed by an object.

3. Displacement is the change in position: $\Delta x = x_2 - x_1$
Path length is greater or equal to the magnitude of the displacement between the same points.

4. An object is said to be in uniform motion in a straight line if its displacement is equal in equal intervals of time. Otherwise, the motion is said to be nonuniform

5. Average velocity is the displacement divided by the time Interval in which the displacement occurs:
$$\bar{v} = \frac{\Delta x}{\Delta t}$$
On an x-t graph. the average velocity over a time Interval is the slope of the line connecting the initial and final positions corresponding to that interval.

6. Average Speed is the ratio of total path length traversed and the corresponding time Interval.

7. Instantaneous velocity or simply velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small
$$v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$
The velocity at a particular instant is equal to the slope of the tangent drawn on position-time graph at that instant.

8. Average acceleration is the change in velocity divided by the time interval during which the change occurs:
$$\vec{a} = \frac{\Delta v}{\Delta t}$$

9. Instantaneous acceleration is defined as the limit of the average acceleration as the time interval at goes to zero:
$$a = \lim_{\Delta t \to 0} \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$
The acceleration of an object at a particular time is the slope of the velocity-time graph at that instant of time. For uniform motion. acceleration is zero and the x-t graph is a straight line inclined to the time axis and the $v-t$ graph is a straight line parallel to the time axis. For motion with uniform acceleration. x - t graph is a parabola while the $v-t$ graph is a straight line inclined to the time axis.

10. The area under the velocity-time curve between times t_1 and t_2 is equal to the displacement of the object during that interval of time.

11. For objects in uniformly accelerated rectilinear motion. the five quantities, displacement x, time taken t. Initial velocity v_0, final velocity v and acceleration a are related by a set of simple equations called kinematic equations of motion:
$$v = v_0 + at$$
$$x = v_0 t + \frac{1}{2} at^2$$
$$v^2 = v_0^2 + 2ax$$

03

MOTION IN A PLANE & RELATIVE MOTION

Chapter 03

Motion in a Plane & Relative Motion

1. Motion in 2D (Plane)

1.1 Position Vector & Displacement

The position vector $\vec{r}$ of a particle P located in a plane with reference to the origin of an x-y coordinate system is given by $\vec{r} = x\hat{i} + y\hat{j}$

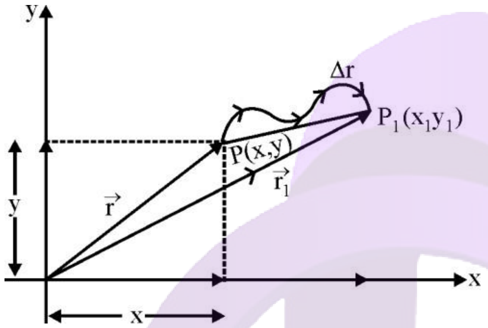


Fig 3.1

Suppose the particle moves along the path as shown to a new position P_1 with the position vector $\vec{r}$

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$

change in position = displacement

$= \vec{r}_1 - \vec{r} = \left(x_1\hat{i} + y_1\hat{j}\right) - \left(x\hat{i} + y\hat{j}\right)$

(By vector addition)

$= \left(x_1 - x\right)\hat{i} + \left(y_1 - y\right)\hat{j}$

$= \Delta x\hat{i} + \Delta y\hat{j}$

from above figure we can see that

$\Delta\vec{r} = \vec{r}_1 - \vec{r}$

1.2 Average Velocity

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$

$$v_{avg} = \Delta v_x\hat{i} + \Delta v_y\hat{j}$$

NOTE:

Direction of the average velocity is same as that of $\Delta\vec{r}$.

1.3 Instantaneous Velocity

$$\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

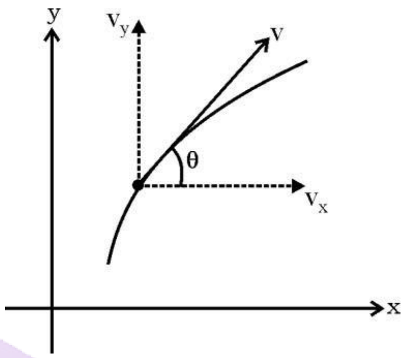


Fig 3.2

where, $v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$,

where $|\vec{v}|$ represents magnitude of velocity

and $\tan\theta = \frac{v_y}{v_x}$

or $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

NOTE:

The instantaneous velocity at any point on the path of an object is tangential to the path at that point and its direction is in the direction of object's motion.

1.4 Average Acceleration

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$\vec{a}_{avg} = a_x\hat{i} + a_y\hat{j}$$

1.5 Instantaneous Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\vec{a} = \vec{a}_x\hat{i} + \vec{a}_y\hat{j}$$

2. Projectile Motion

When a particle is projected obliquely from the earth's surface, it moves simultaneously in horizontal and vertical directions in a curved trajectory as depicted in the diagram below. Motion of such a particle is called **projectile motion.**

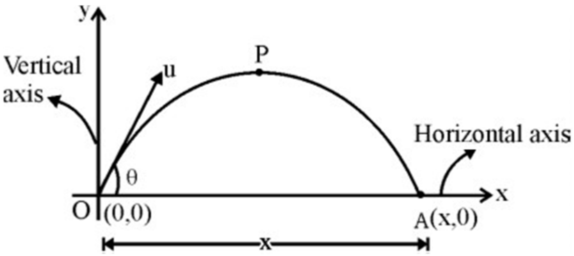


Fig 3.3

2.1 Analysis of Velocity in Case of a Projectile

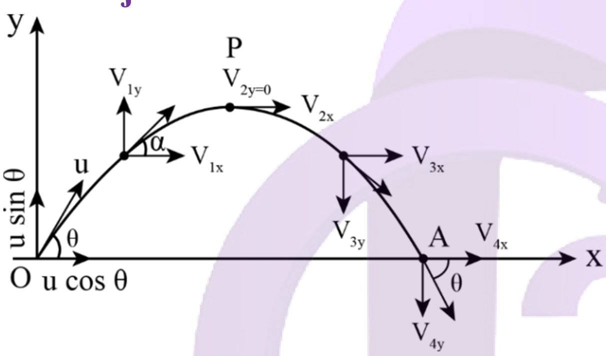


Fig 3.4

(i) $V_{1x} = V_{2x} = V_{3x} = V_{4x} = u_x = u\cos\theta$
which means that the velocity along x-axis remains constant [as there is no external force acting along that direction]

(ii) a) Magnitude of velocity along y-axis first decreases and then it increases after the topmost point P.
b) At topmost point magnitude of velocity is zero.
c) Direction of velocity is in the upward direction while ascending and is in the downward direction while descending.
d) Magnitude of velocity at A is same as magnitude of velocity at O; but the direction is changed
e) Angle which the net velocity makes with the horizontal can be calculated by

$$\tan\alpha = \frac{v_y}{v_x} = \frac{\text{velocity along } y-\text{axis}}{\text{velocity along } x-\text{axis}}$$

& net velocity is always along the tangent.

2.2 Equation of Trajectory

Trajectory is the path traced by the body. To find the trajectory we must find relation between y and x by eliminating time.
[Ref. to the earlier diag.]

Horizontal Motion	Vertical Motion
$u_x = u\cos\theta$ $a_x = 0$ $s_x = u\cos\theta t = x$ $\Rightarrow t = \dfrac{x}{u\cos\theta}$	$u_y = u\sin\theta$ $a_y = -g$ $s_y = u_y t + \dfrac{1}{2}a_y t^2$

So,

$$\left(y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}\right)$$

$$\Rightarrow y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Which resembles to $\left(y = bx - ax^2\right)$

(i) This is an equation of a parabola
(ii) Because the coefficient of x^2 is negative, it is an inverted parabola.

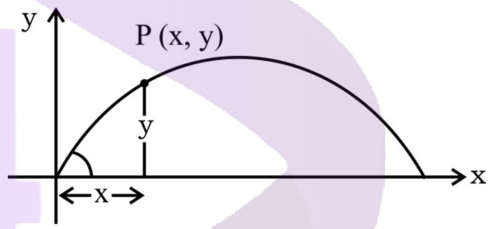


Fig 3.5

Path of the projectile is a parabola

$$R = \frac{2u^2\sin\theta\cos\theta}{g} \quad \text{or} \quad \frac{2u^2}{g} = \frac{R}{\sin\theta\cos\theta}$$

Substituting this value in the above equation we have,

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

In this case a particle is projected at an angle θ with an initial velocity u. For this particular case we will calculate the following:
(a) time taken to reach A from O
(b) horizontal distance covered (OA)
(c) maximum height reached during the motion
(d) velocity at any time 't' during the motion

Horizontal axis	Vertical axis
$u_x = u\cos\theta$ $a_x = 0$ (In the absence of any external force a_x is assumed to be zero)	$u_y = u\sin\theta$ $a_y = -g$ $s_y = u_y t + \frac{1}{2}a_y t^2$ when the particle returns to same horizontal level, vertical displacement is 0 and time taken is called **time of flight**, (T). $0 = u\sin\theta T - \frac{1}{2}gT^2$ $T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$
$s_x = u_x t + 1/2 a_x t^2$ $x - 0 = u\cos\theta\, t$ $x = u\cos\theta \times \frac{2u_y}{g}$ $x = \frac{2u^2\cos\theta\sin\theta}{g}$ ($2\cos\theta\sin\theta = \sin 2\theta$) $x = \frac{u^2\sin 2\theta}{g}$ horizontal distance covered is known as **Range**	$v_y = u_y + a_y t$ It depends on time 't'. Its magnitude first decreases and then becomes zero and then increases.
$v_x = u_x + a_x t$ $v_x = u\cos\theta$ It is independent of t and is constant	**Maximum height attained** by the particle Method 1: using time of ascent Time of ascent, $t = \frac{u\sin\theta}{g}$ $s_y = u_y t + \frac{1}{2}at^2$ $= u\sin\theta \times \frac{u\sin\theta}{g}$ $-\frac{1}{2}g\frac{u^2\sin^2\theta}{g^2}$ $H = \frac{u^2\sin^2\theta}{2g}$
Time of ascent (t_1) =Time of descent (t_2) At topmost point y = 0 $\Rightarrow 0 = u\sin\theta - gt$	**Maximum height attained** Method 2: using third equation of motion $v_y^2 - u_y^2 = 2a_y s_y$ $H = \frac{u^2\sin^2\theta}{2g}$

$t_1 = \frac{u\sin\theta}{g}$ $t_2 = T - t_1 = \frac{u\sin\theta}{g}$ $t_1 = t_2 = \frac{T}{2} = \frac{u\sin\theta}{g}$	

Maximum Range

$$R = \frac{u^2\sin 2\theta}{g} \text{ and } R_{max} = \frac{u^2}{g}$$

Range is maximum when sin 2θ is maximum
Maximum value of sin 2θ = 1.
So, $\theta = 45°$ (for maximum range)

3. Projectile Motion from a Height

3.1 Horizontal Direction:

(i) Initial velocity $u_x = u$

(ii) Acceleration $a_x = 0$

Vertical Direction:

(i) Initial velocity $u_y = 0$

(ii) Acceleration $a_y = -g$ (downward)

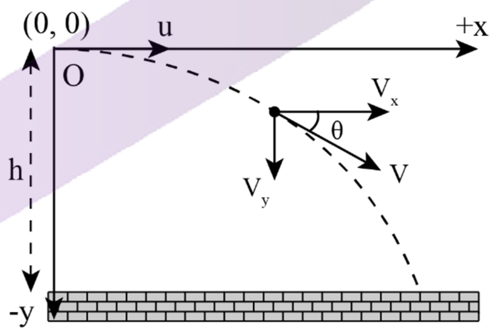


Fig 3.6

The path traced by projectile is called its trajectory.
After time t,
Horizontal displacement x = ut

Vertical displacement $y = -\frac{1}{2}gt^2$

(Negative sign indicates that the direction of vertical displacement is downward.)

So $y = \frac{1}{2}g\frac{x^2}{u^2}\left(\because t = \frac{x}{u}\right)$ this is equation of a parabola

Above equation is called **trajectory equation.**

The equations for this type of motion will be:

- Time of flight

$$T_f = \sqrt{\frac{2h}{g}}$$

- Horizontal Range

$$R = u_x t = u\sqrt{\frac{2h}{g}}$$

- Trajectory Equation

$$y = \frac{1}{2}g\frac{x^2}{u^2}\left(\because t = \frac{x}{u}\right)$$

This is equation of parabola

- Along vertical direction

$$v_y^2 = 0^2 + 2(h_1)(g)$$

$$v_y = \sqrt{2gh_1}$$

Along horizontal direction:

$$v_x = u_x = u$$

So, velocity

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh_1}$$

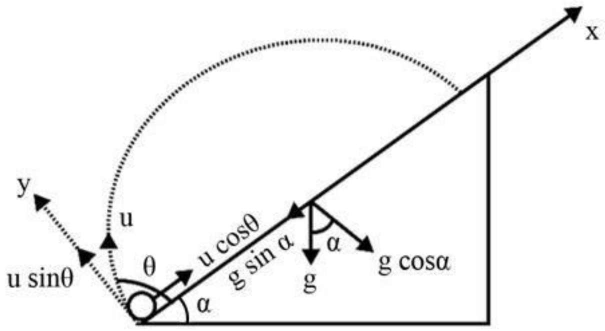


Fig 3.8

Projectile up an inclined plane	
Motion along x-axis	**Motion along y-axis**
$u_x = u\cos\theta$ $a_x = -g\sin\alpha$ $v_x = u\cos\theta - g\sin\alpha t$ $x = u\cos\theta t - \frac{1}{2}g\sin\alpha t^2$	$u_y = u\sin\theta$ $a_y = -g\cos\alpha$ $v_y = u\sin\theta - g\cos\alpha t$ $y = u\sin\theta t - \frac{1}{2}g\cos\alpha t^2$

4. Projectile on an Incline

4.1 The Motion of a Particle along the Inclined Plane in Upward Direction.

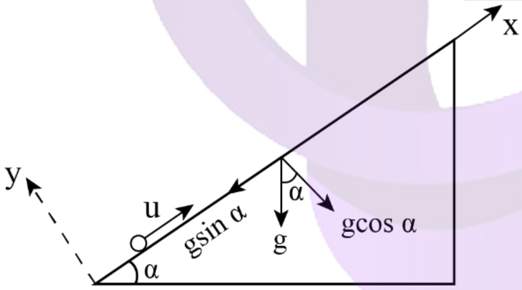


Fig 3.7

$$u_x = u$$

$$a_x = -g\sin\alpha$$

$$v_x = u - (g\sin\alpha)t$$

$$x = ut - \frac{1}{2}(g\sin\alpha)t^2$$

$$u_y = 0$$

$$a_y = -g\cos\alpha$$

$$v_y = 0$$

$$y = 0$$

4.2 The Motion of a Particle along the Inclined Plane in Downward Direction.

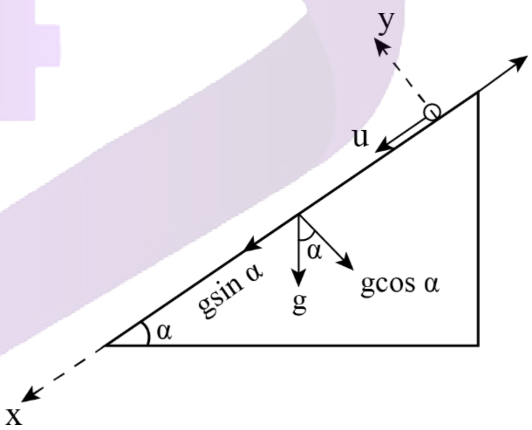


Fig 3.9

$$u_x = u$$

$$a_x = +g\sin\alpha$$

$$v = u + (g\sin\alpha)t$$

$$x = ut + \frac{1}{2}(g\sin\alpha)t^2$$

$$u_y = 0$$

$$a_y = -g\cos\alpha$$

$$v_y = 0$$

$$y = 0$$

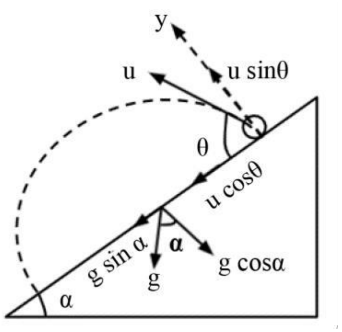


Fig 3.10

Projectile down an inclined plane	
Motion along x-axis	**Motion along y-axis**
$u_x = u\cos\theta$ $a_x = g\sin\alpha$ $v_x = u\cos\theta + g\sin\alpha t$ $x = u\cos\theta t + \frac{1}{2} g\sin\alpha t^2$	$u_y = u\sin\theta$ $a_y = g\cos\alpha$ $v_y = u\sin\theta - g\cos\alpha t$ $y = u\sin\theta t - \frac{1}{2} g\cos\alpha t^2$

$$R = \frac{u^2}{g\cos^2\alpha}\{\sin(2\theta - \alpha) - \sin\alpha\}$$

The maximum range therefore is,

$$\Rightarrow R_{max} = \frac{u^2}{g\cos^2\alpha}(1 - \sin\alpha)$$

5. Relative Motion

Relative is a very general term. In physics we use relative very often. For e.g.

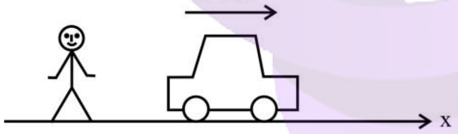


Fig 3.11

- **Case I:** If you are stationary and you observe a car moving on a straight road then you say velocity of car is 20 m/s which means velocity of car relative to you is 20 m/s or, velocity of car relative to the ground is 20 m/s.
(As you are stationary on the ground.)

- **Case II:** If you go inside a car and observe you will find that the car is at rest while the road is moving backwards. You will say: Velocity of car relative to the you is 0 m/s Mathematically, velocity of B relative to A is represented as

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

As this being a vector quantity, direction is very important.

5.1 Velocity of Approach / Separation

- It is the component of relative velocity of one particle with respect to another, along the line joining them.
- If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.
- In one dimension, since relative velocity is along the line joining A and B, hence velocity of approach /separation is just equal to magnitude of relative velocity of A with respect to B.

5.2 Velocity of Approach / Separation in Two Dimensions

- It is the component of relative velocity of one particle with respect to another, along the line joining them.
- If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

6. River-Boat Problems

- In river–boat problems we come across the following three terms:
$\vec{v}_r =$ absolute velocity of river.
$\vec{v}_{br} =$ velocity of boatman with respect to river and
$\vec{v}_b =$ absolute velocity of boatman.
Hence, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground.
Further $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$.
- Now, let us derive some standard results and their special cases.
A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in figure. River is flowing along positive x–direction with velocity $\vec{v}_r$. Width of the river is d.
Then, $\vec{v}_b = \vec{v}_r + \vec{v}_{br}$
Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$

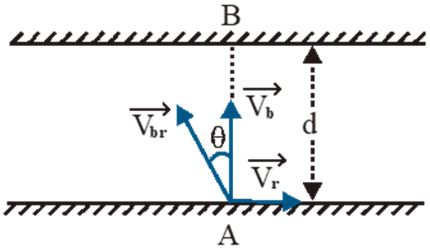


Fig 3.12

Now, time taken by the boatman to cross the river is:

$$t = \frac{d}{v_{by}} = \frac{d}{v_{br}\cos\theta} \text{ or } t = \frac{d}{v_{br}\cos\theta} \quad ...(i)$$

Further, displacement along x–axis when he reaches on the other bank (also called **drift**) is given by-

$$x = (v_r - v_{br}\sin\theta)\frac{d}{v_{br}\cos\theta} \quad ...(ii)$$

6.1 Condition when the Boatman crosses the river in shortest interval of Time

From eq. (i) we can see that time (t) will be minimum when $\theta = 0°$ i.e., the boatman should steer his boat perpendicular to the river current.

6.2 Condition when the Boatman wants to reach point B, i.e., at a point just opposite from where he started (shortest distance)

In this case, the drift (x) should be zero.

$\therefore x = 0$

or, $(v_r - v_{br}\sin\theta)\frac{d}{v_{br}\cos\theta} = 0$

or, $v_r = v_{br}\sin\theta$

or, $\sin\theta = \frac{v_r}{v_{br}}$ or $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

Hence, to reach point B the boatman should row at an angle $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream from AB.

$$t = \frac{d}{v_b} = \frac{d}{\sqrt{v_{br}^2 - v_r^2}}$$

Since $\sin\theta \not> 1$ So, if $v_r \geq v_{br}$, the boatman can never reach at point B. Because if $v_r = v_{br}$, $\sin\theta = 1$ or $\theta = 90°$ and it is just impossible to reach at B if $\theta = 90°$.

Similarly, if $v_r > v_{br}$, $\sin\theta > 1$, i.e., no such angle exists. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

7. Relative Velocity of Rain with Respect to Man

Consider a man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure. To find the relative velocity of rain with respect to man (i.e., $\vec{v}_{rm}$), bring the man at rest by imposing a velocity $-\vec{v}_m$ on man and apply this velocity on rain also.

Now the relative velocity of rain with respect to man will be the resultant velocity of $\vec{v}_r\left(=\overline{OB}\right)$ and $-\vec{v}_m\left(=\overline{OC}\right)$, which will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$$\therefore v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m\cos 90°} = \sqrt{v_r^2 + v_m^2}$$

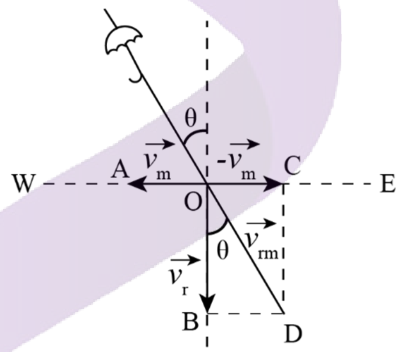


Fig 3.13

If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction, then

$$\tan\theta = \frac{OD}{OB} = \frac{v_m}{v_r} \text{ or } \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Here, angle θ is the angle that $\vec{v}_{rm}$ makes w.r.t vertical.

- In the above problem if the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain with respect to man i.e., the umbrella should be held making an angle $\theta\ (= \tan^{-1}\frac{v_m}{v_r})$ west of vertical.

NCERT CORNER

(Some important points to remember)

1. Scalar quantities are quantities with magnitudes only. Examples are distance, speed, mass and temperature

2. Vector quantities are quantities with magnitude and direction both. Examples are displacement, velocity and acceleration. They obey special rules of vector algebra.

3. A vector $\vec{A}$ multiplied by a real number λ is also a vector, whose magnitude is λ times the magnitude of the vector $\vec{A}$ and whose direction is the same or opposite depending upon whether λ is positive or negative.

4. Two vectors $\vec{A}$ and $\vec{B}$ may be added graphically using head to tail method or parallelogram method.

5. Vector addition is commutative: $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
It also obeys the associative law:
$(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$

6. A null or zero vector is a vector with zero magnitude. Since the magnitude is zero, we don't have to specify its direction. It has the properties:
$\vec{A}+0=\vec{A}$
$\vec{A}0=0$
$0\vec{A}=0$

7. The subtraction of vector $\vec{B}$ from $\vec{A}$ is defined as the sum of $\vec{A}$ and $-\vec{B}$:
$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

8. A vector A can be resolved into component along two given vectors a and b lying in the same plane:
$\vec{A}=\lambda\vec{a}+\mu\vec{b}$ where λ and μ are real numbers.

9. A unit vector associated with a vector $\vec{A}$ has magnitude one and is along the vector $\vec{A}$:

$$\hat{n}=\frac{\vec{A}}{|\vec{A}|}$$

The unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ are vectors of unit magnitude and point in the direction of the *x*-, *y*-, and *z*-axes, respectively in a right-handed coordinate system.

10. A vector $\vec{A}$ can be expressed as $\vec{A}=A_x\hat{i}+A_y\hat{j}$ where A_x, A_y are its components along x-, and y-axes. If vector $\vec{A}$ makes an angle θ with the x-axis, then $A_x=\vec{A}\cos\theta$, $A_y=\vec{A}\sin\theta$ and

$$A=|\vec{A}|=\sqrt{A_x^2+A_y^2},\ \tan\theta=\frac{A_y}{A_x}$$

11. Vectors can be conveniently added using analytical method. If sum of two vectors $\vec{A}$ and $\vec{B}$, that lie in x-y plane, is R, then: $\vec{R}=R_x\hat{i}+R_y\hat{j}$. where, $R_x=A_x+B_x$ and $R_y=A_y+B_y$

12. The position vector of an object in x-y plane is given by $r=x\hat{i}+y\hat{j}$ and the displacement from position ***r*** to position r' is given by

$$\begin{aligned}\Delta\vec{r}&=\vec{r}'-\vec{r}\\&=(x'-x)\hat{i}+(y'-y)\hat{j}\\&=\Delta x\hat{i}+\Delta y\hat{j}\end{aligned}$$

13. If an object undergoes a displacement Δr in time Δt, its average velocity is given by $v=\frac{\Delta r}{\Delta t}$. The velocity of an object at time *t* is the limiting value of the average velocity as Δt tends to zero: $v=\lim_{\Delta t\to 0}\frac{\Delta r}{\Delta t}=\frac{dr}{dt}$.
It can be written in unit vector notation as:
$v=v_x\hat{i}+v_y\hat{i}+v_z\hat{k}$ where $v_x=\frac{dx}{dt}, v_y=\frac{dy}{dt}, v_z=\frac{dz}{dt}$
When position of an object is plotted on a coordinate system, $\vec{v}$ is always tangent to the curve representing the path of the object.

14. If the velocity of an object changes from $\vec{v}$ to $\vec{v}'$ in time Δt, then its average acceleration is given by:

$$\vec{a}=\frac{\vec{v}-\vec{v}'}{\Delta t}=\frac{\Delta\vec{v}}{\Delta t}$$

The acceleration $\vec{a}$ at any time t is the limiting value of $\vec{a}$ as $\Delta t\to 0$,

$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

In component form, we have: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Where, $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$, $a_z = \frac{dv_z}{dt}$

15. Relative motion can be defined as the comparison between the motions of a single object to the motion of another object moving with the same velocity. Relative motion can be easily found out with the help of the concept of relative velocity, relative acceleration or relative speed

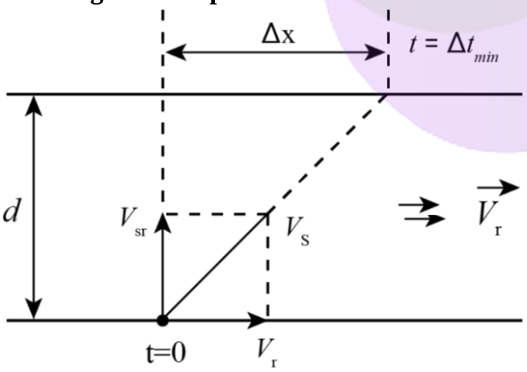
16. The relative velocity of an object A with respect to object B is the rate of position of the object A with respect of object B.

- If V_A and V_B be the velocities of objects A and B with respect to the ground, then:
  (a) The relative velocity of A with respect to B is $V_{AB} = V_A – V_B$
  (b) The relative velocity of B with respect to A is $V_{BA} = V_A – V_B$
- SI unit: m/s
- Dimensional formula: $[LT^{-1}]$

17. Relative Acceleration: The relative acceleration (also a_r) is the acceleration of an object or observer B in the rest frame of another object or observer A.

- Acceleration of B relative to A = a_B - a_A
- SI unit: m/s^2
- Dimensional formula: $[LT^{-2}]$

18. Crossing of River problems:



- **Time of crossing:** Component ($v_r + v_{sr}\cos\boldsymbol{\theta}$) will enable the person to drift along the length of river.

Hence drift Δx will be

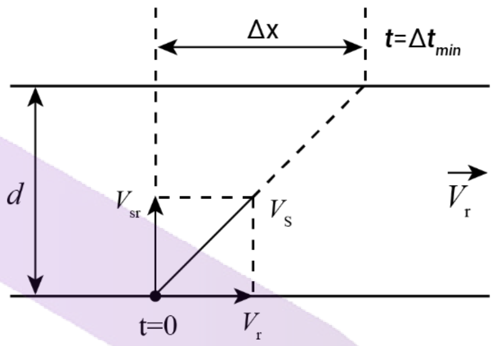
$$\Delta t = \frac{d}{v_{sr}\sin\theta} \text{ ...(i)}$$

$$\Delta x = (v_r + v_{sr}\cos\theta)\Delta t \text{ ...(ii)}$$

- **Minimum time of crossing**

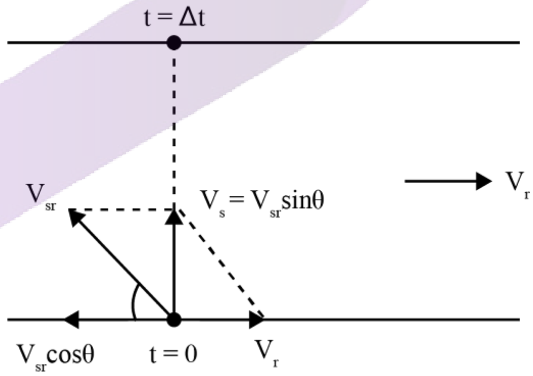
$\therefore \Delta t_{min} = \frac{d}{v_{sr}}$ And hence

Drift $\Delta x = v_r\left(\frac{d}{v_{sr}}\right)$



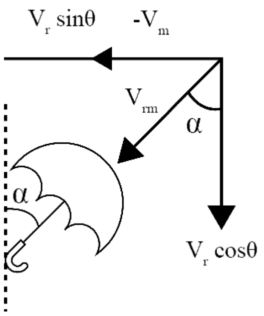
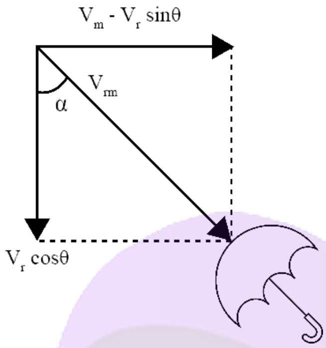
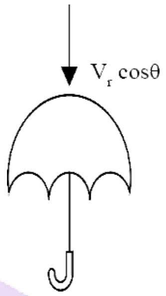
- **Shortest Path:** The person should try to swim such that the resultant velocity becomes perpendicular to the river flow.

$\therefore \Delta t = \frac{d}{\sqrt{v_{sr}^2 - v_r^2}}$



19. Rain-man umbrella problems

- A person standing/running in a particular direction would be needed to be protected by properly directing the axis of the umbrella.
- Here again 3 situations may arise-

Case-I	Case-II	Case-III
$v_r \sin\theta > v_m$ $\Downarrow$ $v_r \sin\theta - v_m$  $\tan\alpha = \dfrac{v_r \sin\theta - v_m}{v_r \cos\theta}$	$v_r \sin\theta < v_m$ $\Downarrow$ $v_m - v_r \sin\theta$  $\tan\alpha = \dfrac{v_m - v \sin\theta}{v_r \cos\theta}$	$v_r \sin\theta = v_m$ $\Downarrow$ $v_r \cos\theta = v_{rm}$  In this case rain appears to fall vertically, which can happen only if the horizontal velocity of rain and man match.

04

LAWS OF MOTION & FRICTION

Laws of Motion and Friction

1. Force

(a) A force is something which changes or tends to change the state of rest or motion of a body. It causes a body to start moving if it is at rest or stop it, if it is in motion or deflect it from its initial path of motion.

(b) Force is also defined as an interaction between two bodies. Two bodies can also exert force on each other even without being in physical contact, e.g., electric force between two charges, gravitational force between any two bodies of the universe.

(c) Force is a vector quantity having SI unit Newton (N) and dimension $[MLT^{-2}]$.

(d) **Superposition of force:** When many forces are acting on a single body, the resultant force is obtained by using the laws of vector addition. $\vec{F} = \vec{F}_1 + \vec{F}_2 + ... \vec{F}_n$

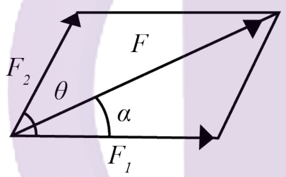


Fig. 4.1

The resultant of the two forces $\vec{F}_1$ *and* $\vec{F}_2$ acting at an angle θ is given by:

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

The resultant force is directed at an angle α with respect to force F_1 where $\tan\alpha = \dfrac{F_2\sin\theta}{F_1 + F_2\cos\theta}$

(e) **Lami's theorem :** If three forces F_1, F_2 and F_3 are acting simultaneously on a body and the body is in equilibrium, then according to Lami's theorem,

$$\frac{F_1}{\sin(\pi-\alpha)} = \frac{F_2}{\sin(\pi-\beta)} = \frac{F_3}{\sin(\pi-\gamma)}$$

where α, β and γ are the angles opposite to the forces F_1, F_2 & F_3 respectively.

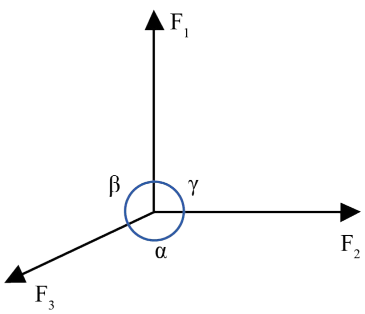


Fig. 4.2

2. Types of Force

There are, basically, four forces, which are commonly encountered in mechanics.

(a) **Weight :** Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

(b) **Contact Force :** When two bodies come in contact they exert forces on each other that are called contact forces.

(i) **Normal Force (N):** It is the component of contact force normal to the surface. It measures how strongly the surfaces in contact are pressed together.

(ii) **Frictional Force (f) :** It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.

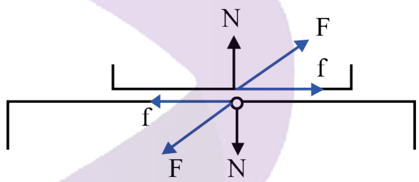


Fig. 4.3

(c) **Tension:** The force exerted by the ends of a taut string, rope or chain is called the tension. The direction of tension is so as to pull the body while that of normal reaction is to push the body.

(d) **Spring Force:** Every spring resists any attempt to change its length; the more you alter its length the harder it resists. The force exerted by a spring is given by F = –kx, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1}).

3. Newton's Laws of Motion

3.1 First law of Motion

(a) Everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by a resultant force to change that state

(b) This law is also known as **law of inertia**. Inertia is the property of inability of a body to change its position of rest or uniform motion in a straight line unless some external force acts on it.

(c) Mass is a measure of inertia of a body.

(d) A frame of reference in which Newton's first law is valid is called **inertial frame**, i.e., if a frame of reference is at rest or in uniform motion it is called **inertial**, otherwise **non-inertial.**

3.2 Second Law of Motion

(a) This law gives the magnitude of force.

(b) According to second law of motion, rate of change of momentum of a body is directly proportional to the resultant force acting on the body, i.e.,

$$\vec{F} \propto \left(\frac{d\vec{p}}{dt}\right)$$

$$\vec{F} = K\frac{d\vec{p}}{dt}$$

Here, the change in momentum takes place in the direction of the applied resultant force. Momentum, $\vec{p} = m\vec{v}$ is a measure of sum of the motion contained in the body.

(c) **Unit force :** It is defined as the force which changes the momentum of a body by unity in unit time. According to this, K=1

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}.$$

If the mass of the system is finite and remains constant w.r.t. time, then (dm/dt) = 0 and

$$\vec{F} = m\left(\frac{d\vec{v}}{dt}\right) = m\vec{a} = \left(\frac{\vec{p}_2 - \vec{p}_1}{t}\right)$$

(d) External force acting on a body may accelerate it either by changing the magnitude of velocity or direction of velocity or both.

(i) **If the force is parallel to the motion**, it changes only the magnitude of velocity but not the direction. So, the path followed by the body is a **straight line**.

(ii) **If the force is acting perpendicular to the motion of body,** it changes only the direction but not the magnitude of velocity. So, the path followed by the body is a **circle** (uniform circular motion).

(iii) **If the force acts at an angle to the motion of a body,** it changes both the magnitude and direction of $\vec{v}$. In this case path followed by the body may be **elliptical, non-uniform circular, parabolic or hyperbolic.**

3.3 Third Law of Motion

(a) According to this law, for every action there is an equal and opposite reaction. When two bodies A and B exert force on each other, the force by A on B (i.e., action represented by $\vec{F}_{AB}$), is always equal and opposite to the force by B on A (i.e., reaction represented $\vec{F}_{BA}$). Thus, $\vec{F}_{AB} = -\vec{F}_{BA}$.

(b) The two forces involved in any interaction between two bodies are called **action and reaction.** But we cannot say that a particular force is action and the other one is reaction.

(c) Action and Reaction force always acts on different bodies.

3.4 Some Important Points Concerning Newton's Laws of Motion

(a) The forces of interaction between bodies composing a system are called **internal forces.** The forces exerted on bodies of a given system by bodies situated outside are called **external forces.**

(b) Whenever one force acts on a body it gives rise to another force called reaction i.e., **a single isolated force** is physically impossible. This is why **total internal force in an isolated system is always zero.**

(c) According to Newton's second law, $\vec{F} = \left(\frac{d\vec{p}}{dt}\right)$.

$$\text{If } \vec{F} = 0, \left(\frac{d\vec{p}}{dt}\right) = 0 \text{ or } \left(\frac{d\vec{v}}{dt}\right) = 0$$

or $\vec{v}$ = constant or zero,

i.e., a body remains at rest or moves with uniform velocity unless acted upon by an external force. This is Newton's I[st] law.

(d) Newton's second law can also be expressed as:

$Ft = p_2 - p_1$. Hence, if a car and a truck are initially moving with the same momentum, then by the application of same breaking force, both will come to rest in the same time.

(e) The second law is a vector law. it is equivalent to three equations : $F_x = ma_x$; $F_y = ma_y$; $F_z = ma_z$. A force can only change the component of velocity in its direction. It has no effect on the component perpendicular to it.

(f) $\vec{F} = m\vec{a}$ is a local relation. The force at a point on space at any instant is related to the acceleration at that instant. Example: An object on an accelerated balloon will have acceleration of balloon. The moment it is dropped, it will have acceleration due to gravity.

3.5 Applications of Newton's Laws of Motion

There are two kinds of problems in classical mechanics :

(a) To find unknown forces acting on a body, given the body's acceleration.

(b) To predict the future motion of a body, given the body's initial position and velocity and the forces acting on it. For either kind of problem, we use Newton's second law . The following general strategy is useful for solving such problems :

(i) Draw a simple, neat diagram of the system.

(ii) Isolate the object of interest whose motion is being analyzed. Draw a **free body diagram** for this object, that is, a diagram showing all external forces acting on the object. For systems containing more than one object, draw separate diagrams for each objects. Do not include forces that the object exerts on its surroundings.

(iii) Establish convenient coordinate axes for each body and find the **components of the forces along these axes.** Now, apply Newton's second law, $\sum \vec{F} = ma$, in component form. Check your dimensions to make sure that all terms have units of force.

(iv) **Solve the component equations** for the unknowns. Remember that you must have as many independent equations as you have unknowns in order to obtain a complete solution.

(v) It is a good idea to check the predictions of your solutions for extreme values of the variables. You can often detect errors in your results by doing so.

4. Linear Momentum

The linear momentum of a body is defined as the product of the mass of the body and its velocity i.e.

Linear momentum = mass × velocity

If a body of mass m is moving with a velocity $\vec{v}$, its linear momentum $\vec{p}$ is given by

$$\boxed{\vec{p} = m\,\vec{v}}$$

Linear momentum is a vector quantity. Its direction is the same as the direction of velocity of the body.

The SI unit of linear momentum is kg ms^{-1} and the cgs unit of linear momentum is g cm s^{-1}. Dimension : $[MLT^{-1}]$

5. Pseudo Force

It is a fictitious force observed only in non-inertial frames of reference. In a non-initial frame, it acts on a body in a direction opposite to the acceleration of the frame of reference.

If observer O is non-inertial with acceleration $\vec{a}_0$ and still wants to apply Newton's Second Law on particle P, then observer has to add a "Pseudo force" in addition to real forces on particle P.

$$\boxed{\vec{F}_{Pseudo} = -m_P\vec{a}_0}$$

Thus, Newton Second Law with respect to O will be

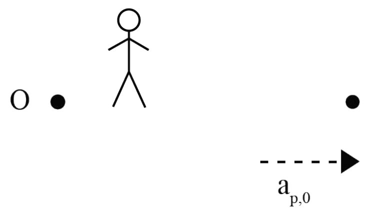


Fig. 4.4

$$\vec{F}_{\text{Real}} + \vec{F}_{Pseudo} = m_P\vec{a}_{P,O}$$

i.e., $\vec{F}_{\text{Real}} - m_P\vec{a}_O = m_P\vec{a}_{P,O}$

Where $\vec{a}_{P,O}$ is acceleration of P with respect to observer O.

NOTE:

If observer is in rotating frame, then Pseudo force is called "Centrifugal force".

Remember : Pseudo force is required only and only if observer is non-inertial. e.g.

(i) Study of motion with respect to accelerating lift.

(ii) Study of motion with respect to accelerating wedge.

6. Apparent Weight in an Accelerated Lift

(a) **When the lift is at rest or moving with uniform velocity, i.e., a = 0 :**

mg– R = 0 or R = mg ∴ $W_{app.} = W_0$

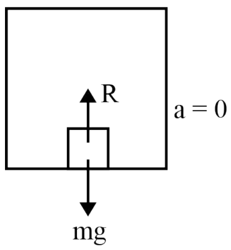


Fig. 4.5

(Where $W_{app.}$ = R = reaction of supporting surface or reading of a weighing machine and W_0= mg = true weight.)

(b) When the lift moves upwards with an acceleration a :

$$R - mg = ma \text{ or } R = m(g + a) = mg\left(1 + \frac{a}{g}\right)$$

$$\therefore W_{app.} = W_0\left(1 + \frac{a}{g}\right)$$

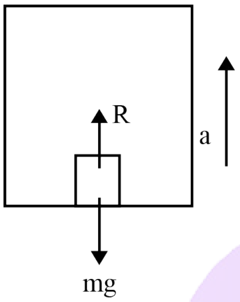


Fig. 4.6

(c) When the lift moves downwards with an acceleration a :

$$mg - R = ma \text{ or } R = m(g - a) = mg\left(1 - \frac{a}{g}\right)$$

$$\therefore W_{app.} = W_0\left(1 - \frac{a}{g}\right)$$

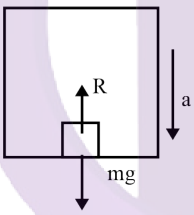


Fig. 4.7

Here, if a > g, $W_{app.}$ will be negative. Negative apparent weight will mean that the body is pressed against the roof of the lift instead of floor.

(d) When the lift falls freely, i.e., a = g :

$R = m(g - g) = 0 \;\therefore\; W_{app.} = 0$

7. Problem of Monkey Climbing a Rope

Let T be the tension in the rope.

(i) When the monkey climbs up with uniform speed : T = mg.

(ii) When the monkey moves up with an acceleration a : T – mg = ma or T = m (g + a).

(iii) When the monkey moves down with an acceleration a : mg – T = ma or T = m (g – a).

8. Problem of a Mass Suspended from a Vertical String

Following cases are possible:

(a) If the carriage (say lift) is at rest or moving uniformly (in translatory equilibrium), then $T_0 = mg$.

(b) If the carriage is accelerated up with an acceleration a, then

$$T = m(g + a) = mg\left(1 + \frac{a}{g}\right) = T_0\left(1 + \frac{a}{g}\right)$$

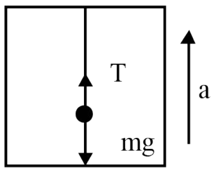


Fig. 4.8

(c) **If the carriage is accelerated down with an acceleration a,** then

$$T = m(g - a) = mg\left(1 - \frac{a}{g}\right) = T_0\left(1 - \frac{a}{g}\right)$$

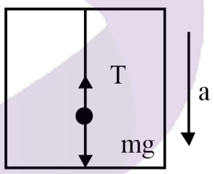


Fig. 4.9

(d) If the carriage begins to fall freely, then the tension in the string becomes zero.

(e) If the carriage is accelerated horizontally, then

(i) mass m experiences a pseudo force ma opposite to acceleration;

(ii) the mass m is in equilibrium inside the carriage and

$T\sin\theta = ma$, $T\cos\theta = mg$, i.e.,

$T = m\sqrt{g^2 + a^2}$;

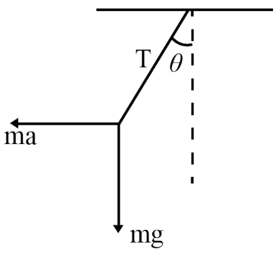


Fig. 4.10

(iii) the string does not remain vertical but inclines to the vertical at an angle $\theta = \tan^{-1}(a/g)$ opposite to acceleration;

(iv) This arrangement is called accelerometer and can be used to determine the acceleration of a moving carriage from inside by noting the deviation of a plumbline suspended from it from the vertical.

9. Constraint Relation

Let us try to visualize this situation

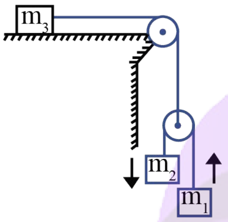


Fig. 4.11

(i) If m_3 was stationary, then magnitude of displacements of m_1 and m_2 would be same and in opposite direction.

Let us say x (displacement of m_1 and m_2 when m_3 is stationary).

(ii) Now consider the case when m_3 displaces by x_1, then

net displacement of $m_1 = x_1 - x$

$m_2 = x_1 + x$

$m_3 = x_1$

(iii) Differentiate it twice we have

$$a_{m_3} = a_1$$

$$a_{m_1} = a_1 - a$$

$$a_{m_2} = a_1 + a$$

This problem can be approached in other way. Which is more mathematical and do not require much of visualisation.

Steps involved to approach problems of multiple pulleys of system having different accelerations:

(i) Define a fixed point/axis.

(ii) Locate positions of all movable points from fixed point/axis.

(iii) (a) Write down the relation between length of the string and the position of different movable points.

(b) No. of relation must be equal to no. of string.

(iv) Differentiate it twice to get the relationship between acceleration of different objects.

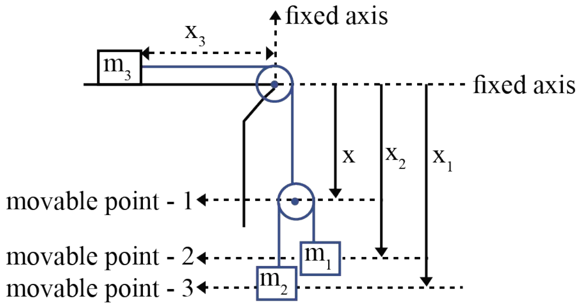


Fig. 4.12

For string connecting m_1 and m_2 :

Let the length of the string be l_1

$$l_1 - (x_2 - x) + (x_1 - x) + \pi r$$

l_1 ↓ constant; πr ↓ constant

Fig. 4.13

On differentiating it twice :

$$0 = (a_2 - a) + (a_1 - a) + 0 \Rightarrow a = \frac{a_1 + a_2}{2}$$

For string connecting m_3 and pulley :

Let the string length be l_2

$$l_2 = x + x_3$$

l_2 ↓ constant; x ↓ length is increasing; x_3 ↓ length is decreasing

Fig. 4.14

NOTE:

If length is decreasing, then differentiation of that length will be negative.

∴ On differentiating twice, we have

$0 = a + (-a_3)$

$a = a_3$

Now, we can apply F = ma for different blocks.

Solve for a_3, a_1, a_2 and Tension.

10. Friction

Friction is an opposing force that comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body.

Thus force of friction is the force that develops at the surfaces of contact of two bodies and impedes (opposes) their relative motion.

(i) Frictional force is independent of the area of contact. This is because with increase in area of contact, force of adhesion also increases (in the same ratio). And the adhesive pressure responsible for friction, remains the same.

(ii) When the surfaces in contact are extra smooth, distance between the molecules of the surfaces in contact decreases, increasing the adhesive force between them. Therefore, the adhesive pressure increases, and so does the force of friction.

10.1 Static Friction, Limiting Friction and Kinetic Friction

The opposing force that comes into play when one body tends to move over the surface of another, but the actual relative motion has yet not started is called Static friction.

Limiting friction is the maximum opposing force that comes into play, when one body is just at the verge of moving over the surface of the other body.

Kinetic friction or dynamic friction is the opposing force that comes into play when one body is actually moving over the surface of another body.

NOTE:

Kinetic friction is always slightly less than the limiting friction.

W_x - applied force

f - friction force

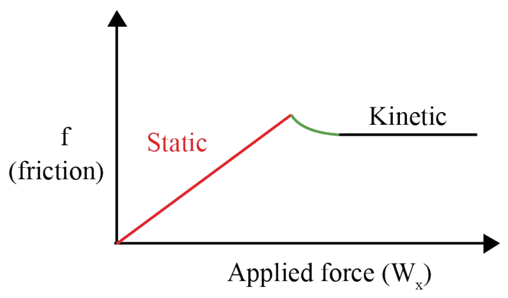


Fig. 4.15

10.2 Laws of Limiting Friction

(a) Static Friction

(i) The force of friction always acts in a direction opposite to the direction of relative motion, i.e., friction is of perverse nature.

(ii) The maximum force of static friction, f_{ms} (called limiting friction) is directly proportional to the normal reaction (R) between the two surfaces in contact. i.e.,

$$f_{ms} \propto N \qquad ...(1)$$

(iii) The force of limiting friction depends upon the nature and the state of polish of the two surfaces in contact and it acts tangential to the interface between the two surfaces.

(iv) The force of limiting friction is independent of the extent of the area of the surfaces in contact as long as the normal reaction remains the same.

10.3 Coefficient of Static Friction

We know that, $f_{ms} \propto N$ or $f_{ms} = \mu_s N$

$$\text{or } \mu_s = \frac{f_{ms}}{N} \qquad ...(2)$$

Here, μ_s is a constant of proportionality and is called the coefficient of static friction. Thus : Coefficient of static friction for any pair of surfaces in contact is equal to the ratio of the limiting friction and the normal reaction. μ_s, being a pure ratio, has got no units and its value depends upon the nature of the surfaces in contact. Further, μ_s, is usually less than unity and is never equal to zero.

Since the force of static friction (f_s) can have any value from zero to maximum (f_{ms}), i.e. $f_s \le f_{ms}$, eqn. (2) is generalised to

$$f_s \le \mu_s N \qquad ...(3)$$

10.4 Kinetic Friction

The laws of kinetic friction are exactly the same as those for static friction. Accordingly, the force of kinetic friction is also directly proportional to the normal reaction, i.e.,

$$f_k \propto N \quad or \quad f_k = \mu_k N \qquad ...(4)$$

μ_k is coefficient of kinetic friction. $\mu_k < \mu_s$.

10.5 Rolling Friction

The opposing force that comes into play when a body rolls over the surface of another body is called the rolling friction.

Cause of rolling friction: Let us consider a wheel which is rolling along a road. As the wheel rolls along the road, it slightly presses into the surface of the road and is itself slightly compressed as shown in Fig.

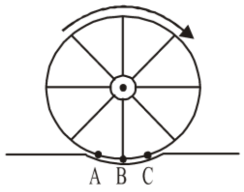


Fig. 4.16

Thus, a rolling wheel:

(i) constantly climbs a 'hill' (BC) in front of it, and

(ii) has to simultaneously get itself detached from the road (AB) behind it. The force of adhesion between the wheel and the road opposes this process.

Both these processes are responsible for rolling friction.

10.6 Angle of Friction

The angle of friction between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction F and normal reaction R makes with the direction of normal reaction R.

It is represented by θ.

In fig. OA represents the normal reaction R which balances the weight mg of the body. OB represent F, the limiting force of sliding friction, when the body tends to move to the right. Complete the parallelogram OACB. Join OC. This represents the resultant of R and F. By definition, ∠AOC = θ is the angle of friction between the two bodies in contact.

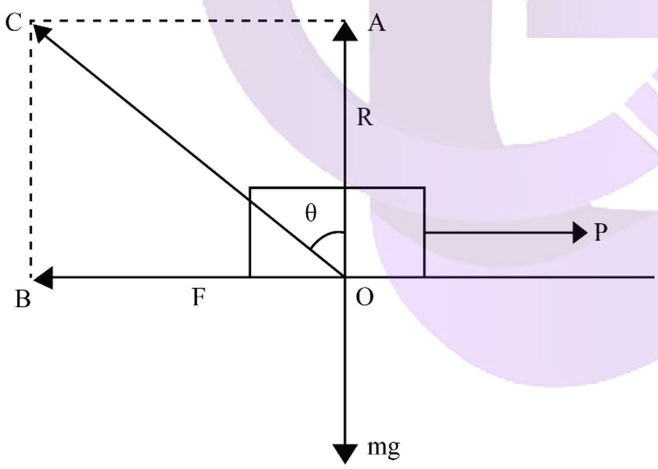


Fig. 4.17

The value of angle of friction depends on the nature of materials of the surfaces in contact and the nature of the surfaces.

Relation between μ and θ

In Δ AOC, $\tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{F}{R} = \mu$ ...(5)

Hence $\boxed{\mu = \tan\theta}$ ...(6)

i.e. coefficient of limiting friction between any two surfaces in contact is equal to tangent of the angle of friction between them.

10.7 Angle of Repose or Angle of Sliding

Angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down.

It is represented by α. Its value depends on material and nature of the surfaces in contact.

In fig., AB is an inclined plane such that a body placed on it just begins to slide down. ∠BAC (α) = angle of repose.

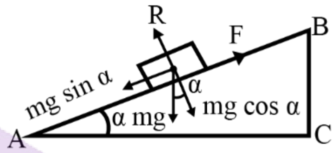


Fig. 4.18

The various forces involved are :

(i) weight, mg of the body, acting vertically downwards,

(ii) normal reaction, R, acting perpendicular to AB,

(iii) Force of friction F, acting up the plane AB.

Now, mg can be resolved into two rectangular components : mg cos α opposite to R and mg sin α opposite to F. In equilibrium,

F = mg sin α ... (7)

R = mg cos α ... (8)

Dividing (7) by (8), we get

$$\frac{F}{R} = \frac{mg\sin\alpha}{mg\cos\alpha}, \text{ i.e., } \boxed{\mu = \tan\alpha}$$

Hence coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

NOTE:

Combining (6) and (9), we obtain

μ = tan θ = tan α

$\therefore \quad \boxed{\theta = \alpha}$

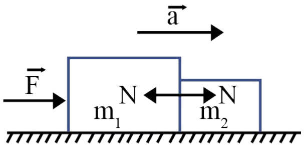
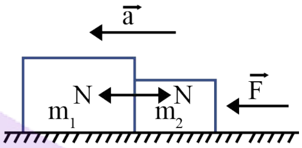
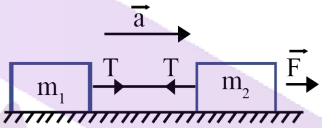
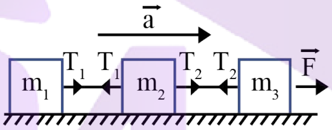
i.e. angle of friction is equal to angle of repose.

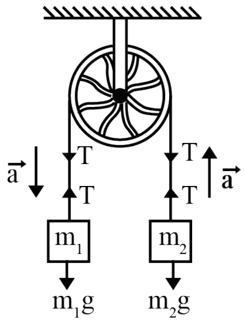
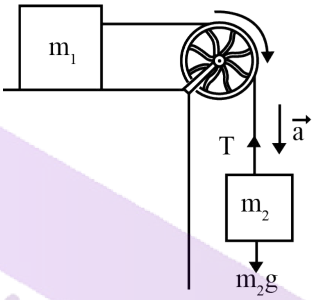
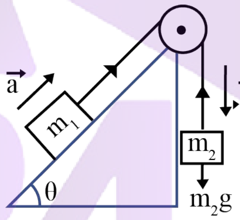
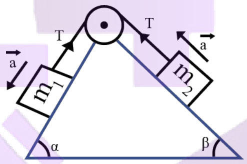
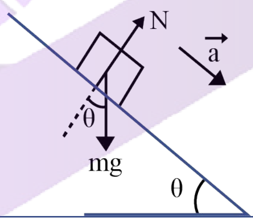
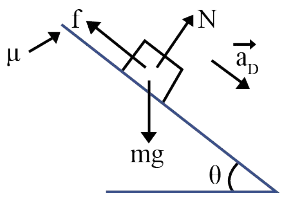
10.8 Method of Changing Friction

Some of the ways of reducing friction are:

(i) By polishing.
(ii) By lubrication.
(iii) By proper selection of materials.
(iv) By Streamlining.
(v) By using ball bearings.

Some Important Cases

Case	Diagram	Result
(a) When two bodies are kept in contact and force is applied on the body of mass m_1.		(i) $a = \frac{F}{m_1 + m_2}$ (ii) $N = \frac{m_2 F}{m_1 + m_2}$
(b) When two bodies are kept in contact and force is applied on the body of mass m_2.		(i) $a = \frac{F}{m_1 + m_2}$ (ii) $N' = \frac{m_1 F}{m_1 + m_2}$
(c) When two bodies are connected by a string and placed on a smooth horizontal surface.		(i) $a = \frac{F}{m_1 + m_2}$ (ii) $T = \frac{m_1 F}{m_1 + m_2}$
(d) When three bodies are connected through strings as shown in fig and placed on a smooth horizontal surface.		(i) $a = \frac{F}{(m_1 + m_2 + m_3)}$ (ii) $T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$ (iii) $T_2 = \frac{(m_1 + m_2) F}{(m_1 + m_2 + m_3)}$

(e) When two bodies of masses m_1 & m_2 are attached at the ends of a string passing over a pulley as shown in the figure		(i) $a = \dfrac{(m_1 - m_2)g}{(m_1 + m_2)}$ (ii) $T = \left(\dfrac{2m_1m_2}{m_1 + m_2}\right)g$
(f) When two bodies of masses m_1 & m_2 are attached at the ends of a string passing over a pulley in such a way that mass m_1 rests on a smooth horizontal table and mass m_2 is hanging vertically.		(i) $a = \dfrac{m_2 g}{(m_1 + m_2)}$, (ii) $T = \dfrac{m_1 m_2 g}{(m_1 + m_2)}$
(g) If in the above case, mass m_1 is placed on a smooth inclined plane making an angle θ with horizontal as shown in		(i) $a = \dfrac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2}$ (ii) $T = \dfrac{m_1 m_2\, g(1 + \sin\theta)}{(m_1 + m_2)}$ (iii) If the system remains in equilibrium, then $m_1 g \sin\theta = m_2 g$
(h) If masses m_1 and m_2 are placed on inclined planes making angles α & β with the horizontal respectively, then		(i) $a = \dfrac{g(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)}$ (ii) $T = \dfrac{m_1 m_2}{(m_1 + m_2)}(\sin\alpha + \sin\beta)g$
(i) When a body is moving on smooth inclined plane.		$a = g \sin\theta$, $N = mg\cos\theta$
(j) When a body is moving down on a rough inclined plane.		$a_D = g(\sin\theta - \mu\cos\theta)$

NCERT CORNER

(Some important points to remember)

1. Aristotle's view that a force is necessary to keep a body in uniform motion is wrong. A force is necessary in practice to counter the opposing force of friction.

2. Newton's first law of motion: "Everybody continues to be in its state of rest or of uniform motion in a straight line, unless compelled by some external force to act otherwise". In simple terms, the First Law is **"If external force on a body is zero, its acceleration is zero".**

3. Momentum (p) of a body is the product of its mass (m) and velocity (v): p = mv

4. Newton's second law of motion: The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. Thus

$$F = k\frac{dp}{dt} = kma$$

where F is the net external force on the body and a its acceleration. We set the constant of proportionality k = 1 in SI units. Then

$$F = \frac{dp}{dt} = ma$$

The SI unit of force is newton : 1 N = 1 kg m s^{-2} .
(a) The second law is consistent with the First Law (F = 0 implies a = 0)
(b) It is a vector equation
(c) It is applicable to a particle, and to a body or a system of particles, provided F is the total external force on the system and a is the acceleration of the system.

5. Impulse is the product of force and time which equals change in momentum. The notion of impulse is useful when a large force acts for a short time to produce a measurable change in momentum. Since the time of action of the force is very short, one can assume that there is no appreciable change in the position of the body during the action of the impulsive force.

6. Newton's third law of motion: To every action, there is always an equal and opposite reaction In simple terms, the law can be stated thus: Forces in nature always occur between pairs of bodies. Force on a body A by body B is equal and opposite to the force on the body B by A. Action and reaction forces are simultaneous forces. There is no cause-effect relation between action and reaction. Any of the two mutual forces can be called action and the other reaction. Action and reaction act on different bodies and so they cannot be cancelled out. The internal action and reaction forces between different parts of a body do, however, sum to zero.

7. Law of Conservation of Momentum The total momentum of an isolated system of particles is conserved. The law follows from the second and third law of motion.

8. Frictional force opposes (impending or actual) relative motion between two surfaces in contact. It is the component of the contact force along the common tangent to the surface in contact. Static friction f_s opposes impending relative motion; kinetic friction f_k opposes actual relative motion. They are independent of the area of contact and satisfy the following approximate laws:

$$f_S \le (f_s)_{max} = \mu_S R$$

$$f_k = \mu_k R$$

μ_s(co-efficient of static friction) and μ_k (co-efficient of kinetic friction) are constants characteristic of the pair of surfaces in contact. It is found experimentally that μ_k is less than μ_s.

Physics

Class11th NEET

04

LAW OF MOTION & FRICTION

Chapter 04

Laws of Motion and Friction

1. Force

(a) A force is something which changes or tends to change the state of rest or motion of a body. It causes a body to start moving if it is at rest or stop it, if it is in motion or deflect it from its initial path of motion.

(b) Force is also defined as an interaction between two bodies. Two bodies can also exert force on each other even without being in physical contact, e.g., electric force between two charges, gravitational force between any two bodies of the universe.

(c) Force is a vector quantity having SI unit Newton (N) and dimension $[MLT^{-2}]$.

(d) **Superposition of force:** When many forces are acting on a single body, the resultant force is obtained by using the laws of vector addition. $\vec{F} = \vec{F_1} + \vec{F_2} + ...\vec{F_n}$

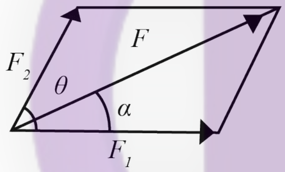


Fig. 4.1

The resultant of the two forces $\vec{F_1}$ and $\vec{F_2}$ acting at an angle θ is given by:

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

The resultant force is directed at an angle α with respect to force F_1 where $\tan\alpha = \dfrac{F_2\sin\theta}{F_1 + F_2\cos\theta}$

(e) **Lami's theorem :** If three forces F_1, F_2 and F_3 are acting simultaneously on a body and the body is in equilibrium, then according to Lami's theorem,

$$\frac{F_1}{\sin(\pi-\alpha)} = \frac{F_2}{\sin(\pi-\beta)} = \frac{F_3}{\sin(\pi-\gamma)}$$

where α, β and γ are the angles opposite to the forces F_1, F_2 & F_3 respectively.

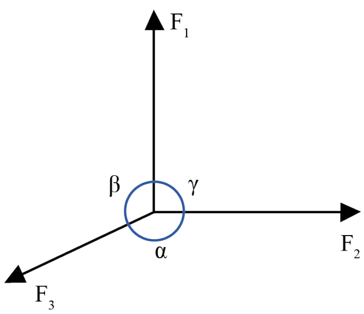


Fig. 4.2

2. Types of Force

There are, basically, four forces, which are commonly encountered in mechanics.

(a) **Weight :** Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

(b) **Contact Force :** When two bodies come in contact they exert forces on each other that are called contact forces.

(i) **Normal Force (N):** It is the component of contact force normal to the surface. It measures how strongly the surfaces in contact are pressed together.

(ii) **Frictional Force (f) :** It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.

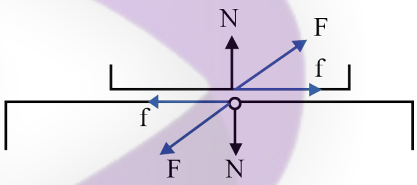


Fig. 4.3

(c) **Tension:** The force exerted by the ends of a taut string, rope or chain is called the tension. The direction of tension is so as to pull the body while that of normal reaction is to push the body.

(d) **Spring Force:** Every spring resists any attempt to change its length; the more you alter its length the harder it resists. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1}).

3. Newton's Laws of Motion

3.1 First law of Motion

(a) Everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by a resultant force to change that state

(b) This law is also known as **law of inertia**. Inertia is the property of inability of a body to change its position of rest or uniform motion in a straight line unless some external force acts on it.

(c) Mass is a measure of inertia of a body.

(d) A frame of reference in which Newton's first law is valid is called **inertial frame**, i.e., if a frame of reference is at rest or in uniform motion it is called **inertial**, otherwise **non-inertial.**

3.2 Second Law of Motion

(a) This law gives the magnitude of force.

(b) According to second law of motion, rate of change of momentum of a body is directly proportional to the resultant force acting on the body, i.e.,

$$\vec{F} \propto \left(\frac{d\vec{p}}{dt}\right)$$

$$\vec{F} = K\frac{d\vec{p}}{dt}$$

Here, the change in momentum takes place in the direction of the applied resultant force. Momentum, $\vec{p} = m\vec{v}$ is a measure of sum of the motion contained in the body.

(c) **Unit force :** It is defined as the force which changes the momentum of a body by unity in unit time. According to this, K=1

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}.$$

If the mass of the system is finite and remains constant w.r.t. time, then (dm/dt) = 0 and

$$\vec{F} = m\left(\frac{d\vec{v}}{dt}\right) = m\vec{a} = \left(\frac{\vec{p}_2 - \vec{p}_1}{t}\right)$$

(d) External force acting on a body may accelerate it either by changing the magnitude of velocity or direction of velocity or both.

(i) **If the force is parallel to the motion**, it changes only the magnitude of velocity but not the direction. So, the path followed by the body is a **straight line**.

(ii) **If the force is acting perpendicular to the motion of body,** it changes only the direction but not the magnitude of velocity. So, the path followed by the body is a **circle** (uniform circular motion).

(iii) **If the force acts at an angle to the motion of a body,** it changes both the magnitude and direction of $\vec{v}$. In this case path followed by the body may be **elliptical, non-uniform circular, parabolic or hyperbolic.**

3.3 Third Law of Motion

(a) According to this law, for every action there is an equal and opposite reaction. When two bodies A and B exert force on each other, the force by A on B (i.e., action represented by $\vec{F}_{AB}$), is always equal and opposite to the force by B on A (i.e., reaction represented $\vec{F}_{BA}$). Thus, $\vec{F}_{AB} = -\vec{F}_{BA}$.

(b) The two forces involved in any interaction between two bodies are called **action and reaction.** But we cannot say that a particular force is action and the other one is reaction.

(c) Action and Reaction force always acts on different bodies.

3.4 Some Important Points Concerning Newton's Laws of Motion

(a) The forces of interaction between bodies composing a system are called **internal forces.** The forces exerted on bodies of a given system by bodies situated outside are called **external forces.**

(b) Whenever one force acts on a body it gives rise to another force called reaction i.e., **a single isolated force** is physically impossible. This is why **total internal force in an isolated system is always zero.**

(c) According to Newton's second law, $\vec{F} = \left(\frac{d\vec{p}}{dt}\right)$.

$$\text{If } \vec{F} = 0, \left(\frac{d\vec{p}}{dt}\right) = 0 \text{ or } \left(\frac{d\vec{v}}{dt}\right) = 0$$

or $\vec{v}$ = constant or zero,

i.e., a body remains at rest or moves with uniform velocity unless acted upon by an external force. This is Newton's I[st] law.

(d) Newton's second law can also be expressed as:

$Ft = p_2 - p_1$. Hence, if a car and a truck are initially moving with the same momentum, then by the application of same breaking force, both will come to rest in the same time.

(e) The second law is a vector law. it is equivalent to three equations : $F_x = ma_x$; $F_y = ma_y$; $F_z = ma_z$. A force can only change the component of velocity in its direction. It has no effect on the component perpendicular to it.

(f) $\vec{F} = m\vec{a}$ is a local relation. The force at a point on space at any instant is related to the acceleration at that instant. Example: An object on an accelerated balloon will have acceleration of balloon. The moment it is dropped, it will have acceleration due to gravity.

3.5 Applications of Newton's Laws of Motion

There are two kinds of problems in classical mechanics :

(a) To find unknown forces acting on a body, given the body's acceleration.

(b) To predict the future motion of a body, given the body's initial position and velocity and the forces acting on it. For either kind of problem, we use Newton's second law . The following general strategy is useful for solving such problems :

(i) Draw a simple, neat diagram of the system.

(ii) Isolate the object of interest whose motion is being analyzed. Draw a **free body diagram** for this object, that is, a diagram showing all external forces acting on the object. For systems containing more than one object, draw separate diagrams for each objects. Do not include forces that the object exerts on its surroundings.

(iii) Establish convenient coordinate axes for each body and find the **components of the forces along these axes.** Now, apply Newton's second law, $\sum \vec{F} = ma$, in component form. Check your dimensions to make sure that all terms have units of force.

(iv) **Solve the component equations** for the unknowns. Remember that you must have as many independent equations as you have unknowns in order to obtain a complete solution.

(v) It is a good idea to check the predictions of your solutions for extreme values of the variables. You can often detect errors in your results by doing so.

4. Linear Momentum

The linear momentum of a body is defined as the product of the mass of the body and its velocity i.e.

Linear momentum = mass × velocity

If a body of mass m is moving with a velocity $\vec{v}$, its linear momentum $\vec{p}$ is given by

$$\boxed{\vec{p} = m\,\vec{v}}$$

Linear momentum is a vector quantity. Its direction is the same as the direction of velocity of the body.

The SI unit of linear momentum is kg ms^{-1} and the cgs unit of linear momentum is g cm s^{-1}. Dimension : $[MLT^{-1}]$

5. Pseudo Force

It is a fictitious force observed only in non-inertial frames of reference. In a non-initial frame, it acts on a body in a direction opposite to the acceleration of the frame of reference.

If observer O is non-inertial with acceleration $\vec{a}_0$ and still wants to apply Newton's Second Law on particle P, then observer has to add a "Pseudo force" in addition to real forces on particle P.

$$\boxed{\vec{F}_{Pseudo} = -m_P\vec{a}_0}$$

Thus, Newton Second Law with respect to O will be

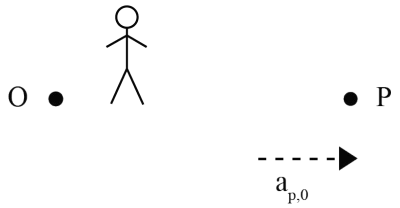


Fig. 4.4

$$\vec{F}_{\text{Real}} + \vec{F}_{Pseudo} = m_P\vec{a}_{P,O}$$

i.e., $\vec{F}_{\text{Real}} - m_P\vec{a}_O = m_P\vec{a}_{P,O}$

Where $\vec{a}_{P,O}$ is acceleration of P with respect to observer O.

NOTE:

> If observer is in rotating frame, then Pseudo force is called "Centrifugal force".
>
> **Remember :** Pseudo force is required only and only if observer is non-inertial. e.g.
>
> (i) Study of motion with respect to accelerating lift.
>
> (ii) Study of motion with respect to accelerating wedge.

6. Apparent Weight in an Accelerated Lift

(a) **When the lift is at rest or moving with uniform velocity, i.e., a = 0 :**

mg– R = 0 or R = mg ∴ $W_{app.} = W_0$

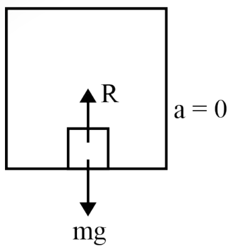


Fig. 4.5

(Where $W_{app.}$ = R = reaction of supporting surface or reading of a weighing machine and W_0= mg = true weight.)

(b) When the lift moves upwards with an acceleration a :

$R - mg = ma$ or $R = m(g + a) = mg\left(1+\frac{a}{g}\right)$

$\therefore W_{app.} = W_0\left(1+\frac{a}{g}\right)$

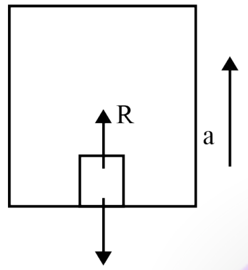


Fig. 4.6

(c) When the lift moves downwards with an acceleration a :

$mg - R = ma$ or $R = m(g-a) = mg\left(1-\frac{a}{g}\right)$

$\therefore W_{app.} = W_0\left(1-\frac{a}{g}\right)$

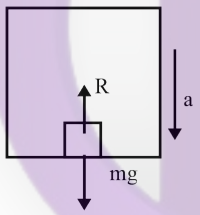


Fig. 4.7

Here, if $a > g$, $W_{app.}$ will be negative. Negative apparent weight will mean that the body is pressed against the roof of the lift instead of floor.

(d) When the lift falls freely, i.e., a = g :

$R = m(g - g) = 0 \quad \therefore \quad W_{app.} = 0$

7. Problem of Monkey Climbing a Rope

Let T be the tension in the rope.

(i) When the monkey climbs up with uniform speed : T = mg.

(ii) When the monkey moves up with an acceleration a : $T - mg = ma$ or $T = m(g + a)$.

(iii) When the monkey moves down with an acceleration a : $mg - T = ma$ or $T = m(g - a)$.

8. Problem of a Mass Suspended from a Vertical String

Following cases are possible:

(a) If the carriage (say lift) is at rest or moving uniformly (in translatory equilibrium), then $T_0 = mg$.

(b) If the carriage is accelerated up with an acceleration a, then

$T = m(g + a) = mg\left(1+\frac{a}{g}\right) = T_0\left(1+\frac{a}{g}\right)$

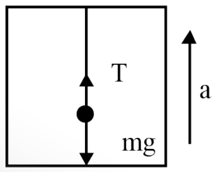


Fig. 4.8

(c) **If the carriage is accelerated down with an acceleration a,** then

$T = m(g - a) = mg\left(1-\frac{a}{g}\right) = T_0\left(1-\frac{a}{g}\right)$

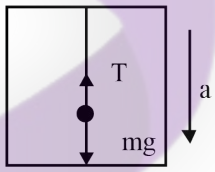


Fig. 4.9

(d) If the carriage begins to fall freely, then the tension in the string becomes zero.

(e) If the carriage is accelerated horizontally, then

(i) mass m experiences a pseudo force ma opposite to acceleration;

(ii) the mass m is in equilibrium inside the carriage and

$T\sin\theta = ma$, $T\cos\theta = mg$, i.e.,

$T = m\sqrt{g^2 + a^2}$;

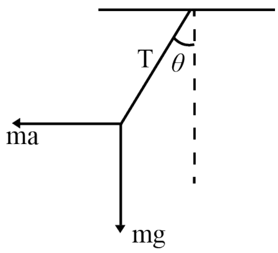


Fig. 4.10

(iii) the string does not remain vertical but inclines to the vertical at an angle $\theta = \tan^{-1}(a/g)$ opposite to acceleration;

(iv) This arrangement is called accelerometer and can be used to determine the acceleration of a moving carriage from inside by noting the deviation of a plumbline suspended from it from the vertical.

9. Constraint Relation

Let us try to visualize this situation

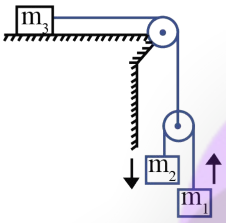


Fig. 4.11

(i) If m_3 was stationary, then magnitude of displacements of m_1 and m_2 would be same and in opposite direction.

Let us say x (displacement of m_1 and m_2 when m_3 is stationary).

(ii) Now consider the case when m_3 displaces by x_1, then

net displacement of $m_1 = x_1 - x$

$m_2 = x_1 + x$

$m_3 = x_1$

(iii) Differentiate it twice we have

$$a_{m_3} = a_1$$

$$a_{m_1} = a_1 - a$$

$$a_{m_2} = a_1 + a$$

This problem can be approached in other way. Which is more mathematical and do not require much of visualisation.

Steps involved to approach problems of multiple pulleys of system having different accelerations:

(i) Define a fixed point/axis.

(ii) Locate positions of all movable points from fixed point/axis.

(iii) (a) Write down the relation between length of the string and the position of different movable points.

(b) No. of relation must be equal to no. of string.

(iv) Differentiate it twice to get the relationship between acceleration of different objects.

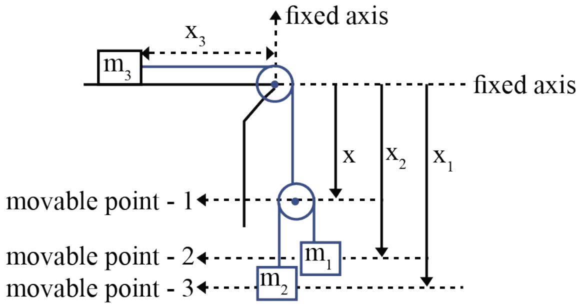


Fig. 4.12

For string connecting m_1 and m_2 :

Let the length of the string be l_1

$$l_1 - (x_2 - x) + (x_1 - x) + \pi r$$

$\downarrow$ constant $\qquad\qquad$ $\downarrow$ constant

Fig. 4.13

On differentiating it twice :

$$0 = (a_2 - a) + (a_1 - a) + 0 \Rightarrow \quad a = \frac{a_1 + a_2}{2}$$

For string connecting m_3 and pulley :

Let the string length be l_2

$$l_2 = x + x_3$$

l_2 $\downarrow$ constant; x $\downarrow$ length is increasing; x_3 $\downarrow$ length is decreasing

Fig. 4.14

NOTE:

If length is decreasing, then differentiation of that length will be negative.

∴ On differentiating twice, we have

$0 = a + (-a_3)$

$a = a_3$

Now, we can apply F = ma for different blocks.

Solve for a_3, a_1, a_2 and Tension.

10. Friction

Friction is an opposing force that comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body.

Thus force of friction is the force that develops at the surfaces of contact of two bodies and impedes (opposes) their relative motion.

(i) Frictional force is independent of the area of contact. This is because with increase in area of contact, force of adhesion also increases (in the same ratio). And the adhesive pressure responsible for friction, remains the same.

(ii) When the surfaces in contact are extra smooth, distance between the molecules of the surfaces in contact decreases, increasing the adhesive force between them. Therefore, the adhesive pressure increases, and so does the force of friction.

10.1 Static Friction, Limiting Friction and Kinetic Friction

The opposing force that comes into play when one body tends to move over the surface of another, but the actual relative motion has yet not started is called Static friction.

Limiting friction is the maximum opposing force that comes into play, when one body is just at the verge of moving over the surface of the other body.

Kinetic friction or dynamic friction is the opposing force that comes into play when one body is actually moving over the surface of another body.

NOTE:

Kinetic friction is always slightly less than the limiting friction.

W_x - applied force

f - friction force

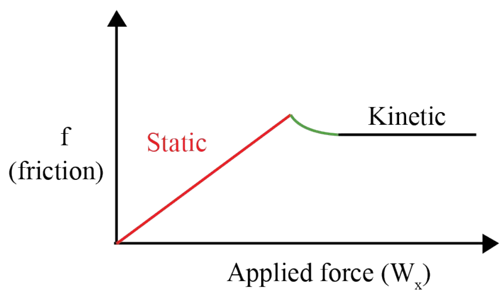


Fig. 4.15

10.2 Laws of Limiting Friction

(a) Static Friction

(i) The force of friction always acts in a direction opposite to the direction of relative motion, i.e., friction is of perverse nature.

(ii) The maximum force of static friction, f_{ms} (called limiting friction) is directly proportional to the normal reaction (R) between the two surfaces in contact. i.e.,

$$f_{ms} \propto N \qquad ...(1)$$

(iii) The force of limiting friction depends upon the nature and the state of polish of the two surfaces in contact and it acts tangential to the interface between the two surfaces.

(iv) The force of limiting friction is independent of the extent of the area of the surfaces in contact as long as the normal reaction remains the same.

10.3 Coefficient of Static Friction

We know that, $f_{ms} \propto N$ or $f_{ms} = \mu_s N$

or $\mu_s = \dfrac{f_{ms}}{N}$ ...(2)

Here, μ_s is a constant of proportionality and is called the coefficient of static friction. Thus : Coefficient of static friction for any pair of surfaces in contact is equal to the ratio of the limiting friction and the normal reaction. μ_s, being a pure ratio, has got no units and its value depends upon the nature of the surfaces in contact. Further, μ_s, is usually less than unity and is never equal to zero.

Since the force of static friction (f_s) can have any value from zero to maximum (f_{ms}), i.e. $f_s \leq f_{ms}$, eqn. (2) is generalised to

$f_s \leq \mu_s N$ ...(3)

10.4 Kinetic Friction

The laws of kinetic friction are exactly the same as those for static friction. Accordingly, the force of kinetic friction is also directly proportional to the normal reaction, i.e.,

$$f_k \propto N \quad or \quad f_k = \mu_k N \qquad ...(4)$$

μ_k is coefficient of kinetic friction. $\mu_k < \mu_s$.

10.5 Rolling Friction

The opposing force that comes into play when a body rolls over the surface of another body is called the rolling friction.

Cause of rolling friction: Let us consider a wheel which is rolling along a road. As the wheel rolls along the road, it slightly presses into the surface of the road and is itself slightly compressed as shown in Fig.

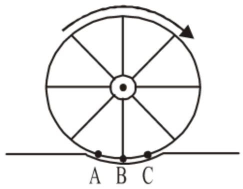


Fig. 4.16

Thus, a rolling wheel:

(i) constantly climbs a 'hill' (BC) in front of it, and

(ii) has to simultaneously get itself detached from the road (AB) behind it. The force of adhesion between the wheel and the road opposes this process.

Both these processes are responsible for rolling friction.

10.6 Angle of Friction

The angle of friction between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction F and normal reaction R makes with the direction of normal reaction R.

It is represented by θ.

In fig. OA represents the normal reaction R which balances the weight mg of the body. OB represent F, the limiting force of sliding friction, when the body tends to move to the right. Complete the parallelogram OACB. Join OC. This represents the resultant of R and F. By definition, ∠AOC = θ is the angle of friction between the two bodies in contact.

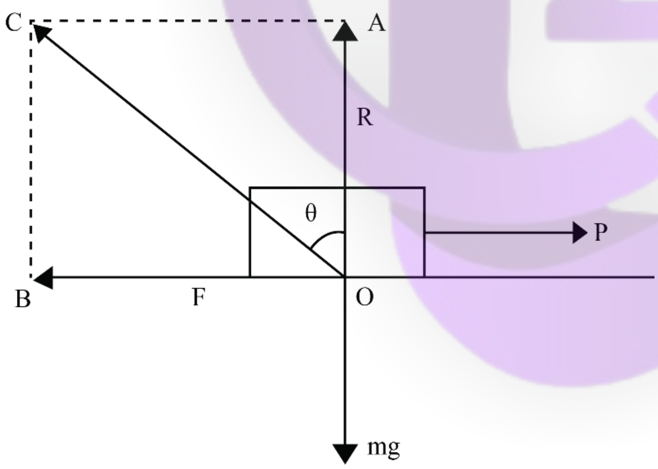


Fig. 4.17

The value of angle of friction depends on the nature of materials of the surfaces in contact and the nature of the surfaces.

Relation between μ and θ

In Δ AOC, $\tan\theta = \dfrac{AC}{OA} = \dfrac{OB}{OA} = \dfrac{F}{R} = \mu$ ...(5)

Hence $\boxed{\mu = \tan\theta}$ ...(6)

i.e. coefficient of limiting friction between any two surfaces in contact is equal to tangent of the angle of friction between them.

10.7 Angle of Repose or Angle of Sliding

Angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down.

It is represented by α. Its value depends on material and nature of the surfaces in contact.

In fig., AB is an inclined plane such that a body placed on it just begins to slide down. ∠BAC (α) = angle of repose.

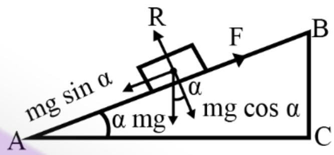


Fig. 4.18

The various forces involved are :

(i) weight, mg of the body, acting vertically downwards,

(ii) normal reaction, R, acting perpendicular to AB,

(iii) Force of friction F, acting up the plane AB.

Now, mg can be resolved into two rectangular components : mg cos α opposite to R and mg sin α opposite to F. In equilibrium,

F = mg sin α ... (7)

R = mg cos α ... (8)

Dividing (7) by (8), we get

$\dfrac{F}{R} = \dfrac{mg\sin\alpha}{mg\cos\alpha}$, i.e., $\boxed{\mu = \tan\alpha}$

Hence coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

NOTE:

Combining (6) and (9), we obtain

μ = tan θ = tan α

∴ $\boxed{\theta = \alpha}$

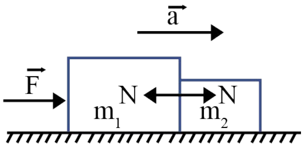
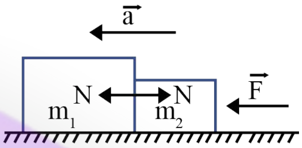
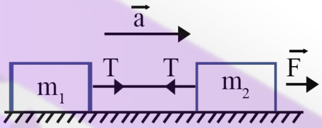
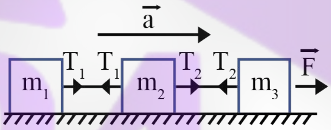
i.e. angle of friction is equal to angle of repose.

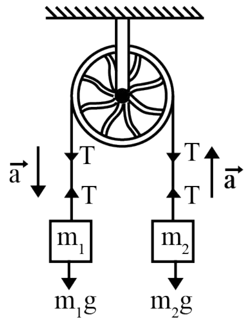
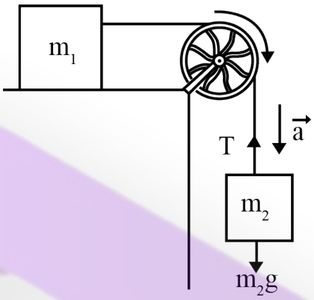
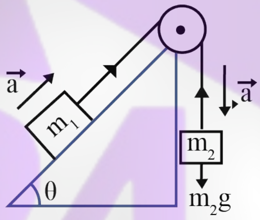
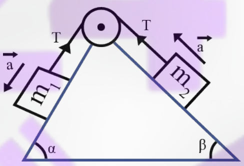
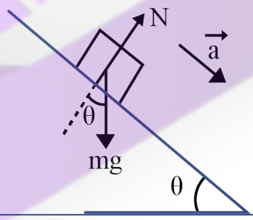
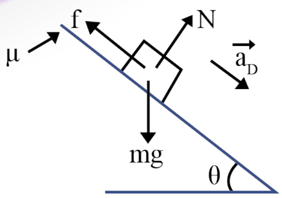
10.8 Method of Changing Friction

Some of the ways of reducing friction are:

(i) By polishing.
(ii) By lubrication.
(iii) By proper selection of materials.
(iv) By Streamlining.
(v) By using ball bearings.

Some Important Cases

Case	Diagram	Result
(a) When two bodies are kept in contact and force is applied on the body of mass m_1.		(i) $a = \dfrac{F}{m_1 + m_2}$ (ii) $N = \dfrac{m_2 F}{m_1 + m_2}$
(b) When two bodies are kept in contact and force is applied on the body of mass m_2.		(i) $a = \dfrac{F}{m_1 + m_2}$ (ii) $N' = \dfrac{m_1 F}{m_1 + m_2}$
(c) When two bodies are connected by a string and placed on a smooth horizontal surface.		(i) $a = \dfrac{F}{m_1 + m_2}$ (ii) $T = \dfrac{m_1 F}{m_1 + m_2}$
(d) When three bodies are connected through strings as shown in fig and placed on a smooth horizontal surface.		(i) $a = \dfrac{F}{(m_1 + m_2 + m_3)}$ (ii) $T_1 = \dfrac{m_1 F}{(m_1 + m_2 + m_3)}$ (iii) $T_2 = \dfrac{(m_1 + m_2) F}{(m_1 + m_2 + m_3)}$

(e) When two bodies of masses m_1 & m_2 are attached at the ends of a string passing over a pulley as shown in the figure		(i) $a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$ (ii) $T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$
(f) When two bodies of masses m_1 & m_2 are attached at the ends of a string passing over a pulley in such a way that mass m_1 rests on a smooth horizontal table and mass m_2 is hanging vertically.		(i) $a = \frac{m_2 g}{(m_1 + m_2)}$, (ii) $T = \frac{m_1 m_2 g}{(m_1 + m_2)}$
(g) If in the above case, mass m_1 is placed on a smooth inclined plane making an angle θ with horizontal as shown in		(i) $a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2}$ (ii) $T = \frac{m_1 m_2 g(1 + \sin\theta)}{(m_1 + m_2)}$ (iii) If the system remains in equilibrium, then $m_1 g \sin\theta = m_2 g$
(h) If masses m_1 and m_2 are placed on inclined planes making angles α & β with the horizontal respectively, then		(i) $a = \frac{g(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)}$ (ii) $T = \frac{m_1 m_2}{(m_1 + m_2)}(\sin\alpha + \sin\beta)g$
(i) When a body is moving on smooth inclined plane.		$a = g \sin\theta$, $N = mg \cos\theta$
(j) When a body is moving down on a rough inclined plane.		$a_D = g(\sin\theta - \mu\cos\theta)$

NCERT CORNER

(Some important points to remember)

1. Aristotle's view that a force is necessary to keep a body in uniform motion is wrong. A force is necessary in practice to counter the opposing force of friction.

2. Newton's first law of motion: "Everybody continues to be in its state of rest or of uniform motion in a straight line, unless compelled by some external force to act otherwise". In simple terms, the First Law is **"If external force on a body is zero, its acceleration is zero".**

3. Momentum (p) of a body is the product of its mass (m) and velocity (v): p = mv

4. Newton's second law of motion: The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. Thus

$$F = k\frac{dp}{dt} = kma$$

where F is the net external force on the body and a its acceleration. We set the constant of proportionality k = 1 in SI units. Then

$$F = \frac{dp}{dt} = ma$$

The SI unit of force is newton : 1 N = 1 kg m s^{-2} .
(a) The second law is consistent with the First Law (F = 0 implies a = 0)
(b) It is a vector equation
(c) It is applicable to a particle, and to a body or a system of particles, provided F is the total external force on the system and a is the acceleration of the system.

5. Impulse is the product of force and time which equals change in momentum. The notion of impulse is useful when a large force acts for a short time to produce a measurable change in momentum. Since the time of action of the force is very short, one can assume that there is no appreciable change in the position of the body during the action of the impulsive force.

6. Newton's third law of motion: To every action, there is always an equal and opposite reaction In simple terms, the law can be stated thus: Forces in nature always occur between pairs of bodies. Force on a body A by body B is equal and opposite to the force on the body B by A. Action and reaction forces are simultaneous forces. There is no cause-effect relation between action and reaction. Any of the two mutual forces can be called action and the other reaction. Action and reaction act on different bodies and so they cannot be cancelled out. The internal action and reaction forces between different parts of a body do, however, sum to zero.

7. Law of Conservation of Momentum The total momentum of an isolated system of particles is conserved. The law follows from the second and third law of motion.

8. Frictional force opposes (impending or actual) relative motion between two surfaces in contact. It is the component of the contact force along the common tangent to the surface in contact. Static friction f_s opposes impending relative motion; kinetic friction f_k opposes actual relative motion. They are independent of the area of contact and satisfy the following approximate laws:

$$f_S \le (f_s)_{max} = \mu_S R$$

$$f_k = \mu_k R$$

μ_s(co-efficient of static friction) and μ_k (co-efficient of kinetic friction) are constants characteristic of the pair of surfaces in contact. It is found experimentally that μ_k is less than μ_s.

Physics 24

05

WORK, ENERGY AND POWER

Chapter 05

Work, Energy and Power

1. Work

Introduction to Work:

In Physics, work stands for ‘mechanical work’.

Work is said to be done by a force when the body is displaced actually through some distance in the direction of the applied force.

However, when there is no displacement in the direction of the applied force, no work is said to be done, i.e., work done is zero, when displacement of the body in the direction of the force is zero.

Suppose a constant force $\vec{F}$ acting on a body produces a displacement $\vec{s}$ in the body along the positive x-direction, as shown in the figure

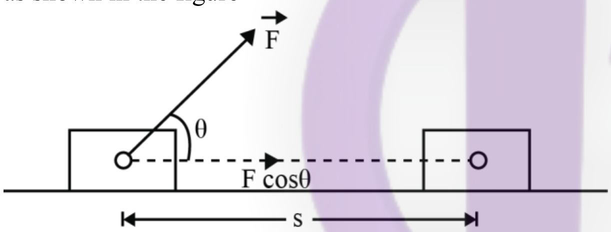


Fig.5.1

If θ is the angle which force makes with the positive x–direction of the displacement, then the component of in the direction of displacement is (F cos θ). As work done by the force is the product of component of force in the direction of the displacement and the magnitude of the displacement,

$$\boxed{W = (F\cos\theta)\,s} \qquad ...(1)$$

If displacement is in the direction of force applied,

$\theta = 0°$. Then from (1), W = (F cos 0°) s = F s

Equation (1) can be rewritten as $\boxed{W = \vec{F}.\vec{s}}$ ...(2)

Thus, work done by a force is the dot product of force and displacement.

In terms of rectangular component, $\vec{F}$ and $\vec{s}$ may written as

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$

From (2), $W = \vec{F}.\vec{s}$

$$W = \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right).\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$\boxed{W = xF_x + yF_y + zF_z}$$

Obviously, work is a scalar quantity, i.e., it has magnitude only and no direction. However, work done by a force can be positive or negative or zero.

Note:

Work done is positive, negative or zero depending upon the angle between force & displacement

1.1. Dimensions and Units of Work

As work = force × distance

$$W = \left(M^1L^1T^{-2}\right) \times L$$

$$W = (M^1\,L^1\,T^{-2}) \times L$$

$$\boxed{W = \left[M^1\,L^2\,T^{-2}\right]}$$

This is the dimensional formula of work.

The **units** of work are of two types:

1. Absolute units 2. Gravitational units

(a) Absolute unit

1. Joule. It is the absolute unit of work in SI.

Work done is said to be one joule, when a force of one newton actually moves a body through a distance of one metre in the direction of applied force.

From $W = Fs\cos\theta$

1 joule = 1 newton × 1 metre × cos 0° = 1 N–m

2. Erg. It is the absolute unit of work in cgs system.

Work done is said to be one erg, when a force of one dyne actually moves a body through a distance of one cm in the direction of applied force.

From $W = Fs\cos\theta$

$$1\,\text{erg} = 1\,\text{dyne} \times 1\,\text{cm} \times \cos 0° = 10^{-5}\,N \times 10^{-2}\,m \times 1$$

$$1\,\text{erg} = 10^{-7}\,J$$

(b) Gravitational units

These are also called the practical units of work.

1. Kilogram-metre (kg–m). It is the gravitational unit of work in SI.

Work done is said to be one kg–m, when a force of 1 kgf moves a body through a distance of 1 m in the direction of the applied force.

From $W = F\cos\theta$

1 kg–m = 1kgf × 1 m × cos 0° = 9.8 N × 1 m = 9.8 joule, i.e.,

$\boxed{1\,kg - m = 9.8\,J}$

2. Gram-centimetre (g-cm). It is the gravitational unit of work in cgs system.

Work done is said to be one g-cm, when a force of 1g f moves a body through a distance of 1 cm in the direction of the applied force.

From $W = Fs\cos\theta$

1 g-cm = 1 g f × 1 cm × cos 0°

1 g-cm = 980 dyne × 1 cm × 1

$\boxed{1g - cm = 980\,erg}$

1.2. Nature of Work Done

Although work done is a scalar quantity, its value may be positive, negative or even zero, as described below:

(a) Positive work

As $W = \vec{F}.\vec{s} = Fs\cos\theta$

∴ when q is acute (< 90°), cos q is positive. Hence, work done is positive.

For example:

When a body falls freely under the action of gravity, $\theta = 0°$, $\cos\theta = \cos 0° = +1$. Therefore, work done by gravity on a body falling freely is positive.

(b) Negative work

As $W = \vec{F}.\vec{s} = Fs\cos\theta$

\ When q is obtuse (> 90°), cos q is negative. Hence, work done is negative.

For example:

When a body is thrown up, its motion is opposed by gravity. The angle θ between gravitational force and the displacement is 180°. As $\cos\theta = \cos 180° = -1$ therefore, work done by gravity on a body moving upwards is Note negative.

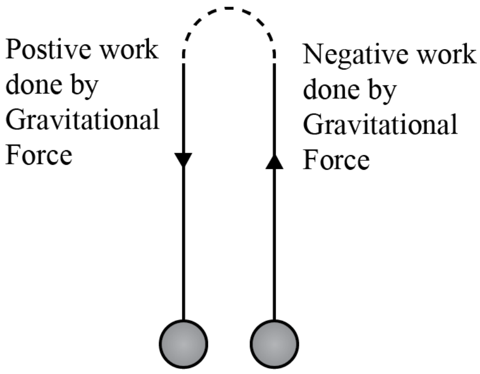


Fig.5.2

(c) Zero work

When force applied $\vec{F}$ or the displacement $\vec{s}$ or both are zero, work done W = F s cos q is zero. Again, when angle q between $\vec{F}$ and $\vec{s}$ is 90°, $\cos\theta = \cos 90° = 0$. Therefore, work done is zero.

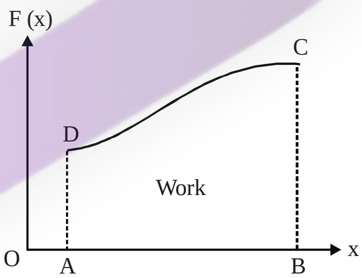
For example:

When we push hard against a wall, the force we exert on the wall does no work, because $\vec{s} = 0$. However, in this process, our muscles are contracting and relaxing alternately and internal energy is being used up. That is why we do get tired.

1.3. Work done by a Variable Force

If the force is variable then the work done is

$$W = \int_{x_A}^{x_B} F(x).dx$$



(b)

Fig.5.3

$\boxed{W = \text{Area ABCDA}}$

Hence, work done by a variable force is numerically equal to the area under the force curve and the displacement axis.

Note:

NOTE:

Energy of a body is defined as the capacity or ability of the body to do the work

Work done is equal to energy consumed.

2. Kinetic Energy

Introduction to Kinetic Energy:

The kinetic energy of a body is the energy possessed by the body by virtue of its motion.

For example:

(i) A bullet fired from a gun can pierce through a target on account of kinetic energy of the bullet.

(ii) Windmills work on the kinetic energy of air.

(iii) For example, sailing ships use the kinetic energy of wind.

(iv) Water mills work on the kinetic energy of water. For example, fast flowing stream has been used to grind corn.

(iv) A nail is driven into a wooden block on account of kinetic energy of the hammer striking the nail.

Formula for Kinetic Energy

$$\boxed{\text{K.E. of body} = \frac{1}{2} m v^2}$$

2.1. Relation Between Kinetic Energy and Linear Momentum

Let m = mass of a particle, $\vec{v} =$ velocity of the particle.

$\therefore$ Linear momentum of the particle, $\vec{p} = \overrightarrow{mv}$

and K.E. of the particle $= \frac{1}{2} mv^2 = \frac{1}{2m}(m^2v^2)$

$$\therefore \boxed{K.E = \frac{p^2}{2m}}$$

This is an important relation. It shows that a particle cannot have K.E. without having linear momentum. The reverse is also true.

Further, if p = constant, $K.E \propto \frac{1}{m}$

This is shown in figure (a)

If K.E. = constant, $p^2 \propto m$ or

This is shown in figure (b).

If m = constant, $p^2 \propto K.E$ or

This is shown in figure (c)

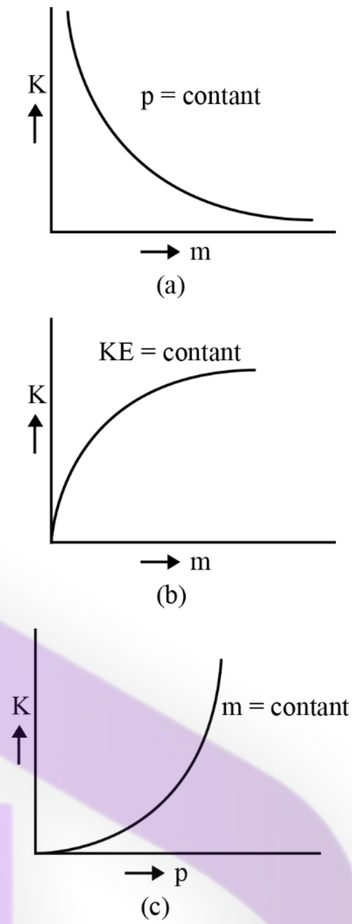


Fig.5.4

3. Work Energy Theorem

According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body.

Thus, when a force does some work on a body, the kinetic energy of the body increases by the same amount. Conversely, when an opposing (retarding) force is applied on a body, its kinetic energy decreases. The decrease in kinetic energy of the body is equal to the work done by the body against the retarding force. Thus, according to work energy principle, work and kinetic energy are equivalent quantities.

Proof: To prove the work-energy theorem, we confine ourselves to motion in one dimension.

Suppose m = mass of a body, u = initial velocity of the body,

F = force applied on the body along it's direction of motion,

a = acceleration produced in the body, v = final velocity of the body after t second.

Small amount of work done by the applied force on the body,

$dW = F(ds)$ when ds is the small distance moved by the body in the direction of the force applied.

Now, $F = ma = m\left(\frac{dv}{dt}\right)$

$$dW = F(ds) = m\left(\frac{dv}{dt}\right)ds = m\left(\frac{dv}{dt}\right)$$

$$dV = mv\,dv\left(\frac{ds}{dt} = v\right)$$

$$\left(\because \frac{ds}{dt} = v\right)$$

Total work done by the applied force on the body in increasing its velocity from u to v is

$$W = \int_u^v mv\,dv = m\int_u^v v\,dv = m\left[\frac{v^2}{2}\right]_u^v$$

$$W = \frac{1}{2}m\left(v^2 - u^2\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

But $\frac{1}{2}mv^2 = K_f$ = final K.E. of the body and $\frac{1}{2}mu^2 = K_i$ = initial K.E. of the body

$W = K_f - K_i$ = change in K.E. of body

i.e., Net work done on the body = increase in K.E. of body

4. Potential Energy

4.1. Conservative and Non-Conservative Forces

Conservative force

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body, and not on the trajectory of path followed between the initial and the final positions.

This means, work done by or against a conservative force in moving a body over any path between fixed initial and final positions will be the same.

For example, gravitational force is a conservative force.

Properties of Conservative forces :

1. Work done by or against a conservative force, in moving a body from one position to the other depends only on the initial position and final position of the body.
2. Work done by or against a conservative force does not depend upon the trajecrory of the path followed by the body in going from initial position to the final position.
3. Work done by or against a conservative force in moving a body through any round trip (i.e., closed path, where final position coincides with the initial position of the body) is always zero.

Non-conservative Forces

A force is said to be non-conservative, if work done by or against the force in moving a body from one position to another, depends on the path followed between initial and final position.

For example, frictional forces are non-conservative forces.

Potential Energy and the Associated Conservative Force:

We know how to find potential energy associated with a conservative force. Now we learn how to obtain the conservative force if potential energy function is known. Consider work done dW by a conservative force in moving a particle through an infinitely small path length $d\vec{s}$ as shown in the figures.

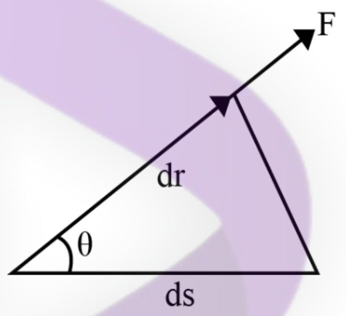


Fig.5.5

$$dU = -dW = -\vec{F}.\vec{ds} = -Fds\cos\theta$$

From the above equation, the magnitude F of the conservative force can be expressed.

$$F = -\frac{dU}{ds\cos\theta} = -\frac{dU}{dr}$$

If we assume an infinitely small displacement in the direction of the force, magnitude of the force is given by the following equation.

$$F = -\frac{dU}{dr}$$

Here minus sign suggest that the force acts in the direction of decreasing potential energy.

Also, $F_x = \frac{-\partial U}{\partial x}, F_y = \frac{\partial U}{\partial y}, F_Z = -\frac{\partial U}{\partial z}$

4.2 Introduction to Potential Energy

The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration in some conservative field.

Thus, potential energy is the energy that can be associated with the configuration (or arrangement) of a system of objects that exert forces on one another. Obviously, if configuration of the system changes, then its potential energy changes.

Two important types of potential energy are :

1. Gravitational potential energy
2. Elastic potential energy.

Also $W_C = -\Delta U$

W_C : Work done by conservative force

4.3. Gravitational Potential Energy

Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

To calculate gravitational potential energy, suppose

m = mass of a body

g = acceleration due to gravity on the surface of earth.

h = height through which the body is raised, as shown in the figure.

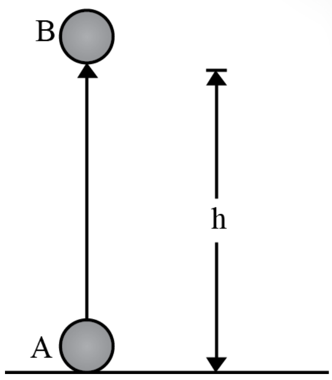


Fig.5.6

If we assume that height h is not too large and the value of g is practically constant over this height,

$W_g = mg \times \cos 180°$

$W_g = -mgh$

$\Delta U = -W_g$

$\Delta U = mgh$

$U_B - U_A = mgh$

Considering $U_A = 0, U_B = U = mgh$

4.4. Spring Potential Energy

Potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

To calculate it, consider an elastic spring OA of negligible mass. The end O of the spring is fixed to a rigid support and a body of mass m is attached to the free end A. Let the spring be oriented along x–axis and the body of mass m lies on a perfectly frictionless horizontal table.

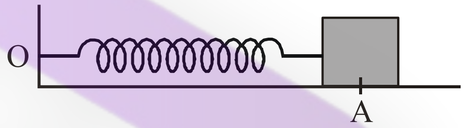


Fig.5.7

The position of the body A, when spring is unstretched is chosen as the origin.

When the spring is compressed or elongated, it tends to regain to its original length, on account of elasticity. The force trying to bring the spring back to its original configuration is called restoring force or spring force.

For a small stretch or compression, spring obeys Hooke's law.

Restoring Force $\propto$ stretch or compression

$\vec{F} \propto -\vec{x}, \vec{F} = -k\vec{x}$

where k is a constant of the spring and is called spring constant.

It is established that for a spring, $k \propto \frac{1}{\ell}$,

ℓ : Natural length of spring

i.e., smaller the length of the spring, greater will be the force constant and vice-versa.

The negative sign in equation indicates that the restoring force is directed always towards the equilibrium position.

Let the body be displaced further through an infinitesimally small distance dx, against the restoring force.

Small amount of work done in increasing the length of the spring by dx is dW = – F dx = kx dx

Total work done in giving displacement x to the body can be obtained by integrating from x = 0 to x = x, i.e.,

$$W = \int_{x=0}^{x=x} k\,x\,dx = k\left[\frac{x^2}{2}\right]_{x=0}^{x=x} = k\left[\frac{x^2}{2} - 0\right] = \frac{1}{2}kx^2$$

This work done is stored in the spring at the point B in the form of P.E

$\therefore$ P.E. at $B = W = \frac{1}{2}kx^2$

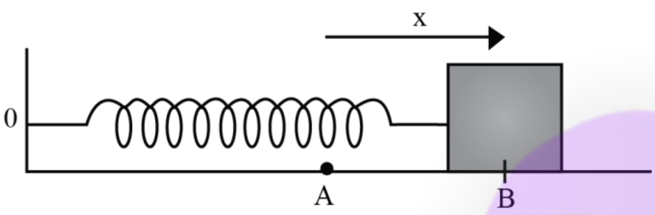


Fig.5.8

The variation of potential energy with distance x is shown in figure

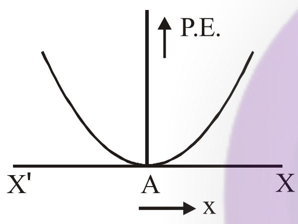


Fig.5.9

5. Mechanical Energy and Its Conservation

The mechanical energy (E) of a body is the sum of kinetic energy (K) and potential energy (U) of the body

i.e., $\boxed{E = K + U}$

Obviously, mechanical energy of a body is a scalar quantity measured in joule.

We can show that the total mechanical energy of a system is conserved if the force, doing work on the system are conservative.

This is called the principle of conservation of total mechanical energy.

For simplicity, we assume the motion to be one dimensional only. Suppose a body undergoes a small displacement Dx under the action of a conservative force $F(x)$. According to work energy theorem,

change in K.E. = work done

$\Delta K = F(x).x$

As the force is conservative, the potential energy function $U(x)$ is defined as

$-\Delta U = F(x).x$ or $\Delta U = -F(x).x$

Adding, we get $\Delta K = F(x).dx$

$\Delta K = -\Delta U, \Delta(K+U) = 0$

which means $\boxed{(K+U) = E = \text{constant}}$

5.1 Illustration of the Law of Conservation of Mechanical Energy

To illustrate the law further, let us calculate kinetic energy K.E., potential energy P.E. and total energy T.E. of a body falling freely under gravity.

Let m be the mass of the body held at A, at a height h above the ground, figure.

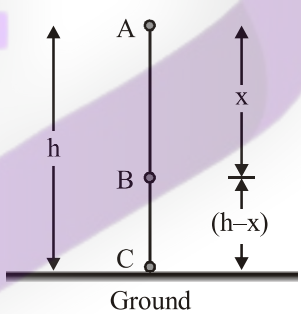


Fig.5.10

As the body is at rest at A, therefore,

At A : K. E. of the body = 0

P.E. of the body = mgh where g is acceleration due to gravity at A.

T.E. of the body $= K.E + P.E = 0 + mgh$

$E_1 = mgh \quad ....(1)$

Let the body be allowed to fall freely under gravity, when it strikes the ground at C with a velocity v.

From $v^2 - u^2 = 2as$

$v^2 - 0 = 2gh$

$v^2 = 2gh \;...(2)$

$\therefore$ At C : K.E. of the body $= \frac{1}{2}mv^2 = \frac{1}{2}m(2\,gh) = mgh$

P.E. of the body = mgh = mg (0) = 0

Total energy of the body = K.E. + P.E.

$E_2 = mgh + 0 = mgh \;...(3)$

In the free fall, let the body cross any point B with a velocity v_1, where AB = x

From $v^2 - u^2 = 2\,as$

$v_1^2 - 0 = 2(g)x \quad ....(4)$

At B : K.E. of the body $= \frac{1}{2}mv^2 = \frac{1}{2}m(2gx) = mgx$

Height of the body at B above the ground = CB = (h – x)

P.E. of the body at B = mg (h – x)

Total energy of the body at B = K.E. + P.E.

$E_B = mgx + mg(h - x) = mgx + mgh - mgx$

$E_B = mgh \;...(5)$

From (1), (3), (5) we find that

$\boxed{E_A = E_B = E_C = mgh}$ which proves conservation of mechanical energy

6. Potential Energy and Nature of Equilibrium

As we know $f = -\frac{du}{dr}$ So, Force = negative of slope of u versus r graph.

The state of stable and unstable equilibrium is associated with a point location, where the potential energy function assumes a minimum and maximum value respectively, and the neutral equilibrium is associated with region of space, where the potential energy function assumes a constant value.

For the sake of simplicity, consider a one dimensional potential energy function U of a central force F. Here r is the radial coordinate of a particle. The central force F experienced by the particle equals to the negative of the slope of the potential energy function. Variation in the force with r is also shown in the figure.

At locations $r = r_1, r = r_2$, and in the region $r \geq r_3$, where potential energy function assumes a minimum, a maximum, and a constant value respectively, the force becomes zero and the particle is in the state of equilibrium

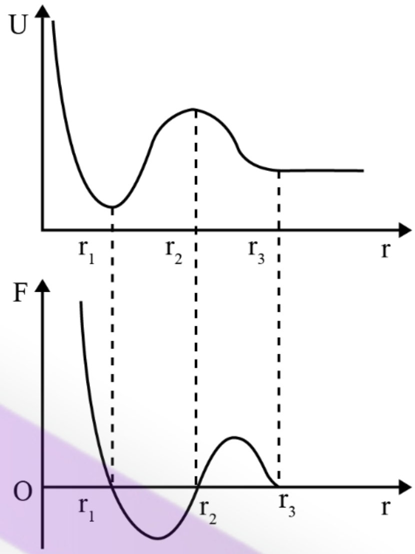


Fig.5.11

Force is negative of the slope of the potential energy function

6.1 Stable Equilibrium

At $r = r_1$ the potential energy function is a minima and the force on either side acts towards the point $r = r_1$. If the particle is displaced on either side and released, the force tries to restore it at $r = r_1$. At this location the particle is in the state of stable equilibrium. The dip in the potential energy curve at the location of stable equilibrium is known as potential well. A particle when disturbed from the state of stable equilibrium starts oscillations about the location of stable equilibrium. At the locations of stable equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0;\text{ and } \frac{\partial F}{\partial r} < 0;\text{ and } \frac{\partial^2 U}{\partial r^2} > 0$$

6.2 Unstable Equilibrium

At $r = r_2$ the potential energy function is a maxima, the force acts away from the point $r = r_2$. If the particle is displaced slightly on either side, it will not return to the location $r = r_2$. At this location, the particle is in the state of unstable equilibrium. At the locations of unstable equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0;\text{ therefore } \frac{\partial F}{\partial r} > 0;\text{ and } \frac{\partial^2 U}{\partial r^2} < 0$$

6.3 Neutral Equilibrium

In the **region** $r = r_3$, the potential energy function is constant and the force is zero everywhere. In this region, the particle is in the state of neutral equilibrium. At the locations of neutral equilibrium we have

$$F(r) = -\frac{\partial U}{\partial r} = 0 \text{ therefore } \frac{\partial F}{\partial r} = 0 \text{ and } \frac{\partial^2 U}{\partial r^2} = 0$$

7. Power

Power of a person or machine is defined as the time rate at which work is done by it.

i.e., Power = Rate of doing work = $\frac{\text{work done}}{\text{time taken}}$

Thus, power of a body measurement how fast it can do the work.

Units of power

The absolute unit of power in SI is **watt**, which is denoted by W.

From P = W/t

$1 \text{ watt} = \frac{1\text{ joule}}{1\text{ sec}}$, i.e., $\boxed{1\text{W} = 1\text{Js}^{-1}}$

Power of a body is said to be one watt, if it can do one joule of work in one second.

$\boxed{1 \text{ h.p.} = 746 \text{ W}}$

NOTE:

Power is also described in terms of rate at which energy is consumed.

$$P = \frac{dW}{dt}$$

* Now, $dW = \vec{F}.\vec{ds}$, where $\vec{F}$ is the force applied and $\vec{ds}$ is the small displacement.

$$P = \frac{\vec{F}.\vec{ds}}{dt}$$

$$P = \vec{F}.\frac{d\vec{s}}{dt}$$

But $\frac{d\vec{s}}{dt} = \vec{v}$, the instantaneous velocity.

$$\boxed{P = \vec{F}.\vec{v}}$$

Dimensions of power can be deduced as :

$$\boxed{P = \frac{W}{t} = \frac{M^1L^2T^{-2}}{T^1} = \left[M^1L^2T^{-3}\right]}$$

NCERT Corner

Important Points to Remember

1. Work done is a scalar quantity. It can be positive or negative unlike mass and kinetic energy which are positive scalar quantities. The work done by the friction or viscous force on a moving body is negative.
2. A force is conservative if (i) work done by it on an object is path independent and depends only on the initial and final position, or (ii) the work done by the force is zero for an arbitrary closed path taken by the object such that it returns to its initial position.
3. The gravitational potential energy of a particle of mass m at a height x about the earth's surface is $U(x) = m g x$ where the variation of g with height is ignored.
4. The elastic potential energy of a spring of force constant k and extension x is $U(x) = \frac{1}{2}kx^2$
5. The potential energy of a body subjected to a conservative force is always undetermined upto a constant. For example, the point where the potential energy is zero is a matter of choice. For the gravitational potential energy mgh, the zero of the potential energy is chosen to be the ground. For the spring potential energy $kx^2/2$, the zero of the potential energy is the equilibrium position of the oscillating mass.
6. For a conservative force in one dimension, we may define a potential energy function U(x) such that

$$F(x) = -\frac{dU(x)}{dx}$$

or

$$U_i - U_f = \int_{x_i}^{x_f} F(x)dx$$

7. For equilibrium $F = -\frac{dU}{dx} = 0$
8. The work – energy theorem states that the change in kinetic energy of a body is the work done by the net force on the body.

$$K_f - K_i = W_{net}$$

9. The work done by a force can be calculated sometimes even if the exact nature of the force is not known. This is calculated with the help of work energy theorem by using change in kinetic energy
10. The WE theorem holds in all inertial frames. It can also be applied in non internal frames provided we include the pseudo forces in the calculation of the net force acting on the body under consideration.
11. Every force encountered in mechanics does not have an associated potential energy. For example, work done by friction over a closed path is not zero and no potential energy can be associated with friction.
12. The principle of conservation of mechanical energy states that the total mechanical energy of a body remains constant if the only forces that act on the body are conservative.

Physics 24

Class11th NEET

06

CIRCULAR MOTION

SSP
SACHIN SIR PHYSICS

Circular Motion

1. Characteristics of Circular Motion

1.1 Circular motion

It is the movement of particles along the circumference of the circle.

2. Various Parameters in Circular Motion

2.1 Radius Vector

The vector joining the centre of the circle and centre of the particle performing circular motion is called radius vector. It has constant magnitude and variable directions.

2.2 Angular Displacement

Introduction: Angle subtended by position vectors of a particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.

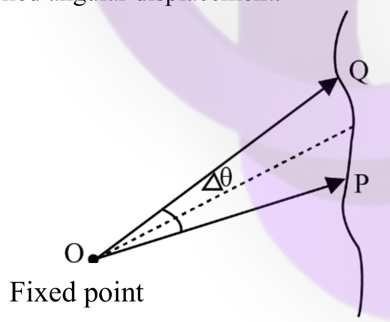


Fig. 6.1

(a) Particle moving in an arbitrary path

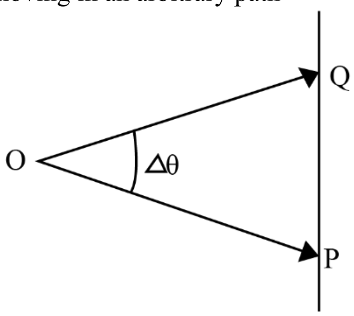


Fig. 6.2

(b) Particle moving in a straight line

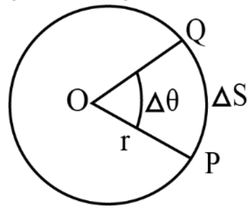


Fig. 6.3

(c) Particle moving in circular path

i) Angular displacement is a vector quantity.

ii) Its direction is perpendicular to the plane of rotation and is given by right hand screw rule.

NOTE:

Clockwise angular displacement is taken as negative and anticlockwise displacement as positive.

$$\text{angle} = \frac{\text{arc}}{\text{radius}} = \frac{\text{linear displacement}}{\text{radius}}$$

iii) For circular motion $\Delta S = r \times \Delta\theta$

iv) Its unit is radian (in M.K.S)

NOTE:

Always change degree into radian, if it occurs in numerical problems.

v) It is a dimensionless quantity, i.e. dimension is $[M^0L^0T^0]$

2.3 Angular Velocity

It is defined as the rate of change of angular displacement of a body or particle moving in a circular path.

i) It is a vector quantity.

ii) Its direction is the same as that of angular displacement i.e. perpendicular to the plane of rotation.

NOTE:

If the particle is revolving in the clockwise, direction then the direction of angular velocity is perpendicular to the revolutionary plane downwards. Whereas in case of anticlockwise direction the direction will be upwards.

iii) Its unit is Radian/sec.

iv) Its dimension is $[M^0L^0T^{-1}]$.

Types of Angular Velocity

Average Angular Velocity

$$\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}}$$

Instantaneous Angular Velocity

The instantaneous angular velocity is defined as the angular velocity at some particular instant of time.

- Instantaneous angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

NOTE:

Instantaneous angular velocity can also be called simply angular velocity.

2.4 Relation Between Linear Velocity And Angular Velocity

We have $\omega = \frac{d\theta}{dt} = \frac{d\theta}{ds}\cdot\frac{ds}{dt} = \frac{1}{r}\cdot v$

$$\left| \because d\theta = \frac{ds}{dr},\ \text{angle} = \frac{\text{arc}}{\text{radius}} \text{ and } v = \frac{ds}{dt} = \text{linear velocity} \right|$$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

NOTE:

i) When a particle moves along a curved path, its linear velocity at a point is along the tangent drawn at that point.

ii) When a particle moves along a curved path, its velocity has two components. One along the radius, which increases or decreases the radius and another one perpendicular to the radius, which makes the particle revolve about the point of observation.

iii) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v\sin\theta}{r}$

3. Angular Acceleration

- The rate of change of angular velocity is defined as angular acceleration.
- If $\Delta\omega$ be change in angular velocity in time Δt, then angular acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

i) It is a vector quantity

ii) Its direction is that of change in angular velocity

iii) Unit : rad/sec^2

iv) Dimension : $M^0L^0T^{-2}$

3.1 Relation Between Angular Acceleration And Linear Acceleration

Linear acceleration = Rate of change of linear velocity

$$\Rightarrow a = \frac{dv}{dt} \quad ...(i)$$

Angular acceleration = Rate of change of angular velocity

$$\Rightarrow \alpha = \frac{d\omega}{dt} \quad ...(ii)$$

From (i) & (ii)

$$\frac{a}{\alpha} = \frac{dv}{d\omega} = \frac{d(r\omega)}{d\omega} = \frac{d\omega}{d\omega}\cdot r \quad [r \text{ is constant}] = r$$

$\Rightarrow a = \alpha r$

In vector form, $\vec{a} = \vec{\alpha} \times \vec{r}$

4. Radial and Tangential Acceleration

a) **Radial Acceleration** is the change in direction of linear velocity and acts along the radius towards the centre of circle. It is given by

$$\alpha_r = \frac{v^2}{r} = \omega^2 r$$

It is also called **centripetal acceleration.**

b) **Tangential acceleration** is the change in magnitude of linear velocity, that act along the tangent to the circular path. It is given by:

$$\alpha_\tau = r\alpha$$

5. Uniform & Non-Uniform Circular Motion

- The **uniform circular motion** is that in which the particle is moving at a constant speed on circular path.
- The **non-uniform circular motion** is that in which the particles move with variable speed on its circular path.

6. Kinematics of Circular Motion

If,
m = mass of body,
r = radius of circular orbit,
v = magnitude of velocity
a_c = centripetal acceleration,
a_t = tangential acceleration

In uniform circular motion :

i) $|\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3| =$ constant i.e., speed is constant

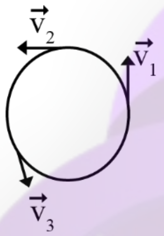


Fig 6.4

ii) As $|\vec{v}|$ is constant, so tangential acceleration $a_t = 0$

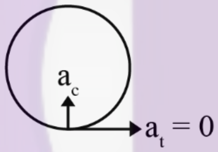


Fig. 6.5

iii) Tangential, force $F_t = 0$

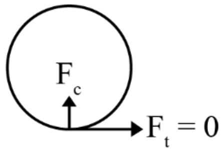


Fig. 6.6

iv) Total acceleration

$$a = \sqrt{a_c^2 + a_t^2} = a_c = \frac{v^2}{r} \text{(towards the centre)}$$

NOTE:

- Because F_c is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.
- Circular motion in a horizontal plane is usually uniform circular motion.
- There is an important difference between projectile motion and circular motion:
  In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in circular motion the magnitude remains constant but the direction continuously changes.
  Hence, equations of motion are not applicable for circular motion.
- Remember that equations of motion remain valid only when both the magnitude & direction of acceleration are constant.

6.1 Equations for Linear and Rotational Motion

S.No	Condition	Linear Motion	Rotational Motion
i.	**With constant velocity**	$a = 0, s = ut$ (i) Average velocity $v_{av} = \frac{v+u}{2}$	$\alpha = 0$, $\theta = \omega t$ (i) Average angular velocity $\omega_{av} = \frac{\omega_1 + \omega_2}{2}$
ii.	**With constant acceleration**	(ii) Average acceleration $a_{av} = \frac{v-u}{t}$	(ii) Average angular acceleration $a_{av} = \frac{\omega_2 - \omega_1}{t}$
		(iii) $s = v_{av}\, t$ $= \frac{v+u}{2} t$	(iii) $\theta = \omega_{av}.\, t$ $= \frac{\omega_1 + \omega_2}{2} t$
		(iv) $v = u + at$	(iv) $\omega_2 = \omega_1 + \alpha t$
		(v) $s = ut + \frac{1}{2}at^2$	(v) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
		(vi) $s = vt - \frac{1}{2}at^2$	(vi) $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$

		(vii) $v^2 = u^2 + 2as$	(vii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$
		(viii) displacement in n^{th} sec $S_n = u + \frac{1}{2}(2n-1)a$	(viii) Angular displacement in n^{th} sec $\theta_n = \omega_1 + \frac{1}{2}(2n-1)\alpha$
iii.	**With variable acceleration**	(i)	(i) $\omega = d\theta/dt$
		(ii) $\int ds = \int v\,dt$	(ii) $\int d\theta = \int \omega\,dt$
		(iii) $a = \frac{dv}{dt} = v\frac{dv}{ds}$	(iii) $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$
		(iv) $\int dv = \int a\,dt$	(iv) $\int d\omega = \int \alpha\,dt$
		(v) $\int v\,dv = \int a\,ds$	(v) $\int \omega\,d\omega = \int \alpha\,d\theta$

7. Non-Uniform Circular Motion

i) In non-uniform circular motion :

$|\vec{v}| \neq$ constant, $\omega \neq$ constant

i.e. speed is not constant

& angular velocity is not constant

ii) If at any instant,

v = magnitude of velocity of particle,

r = radius of circular path,

ω = angular velocity of particle,

then

iii) Tangential acceleration:

$$a_t = \frac{dv}{dt}$$

where, $v = \frac{ds}{dt}$ and s = arc (length)

iv) Tangential Force:

$F_t = ma_t$

v) Centripetal Force:

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

vi) Net Force on the particle

$$\vec{F} = \vec{F}_c + \vec{F}_t$$

$$\Rightarrow F = \sqrt{F_c^2 + F_t^2}$$

If θ is the angle made by [Note: angle between F_c and F_t is 90°] F with F_c, then

$$\tan\theta = \frac{F_t}{F_c}$$

$$\Rightarrow \theta = \tan^{-1}\left[\frac{F_t}{F_c}\right]$$

Angle between F & F_t is $(90° - \theta)$

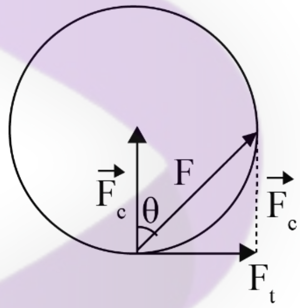


Fig. 6.7

vii) Net acceleration towards the centre = centripetal acceleration

$$\Rightarrow a_c = \frac{v^2}{r} = \omega^2 r = \frac{F_c}{m}$$

viii) Net Acceleration $a = \sqrt{a_c^2 + a_t^2} = \frac{F_{net}}{m}$

The angle made by 'a' with a_c, $\tan\theta = \frac{a_t}{a_c} = \frac{F_t}{F_c}$

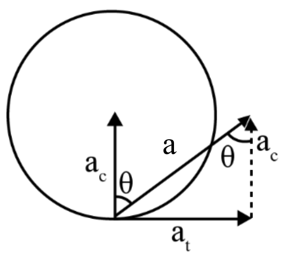


Fig. 6.8

NOTE:

- In both uniform & non-uniform circular motion, F_c is perpendicular to velocity ; so work done by centripetal force will be zero in both the cases.
- In uniform circular motion, $F_t = 0$, as $a_t = 0$, so work done will be zero by tangential force.
- But in non-uniform circular motion $F_t \neq 0$, thus there will be work done by tangential force in this case.
- Rate of work done by net force in non-uniform circular motion = Rate of work done by tangential force

$$\Rightarrow P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v} = \vec{F}_t \cdot \frac{d\vec{x}}{dt}$$

8. Centripetal & Centrifugal Force

8.1 Centripetal

i) A body or particle moving in a curved path always moves effectively in a circle at any instant.

ii) The velocity of the particle changes moving on the curved path, this change in velocity is brought by a force, known as centripetal force and the acceleration produced in the body, is known as centripetal acceleration.

iii) The direction of centripetal force or acceleration is always towards the centre of the circular path.

8.2 Expression for Centripetal Acceleration

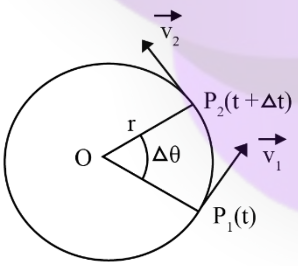


Fig. 6.9

(a) Particle moving in circular path of radius r

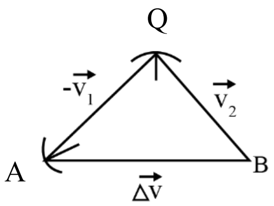


Fig. 6.10

(b) Vector diagram of velocities

The triangle OP_1P_2 and the velocity triangle are similar

$$\therefore \frac{P_1P_2}{P_1O} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{\Delta s}{r} = \frac{AB}{AQ} \left[|\vec{v}_1| = |\vec{v}_2| = v\right]$$

$$\Rightarrow \Delta v = \frac{v}{r}\Delta s$$

$$\Rightarrow \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t}$$

$$\Rightarrow \lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \frac{v}{r}\lim_{\Delta t \to 0}\left(\frac{\Delta s}{\Delta t}\right)$$

$$\Rightarrow a_c = \frac{v}{r}v = \frac{v^2}{r} = r\omega^2$$

$$a_c = r\omega^2$$

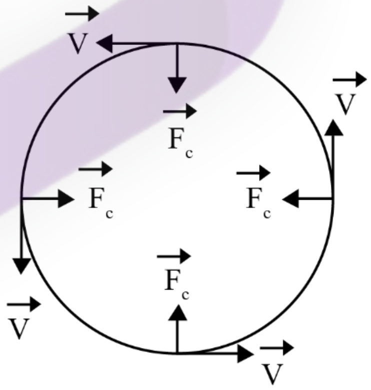
This is the magnitude of centripetal acceleration of particle

i) It is a vector quantity. In vector form $\vec{a}_c = \vec{\omega} \times \vec{v}$

ii) The direction of $\vec{a}_c$ would be the same as that of $\Delta\vec{v}$.

iii) Because the velocity vector at any point is tangential to the circular path at that point, the acceleration vector acts along the radius of the circle at that point and is directed towards the centre. This is the reason that it is called centripetal acceleration.

8.3 Expression for Centripetal Force



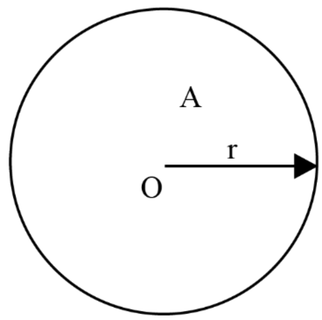


Fig. 6.11

If v = velocity of particle,

r = radius of path
Then necessary centripetal force
F_c = mass × acceleration

$$F_c = m\frac{v^2}{r}$$

This is the expression for centripetal force

i) It is a vector quantity
ii) In vector form

$$\vec{F}_c = -\frac{mv^2}{r}$$

$$\vec{F}_c = -\frac{mv^2}{r^2}\vec{r}$$

$$\vec{F}_c = -m\omega^2 r$$

$$\vec{F}_c = -m(\vec{v}\times\vec{\omega})$$

Negative sign indicates direction only.

$$|\vec{F}_c| = m(\vec{v}\times\vec{\omega})$$

iii) For circular motion:

$$|\vec{F}_c| = m(v\omega\sin 90°) = mv\omega$$

NOTE:

- Centripetal force is not a real force. It is only the requirement for circular motion.
- It is not a new kind of force. Any of the forces found in nature such as gravitational force, electric friction force, tension in string, reaction force may act as centripetal force.

8.4 Centrifugal Force

The natural tendency of a body is to move uniformly along a straight line. When we apply centripetal force on the body, it is forced to move along a circle. While moving actually along a circle, the body has a constant tendency to regain its natural straight line path. This tendency gives rise to a force called centrifugal force.

Hence, **Centrifugal force is a force that arises when a body is moving actually along a circular path, by virtue of the tendency of the body to regain its natural straight line path.**

- Centrifugal forces can be regarded as the reaction of centripetal force. As forces of action and reaction are always equal and opposite, therefore, magnitude of centrifugal force = m v^2/r, which is same as that of centripetal force.
- However, the direction of centrifugal force is opposite to the direction of centripetal force i.e. **centrifugal force acts along the radius and away from the centre of the circle.**

NOTE:

Both centripetal and centrifugal forces, being the forces of action and reaction, act always on different bodies. For example, when a piece of stone tied to one end of a string is rotated in a circle, centripetal force F_1 is applied on the stone by the hand. In turn, the hand is pulled outwards by centrifugal force F_2 acting on it, due to the tendency of the stone to regain its natural straight line path. The centripetal and centrifugal forces are shown in Fig.

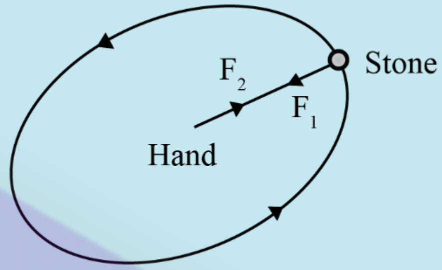


Fig. 6.12

9. Applications of Circular Motion

9.1 Hint to Solve Numerical Problem

- Write down the required centripetal force.
- Draw the free body diagram of each component of the system.
- Resolve the forces acting on the rotating particle along radius and perpendicular to radius.
- Calculate net radial force acting towards the centre of the circular path.
- Make it equal to required centripetal force.
- For remaining components see according to the question.

NOTE:

When a system of particles rotates about an axis, the angular velocity of all the particles will be the same, but their linear velocity will be different, because of different distances from the axis of rotation i.e. $v = r\omega$.

9.2 Motion in Horizontal Circle : Conical Pendulum

This is the best example of uniform circular motion. A conical pendulum consists of a body attached to a string of length ℓ, such that it can revolve in a horizontal circle with uniform speed. The string traces out a cone in space.

i) The force acting on the bob are
(a) Tension F (b) weight mg

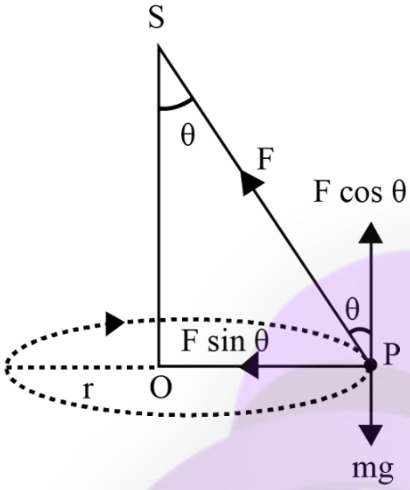


Fig. 6.13

ii) The horizontal component F sin θ of the tension F provides the centripetal force and the vertical component F cos θ balances the weight of bob

$\therefore F\sin\theta = \frac{mv^2}{r}$ and $F\cos\theta = mg$

From these equations

$$F = mg\sqrt{1+\frac{v^4}{r^2g^2}} \quad .....(i)$$

and $\tan\theta = \frac{v^2}{rg}$ ....(ii)

Also if h = height of conical pendulum

$$\tan\theta = \frac{OP}{OS} = \frac{r}{h} \quad ....(iii)$$

From (ii) & (iii)

$$\omega^2 = \frac{v^2}{r^2} = \frac{g}{h}$$

The time period of revolution

As, $T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{\ell\cos\theta}{g}} \quad [\text{where } OS = \ell]$$

9.3 Rounding a Level Curved Road

- When a vehicle goes round a curved road, it requires some centripetal force. While rounding the curve, the wheels of the vehicle have a tendency to leave the curved path and regain the straight line path. Force of friction between the wheels and the road opposes this tendency of the wheels. This force (of friction) therefore, acts, towards the centre of the circular track and provides the necessary centripetal force.
- Three forces are acting on the car, fig.

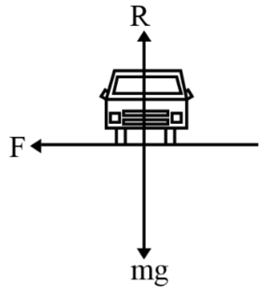


Fig. 6.14

i) The weight of the car, mg, acting vertically downwards,
ii) Normal reaction R of the road on the car, acting vertically upwards,
iii) Frictional Force F, along the surface of the road, towards the centre of the turn.

- As there is no acceleration in the vertical direction,
R–mg=0 or R=mg ...(1)
The centripetal force required for circular motion is along the surface of the road, towards the centre of the turn.
As, it is the static friction that provides the necessary centripetal force. Clearly,

$$\frac{mv^2}{r} \le F \quad ...(2)$$

where v is the velocity of the car while turning and r is the radius of a circular track.
As $F = \mu_s R = \mu_s mg$, [using (1)]
where μ_s is the coefficient of static friction between the tyres and the road. Therefore, from (2),

$$\frac{mv^2}{r} \le \mu_s mg$$

or $v \le \sqrt{\mu_s rg}$

$$\therefore \boxed{v_{max} = \sqrt{\mu_s rg}} \quad ....(3)$$

Hence the maximum velocity with which a vehicle can go round a level curve, without skidding is

$$\boxed{v = \sqrt{\mu_s rg}}$$

- The value depends on the radius r of the curve and on the coefficient of static friction (μ_s) between the tyres and the road. Clearly, v is independent of the mass of the car.

9.4 Banking of Roads

- The maximum permissible velocity with which a vehicle can go around a level curved road without skidding depends on μ, the coefficient of friction between the tyres and the road. The value of μ decreases when the road is smooth or tyres of the vehicle are worn out or the road is wet. Thus, the force of friction is not a reliable source for providing the required centripetal force to the vehicle.
- A safer course of action would be to raise the outer edge of the curved road above the inner edge. By doing so, a component of normal reaction of the road shall be spared to provide the centripetal force. ***The phenomenon of raising the outer edge of the curved road above the inner edge is called banking of roads.***
- We can calculate the angle of banking θ, as detailed below:
  In Fig., OX is a horizontal line. OA is the level of banked curved road whose outer edge has been raised.
  $\angle XOA = \theta$ = angle of banking.

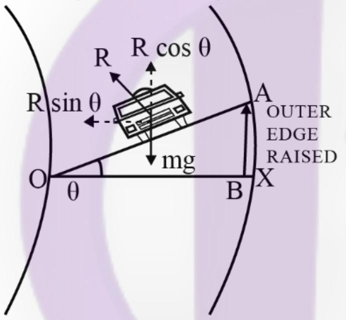


Fig. 6.15

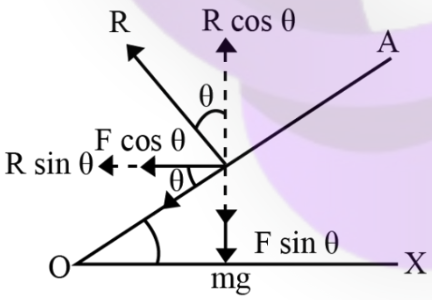


Fig. 6.16

Three forces are acting on the vehicle as shown in Fig.

- Weight mg of the vehicle acting vertically downwards.
- Normal reaction R of the banked road acting upwards in a direction perpendicular to OA.
- Force of friction F between the banked road and the tyres, acting along AO.

R can be resolved into two rectangular components :-

- R cos θ, along vertically upward direction.
- R sin θ, along the horizontal, towards the centre of the curved road.

F can also be resolved into two rectangular components :

- F cos θ, along the horizontal, towards the centre of curved road.
- F sin θ, along vertically downward direction.

As there is no acceleration along the vertical direction, the net force along this direction must be zero. Therefore,
$R\cos\theta = mg + F\sin\theta$ ...(1)

If v is the velocity of the vehicle over the banked circular road of radius r, then centripetal force required = mv^2/r. This is provided by the horizontal components of R and F as shown in Fig.

$$\therefore R\sin\theta + F\cos\theta = \frac{mv^2}{r} \quad ...(2)$$

But $F \le \mu_s R$, where μ_s is coefficient of static friction between the banked road and the tyres.

To obtain v_{max}, we put $F = \mu_s R$ in (1) and (2).

$$R\cos\theta = mg + \mu_s R\sin\theta \quad ...(3)$$

and $$R\sin\theta + \mu_s R\cos\theta = \frac{mv^2}{r} \quad ....(4)$$

From (3), $R(\cos\theta - \mu_s \sin\theta) = mg$

$$R = \frac{mg}{\cos\theta - \mu_s\sin\theta} \quad ....(5)$$

From (4), $$R(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{r}$$

Using (5), $$\frac{mg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{mv^2}{r}$$

$$\therefore v^2 = \frac{rg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{rg\cos\theta(\tan\theta + \mu_s)}{\cos\theta(1 - \mu_s\tan\theta)}$$

$$\boxed{v = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}\right]^{1/2}} \quad ....(6)$$

This is the max. velocity of vehicle on a banked road.

Special Case:
If $\mu_s = 0$, i.e., if banked road is perfectly smooth, then,

$$\boxed{v_0 = (rg\tan\theta)^{1/2}} \quad ....(7)$$

This is the speed at which a banked road can be rounded even when there is no friction. Driving at this speed on a banked road will cause almost no wear and tear of the tyres.
From (7), $v_0^2 = rg\tan\theta$

or $\boxed{\tan\theta = v_0^2 / rg}$ ....(8)

- If the speed of the vehicle is less than v_0, frictional force will be up the slope. Therefore, the vehicle can be parked only if $\tan\theta \le \mu_s$.
- Roads are usually banked for the average speed of vehicles passing over them. However, if the speed of a vehicle is somewhat less or more than this, the self adjusting static friction will operate between the tyres and the road, and the vehicle will not skid.
- The speed limit at which the curve can be negotiated safely is clearly indicated on the sign boards erected along the curved roads.

NOTE:

The curved railway tracks are also banked for the same reason. The level of outer rail is raised a little above the level of inner rail, while laying a curved railway track.

9.5 Bending of a Cyclist

- When a cyclist takes a turn, he also requires some centripetal force. If he keeps himself vertical while turning, his weight is balanced by the normal reaction of the ground.
- In that event, he has to depend upon force of friction between the tyres and the road for obtaining the necessary centripetal force. As the force of friction is small and uncertain, dependence on it is not safe.
- To avoid dependence on force of friction for obtaining centripetal force, the cyclist has to bend a little inwards from his vertical position, while turning. By doing so, a component of normal reaction in the horizontal direction provides the necessary centripetal force. To calculate the angle of bending with vertical,
  Let,
  m = mass of the cyclist,
  v = velocity of the cyclist while turning,
  r = radius of the circular path,
  θ = angle of bending with vertical.

In Fig., we have shown weight of the cyclist (mg) acting vertically downwards at the centre of gravity C. R is force of reaction of the ground on the cyclist. It acts at an angle θ with the vertical.

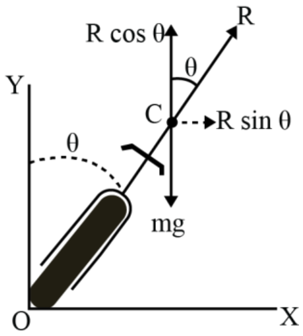


Fig. 6.17

R can be resolved into two rectangular components:

- R cos θ, along the vertical upward direction,
- R sin θ, along the horizontal, towards the centre of the circular track.

In equilibrium, R cos θ balances the weight of the cyclist i.e.

R cos θ = mg ...(1)

and R sin θ provides the necessary centripetal force ($m v^2/r$)

$$\therefore R\sin\theta = \frac{mv^2}{r} \text{....(2)}$$

Dividing (2) by (1), we get

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r\,mg}$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

Clearly, θ would depend on v and r.

- For a safe turn, θ should be small, for which v should be small and r should be large i.e. turning should be at a slow speed and along a track of larger radius. This means, a safe turn should neither be fast nor sharp.

10. Motion in Vertical Circle

Motion of a body suspended by string: This is the best example of non-uniform circular motion.

When the body rises from the bottom to the height h, a part of its kinetic energy converts into potential energy

Total mechanical energy remains conserved

Total (P.E. + K.E.) at A = Total (P.E. + K.E.) at P

$$\Rightarrow 0 + \frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{u^2 - 2gh}$$

as $h = \ell - \ell\cos\theta$

$= \ell(1 - \cos\theta)$

$\Rightarrow v=\sqrt{u^2-2g\ell(1-\cos\theta)}$

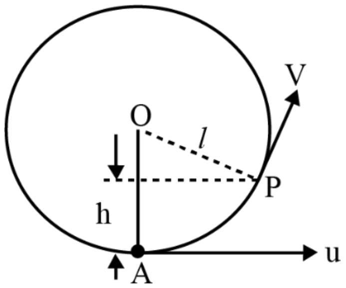


Fig. 6.18

[Where ℓ is length of the string]

10.1 Tension at a Point P

i) At point P, required centripetal force $=\frac{mv^2}{\ell}$

a) Net force towards the centre :

T – mg cos θ, which provides required centripetal force.

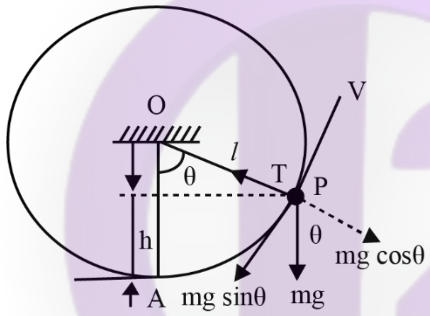


Fig. 6.19

$\therefore T-mg\cos\theta=\frac{mv^2}{\ell}$

$T=m\left[g\cos\theta+\frac{v^2}{\ell}\right]$

$=\frac{m}{\ell}\left[u^2-g\ell(2-3\cos\theta)\right]$

b) Tangential force for the motion

$F_t = mg\sin\theta$

This force retards the motion

ii) Results:

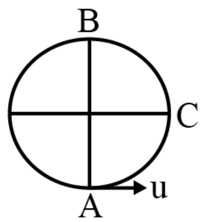


Fig 6.20

a) Tension at the lowest point A :

$T_A=\frac{mv_A^2}{\ell}+mg$

Here $\theta=0°$

$T_A=\frac{mu^2}{\ell}+mg$

b) Tension at point B :

$T_B=\frac{mv_B^2}{\ell}-mg$

$T_B=\frac{mu^2}{\ell}-5mg\ (\because \theta=180°)$

c) Tension at point C :

$T_C=\frac{mv_C^2}{\ell}$

$T_C=\frac{mu^2}{\ell}-2mg\ (\text{Here } \theta=90°)$

Thus, we conclude that

$T_A > T_C > T_B$

and also

$T_A - T_B = 6$ mg

$T_A - T_C = 3$ mg

$T_C - T_B = 3$ mg

iii) Cases:

a) If $u>\sqrt{5g\ell}$

In this case tension in the string will not be zero at any of the points, which implies that the particle will continue the circular motion.

b) If $u=\sqrt{5g\ell}$

In this case the tension at the top most point (B) will be zero, which implies that the particle will just complete the circular motion.

c) Critical Velocity: The minimum velocity at which the circular motion is possible.

The critical velocity at $A=\sqrt{5g\ell}$

The critical velocity at $B=\sqrt{g\ell}$

The critical velocity at $C=\sqrt{3g\ell}$

Also, $T_A = 6$ mg, $T_B = 0$, $T_C = 3$ mg

d) If $\sqrt{2g\ell}<u<\sqrt{5g\ell}$

In this case particles will not follow circular motion. Tension in string becomes zero somewhere between points C & B whereas velocity remains positive. Particle leaves circular path and follow parabolic trajectory

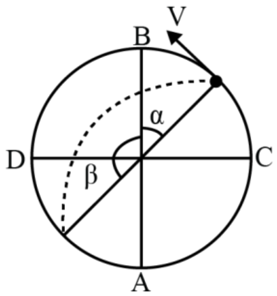


Fig. 6.21

e) If $u = \sqrt{2g\ell}$

In this case both velocity and tension in the string becomes zero between A and C and the particle will oscillate along a semi-circular path.

f) If $u < \sqrt{2g\ell}$

The velocity of the particle remains zero between A and C but tension will not be zero and the particle will oscillate about the point A.

11. Tips & Tricks

1. Centripetal force does not increase the kinetic energy of the particle moving in a circular path, hence the work done by the force is zero.
2. Centrifuges are the apparatuses used to separate small and big particles from a liquid.
3. The physical quantities which remain constant for a particle moving in a circular path are speed, kinetic energy and angular momentum.
4. If a body is moving on a curved road with speed greater than the speed limit, the reaction at the inner wheel disappears and it will leave the ground first.
5. On unbanked curved roads the minimum radius of curvature of the curve for safe driving is $r = v^2/mg$, where v is the speed of the vehicle and m is small.
6. If r is the radius of curvature of the speed breaker, then the maximum speed with which the vehicle can run on it without leaving contact with the ground is $v = \sqrt{(gr)}$
7. While taking a turn on the level road sometimes vehicles overturned due to centrifugal force.
8. If h is the height of centre of gravity above the road, a is half the wheel base distance, then for road safety

   $\frac{mv^2}{r} \cdot h < mg \cdot a,$

   Minimum safe speed for no overturning is $v = \sqrt{gar/h}$

   .
9. On a rotating platform, to avoid the skidding of an object placed at a distance r from axis of rotation, the maximum angular velocity of the platform,

   $\omega = \sqrt{(\mu g/h)}$, where μ is the coefficient of friction between the object and the platform.
10. If an inclined plane ends into a circular loop of radius r, then the height from which a body should slide from the inclined plane in the order to complete the motion in a circular track is h = 5r/2.
11. Minimum velocity that should be imparted to a pendulum to complete the vertical circle is $\sqrt{(5g\ell)}$ where ℓ is the length of the pendulum.
12. While describing a vertical circle when the stone is in its lowest position, the tension in the string is six times the weight of the stone.
13. The total energy of the stone while revolving in a vertical circle is (5/2) mgl .
14. When the stone is in horizontal position then the tension in the string is 3mg and the velocity of the stone is $\sqrt{(3g\ell)}$.
15. If the velocity of the stone at the highest point is X mg, then the tension at the lowest point will be (X + 6)mg.
16. If a body of mass m is tied to a string of length ℓ and is projected with a horizontal velocity u such that it does not complete the motion in the vertical circle, then

    (a) the height at which the velocity vanishes is $h = \frac{u^2}{2g}$

    (b) the height at which the tension vanishes is

    $h = \frac{u^2 + g\ell}{3g}$.
17. The K.E. of a body moving in a horizontal circle is the same throughout the path but the K.E. of the body moving in a vertical circle is different at different places.

NCERT Corner

Important Points to Remember

1. Circular Motion

- It is the movement of particles along the circumference of a circle.
- The **uniform circular motion** is that in which the particle is moving at a constant speed on circular path.
- The **non-uniform circular motion** is that in which the particles move with variable speed on its circular path.

2. Variables in Circular Motion

- **Angular Displacement:** It is the angle subtended by the position vector at the centre of the circular path. Angular displacement, $\Delta\theta = \Delta s / r$ where, Δs is the arc length and r is the radius
- **Angular Velocity:** The time rate of change of angular displacement $(\Delta\theta)$ is called angular velocity.

  Angular velocity, $\omega = \Delta\theta / \Delta t$

  Angular velocity is a vector quantity

  Relation between linear velocity (v) **and and angular velocity** (ω) **is given by**

  $v = r\omega$
- **Angular Acceleration:** The rate of change of angular velocity is called angular acceleration.

  Angular acceleration,

  $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ Its SI unit is rad/s^2 and dimensional formula is $[T^{-2}]$

  Acceleration in a circular motion has two components as given below:

  (a) Tangential acceleration is the change in magnitude of linear velocity and act along tangent to the circular path. It is given by:

  $\alpha_T = r\alpha$

  (b) Radial Acceleration is the change in direction of linear velocity and acts along the radius towards the centre of circle. It is given by $\alpha_r = \frac{v^2}{r} = \omega^2 r$

  It is also called **centripetal acceleration.**

  Relation between linear acceleration (a) **and angular acceleration** (α)

  $a = r\alpha$, where r = radius

  Relation between angular acceleration (α) **and linear velocity** (v)

  $\alpha = \frac{v^2}{r}$

3. Centripetal and Centrifugal Force

- **Centripetal force:** In uniform circular motion the force acting on the particle along the radius and towards the centre keeps the body moving along the circular path. This force is called centripetal force.
- **Centrifugal force:** The pseudo force experienced by a particle performing uniform circular motion due to accelerated frame of reference which is along the radius and directed way from the centre is called centrifugal force.

NOTE:

- Pseudo force acts in non inertial frame i.e. accelerated frame of reference in which Neutron's law's of motion do not hold good.
- When a car moving along a horizontal curve takes a turn, the person in the car experiences a push in the outward direction.
- The coin placed slightly away from the centre of a rotating gramophone disc slips towards the edge of the disc.
- A cyclist moving fast along a curved road has to lean inwards to keep his balance

4. Difference Between Centripetal Force and Centrifugal Force

Centripetal Force	Centrifugal Force
• Centripetal force is directed along the radius. Towards the centre of the circle.	• Centrifugal force is directed along the radius, away from the centre of the circle.
• It is a real force.	• It is a pseudo force.
• It arises in both inertial and non-inertial frame of reference.	• It arises only in non-inertial frame of reference or in rotating frame reference
• Eg. when a satellite is revolving in circular orbit around the earth, the centripetal force is due to gravitational force of attraction.	• Eg. along a curved road the passenger in the vehicle has a feeling of push in the outward direction The push is due to centrifugal force.

5. Equations of Motions:

For constant angular acceleration-

(i) $\omega = \omega_0 + \alpha t$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$

6. Motion of a car on a plane circular road-

For motion without skidding

$$\frac{Mv_{max}^2}{r} = \mu M_g$$

$$\Rightarrow v_{max} \sqrt{\mu rg}$$

7. Motion on a banked road

Angle of banking $= \theta$

$$\tan\theta = \frac{h}{b}$$

Maximum safe speed at the bend

$$v_{max} = \left[\frac{rg(\mu + \tan\theta)}{1 - (\mu \tan\theta)}\right]^{1/2}$$

If friction is negligible

$$v_{max} = \sqrt{rg\tan\theta} = \sqrt{\frac{rhg}{b}} \text{ and } \tan\theta = \frac{v_{max}^2}{rg}$$

8. Motion of cyclist on a curve

In equilibrium angle with vertical is θ, then $\tan\theta = \frac{v^2}{rg}$

Maximum safe speed $= v_{max} = \sqrt{\mu rg}$

9. Motion in a vertical circle (particle tied to string)

At the top position – Tension $T_A = m\left(\frac{v_A^2}{r} - g\right)$

For $T_A = 0$, critical speed $= \sqrt{gr}$

At the bottom – Tension $T_B = m\left(\frac{v_B^2}{r} + g\right)$

For completing the circular motion minimum speed at the bottom $v_B = \sqrt{5gr}$

Tension $T_B = 6mg$

10. Conical Pendulum (Motion in a horizontal circle)

Tension in string $= \frac{mg\ell}{(\ell^2 - r^2)^{1/2}}$

Angular velocity $= \sqrt{\frac{g}{\ell\cos\theta}}$

Periodic time $= 2\pi\sqrt{\frac{\ell\cos\theta}{g}} = 2\pi\sqrt{\frac{r}{g\tan\theta}}$

Physics

Class11th NEET

07

CENTRE OF MASS, MOMENTUM AND COLLISION

Centre of Mass, Momentum & Collisions

1. Centre of Mass

In the first part of mechanics, we have discussed all about point objects. In this chapter, we shall deal with the cases of large objects or systems of point objects. To deal with such scenarios, we need to know about the centre of mass of an object or a system.

1.1 Definition

Centre of mass is a hypothetical point where the whole mass of the object is assumed to be concentrated mathematically. It is the weighted mean of the positions of all the point objects with masses M_1, M_2, M_3, ...,M_n respectively

Example :

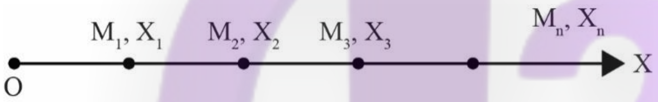


Fig. 7.1

then

$$X_{cm} = \frac{M_1x_1 + M_2x_2 + ...M_nx_n}{M_1 + M_2.... + M_n}$$

Similarly :

$$\vec{r}_{cm} = \frac{\sum M_i\vec{r}_i}{\sum M_i}$$

$$\Rightarrow X_{cm} = \frac{\sum M_ix_i}{\sum M_i} \text{ and } Y_{cm} = \frac{\sum M_iy_i}{\sum M_i}$$

1.2 Location of Centre of Mass

(a) For 2 point objects

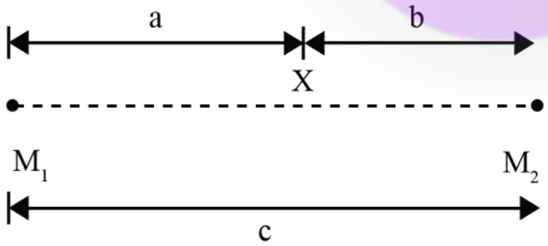


Fig 7.2

$$a = \frac{M_2c}{M_1 + M_2}$$

$$b = \frac{M_1c}{M_1 + M_2}$$

COM will be towards the heavier mass.

(b) For multiple systems of point objects

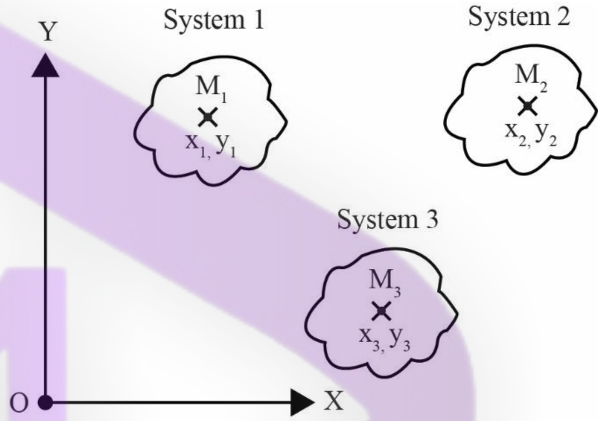


Fig 7.3

Consider three systems, where total mass and position of COM mass of all the particles in system 1, is M_1 and (x_1, y_1) respectively in system 2 is M_2 and (x_2, y_2) and in system 3 is M_3 and (x_3, y_3) respectively.

Then COM of all particles in all combined systems is

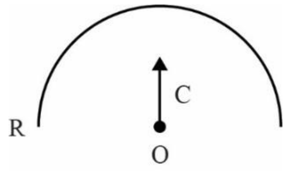
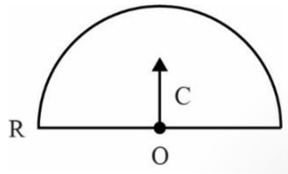
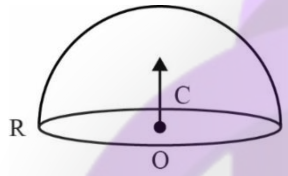
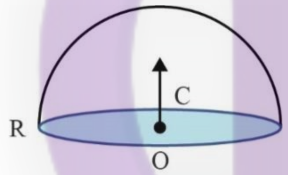
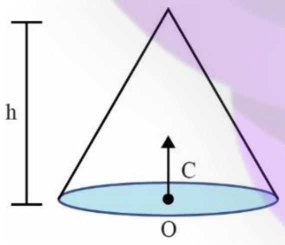
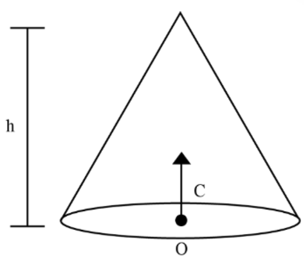
$$X_{cm} = \frac{M_1x_1 + M_2x_2 + M_3x_3}{M_1 + M_2 + M_3} \text{ and}$$

$$Y_{cm} = \frac{M_1y_1 + M_2y_2 + M_3y_3}{M_1 + M_2 + M_3}$$

(c) For objects with continuous mass distributions:

COM of objects with continuous mass distribution can be found with help of integration. Presently we shall focus on the location of COM for some objects whose mass is continuously distributed.

S.No	Shapes	COM
1.	X Uniform Rod	$x = \frac{L}{2}$

2.	 Semicircular ring	$y = \frac{2R}{\pi}$
3.	 Semicircular disk	$y = \frac{4R}{3\pi}$
4.	 Hollow hemisphere	$y = \frac{R}{2}$
5.	 Solid hemisphere	$y = \frac{3R}{8}$
6.	 Solid right circular cone	$y = \frac{h}{4}$
7.	 Hollow right circular cone	$y = \frac{2h}{3}$

1.3 Motion of Centre of Mass

We know

$$\vec{r}_{cm} = \frac{M_1\vec{r}_1 + M_2\vec{r}_2 + M_3\vec{r}_3 + ...M_n\vec{r}_n}{M_1 + M_2 + ...M_n}$$

Differentiating both sides with respect to time,

$$\vec{V}_{cm} = \frac{M_1\vec{v}_1 + M_2\vec{v}_2 + ...M_n\vec{v}_n}{M_1 + M_2 + ...M_n}$$

Again, differentiating both sides w.r.t. time

$$\vec{A}_{cm} = \frac{M_1\vec{a}_1 + M_2\vec{a}_2 + ...M_n\vec{a}_n}{M_1 + M_2 + ...M_n}$$

1.4 Properties and Application of COM

(a) Entire mass is supposed to be concentrated at COM.

(b) If some force is applied on a free object, the body does not rotate if line of action of force passes through centre of mass.

We know

$$\vec{A}_{cm} = \frac{M_1\vec{a}_1 + M_2\vec{a}_2 + ...M_n\vec{a}_n}{M_1 + M_2 + ...M_n}$$

$$\Rightarrow M\vec{A}_{cm} = M_1\vec{a}_1 + M_2\vec{a}_2 + M_n\vec{a}_n$$

By Newton's 2nd law.

$M_1\vec{a}_1 = \vec{F}_1$ and similarly for all objects, with terms having the obvious meanings.

$$\therefore \Rightarrow \vec{F}_1 + \vec{F}_2 + ...\vec{F}_n = M\vec{A}_{cm}$$

$$\Rightarrow \sum \vec{F}_{ext} = M\vec{A}_{cm}$$

The above relation is very useful in solving complicated problems.

(c) We know

$$\vec{V}_{cm} = \frac{M_1\vec{v}_1 + M_2\vec{v}_2 + ...M_n\vec{v}_n}{M_1 + M_2 + ...M_n}$$

$$\Rightarrow M\vec{v}_{cm} = M_1\vec{v}_1 + M_2\vec{v}_2 + ...M_n\vec{v}_n$$

$$\vec{P}_{sys} = \vec{P}_1 + \vec{P}_2 + ...\vec{P}_n$$

Total momentum of system of all particles is total mass times $\vec{V}_{cm}$.

Above relation helps us in applying momentum related equations in complex problems.

1.5 Example of Motion of COM and its Applications

(a) Trajectory of COM remains unchanged on disintegration of an unstable nucleus or a bomb till the time $\sum \vec{F}_{ext}$ on the system does not change.

(b) Motion of binary stars.

(c) Conservation of momentum during disintegration of an unstable nucleus.

(d) Motion of earth-moon system about sun.

2. Linear Momentum

The total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass $\vec{P} = M\vec{v}_{cm}$.

2.1 Linear Momentum Conservation in Presence of External Force

$\vec{F}_{ext} = \frac{d\vec{P}}{dt} \Rightarrow \vec{F}_{ext}dt = d\vec{P}$

$\Rightarrow d\vec{P} = \left(\vec{F}_{ext}\right)_{impulsive} dt$

$\therefore \text{If } \left(\vec{F}_{ext}\right)_{impulsive} = 0$

$\Rightarrow d\vec{P} = 0 \text{ or } \vec{P} \text{ is constant}$

NOTE:

Momentum is conserved if the external force present is non-impulsive. Eg. Gravitation or spring force.

When the vector sum of the external forces acting on a system is zero, the total linear momentum of the system remains constant.

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \ldots\ldots + \vec{P}_n = \text{constant}$

3. Impulse

Impulse of a force $\vec{F}$ acting on a body for the time interval $t = t_1$ to $t = t_2$ is defined as :-

$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt$

$\vec{I} = \int \vec{F}dt = \int m\frac{d\vec{v}}{dt}dt = \int_{v_1}^{v_2} md\vec{v}$

$\vec{I} = m\left(\vec{v}_2 - \vec{v}_1\right) = \Delta\vec{P} = \text{change in momentum due to force } \vec{F}$

.

NOTE:

Impulse applied to an object in a given time interval can also be calculated from the area under force time (F-t) graph in the same time interval.

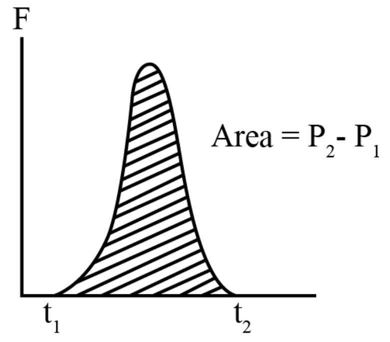


Fig. 7.4

3.1 Instantaneous Impulse

There are many cases when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknow but the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write.

$\vec{I} = \int \vec{F}dt = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$

3.1.1 Important Points

- It is a vector quantity.
- Dimensions = $[MLT^{-1}]$
- SI unit = kg m/s
- Direction is along change in momentum.
- Magnitude is equal to area under the F-t graph.
- $\vec{I} = \int \vec{F}dt = \vec{F}_{av}\int dt = \vec{F}\Delta t$
- It is not a property of a particle, but it a measure of the degree to which an external force changes the momentum of the particle.

3.2 Average Force

We can now define the average force which acts on a particle during a time interval Δt . It is:

$\vec{F} = \frac{\Delta p}{\Delta t} = \frac{\vec{I}}{\Delta t}$

The value of the average force depends on the time chosen.

4. Collisions

We define a collision as an isolated event in which two or more colliding bodies exert relatively strong forces on each other for a relatively short time.
Two key rules of the collision game are :
(i) Law of conservation of linear momentum, and
(ii) Law of conservation of energy.

4.1 Types of Collision

Collisions between particles have been divided broadly into two types :

(a) Elastic collision : A collision in which there is absolutely no loss of kinetic energy is called an elastic collision. For example, collisions between atomic and subatomic particles are elastic collisions. Practically a collision between two ivory balls can also be taken as an elastic collision.

The basic characteristics of an elastic collision are:
(i) The linear momentum is conserved,
(ii) Total energy of the system is conserved,
(iii) The kinetic energy is conserved.
(iv) The forces involved during elastic collisions must be conservative forces.

(b) Inelastic collision : A collision in which there occurs some loss of kinetic energy is called an inelastic collision. As there is always some loss of kinetic energy in most of the collisions, therefore, collisions we come across in daily life are generally inelastic.

The basic characteristics of an inelastic collision are :
(i) The linear momentum is conserved
(ii) Total energy is conserved.
(iii) Kinetic energy is NOT conserved. Obviously, a part of kinetic energy is converted into some other form of energy e.g., heat energy, sound energy etc.
(iv) Some or all the forces involved in an inelastic collision may be non-conservative in nature.

NOTE:

A perfectly inelastic collision is one in which maximum amount of kinetic energy is lost.

4.2 Coefficient of Restitution or Coefficient of Resilience

Coefficient of restitution or coefficient of resilience of a collision is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is represented by 'e'.

$$e = \frac{\text{relative velocity of separation (after collision)}}{\text{relative velocity of approach (before collision)}}$$

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

where u_1, u_2 are velocities of two bodies before collision, and v_1, v_2 are their respective velocities after collision.

- For a perfectly elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.
  $\therefore \boxed{e = 1}$
- For a perfectly inelastic collision, relative velocity of separation after collision = 0
  $\therefore \boxed{e = 0}$
- For all other collisions, e lies between 0 and 1, i.e. $0 < e < 1$.

4.3 Elastic Collision in One Dimension

It involves two bodies moving initially along the same straight line, striking against each other without loss of kinetic energy and continuing to move along the same straight line after collision.
Suppose two balls A and B of masses m_1 and m_2 are moving initially along the same straight line with velocities u_1 and u_2 respectively, figure (a).

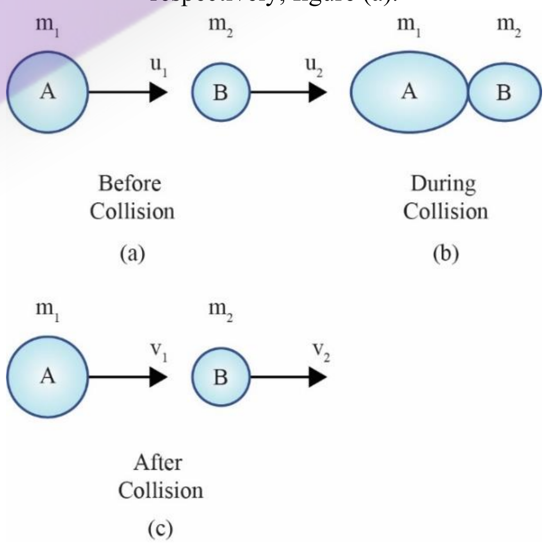


Fig 7.5

When $u_1 > u_2$,
Relative velocity of approach before collision, $= u_1 - u_2$

Therefore, two balls collide, as shown in figure (b). Let the collision be perfectly elastic. After collision, suppose v_1 is the velocity of A and v_2 is the velocity of B along the same straight line, as shown in figure (c).
When $v_2 > v_1$, the bodies separate after collision.
Relative velocity of separation after collision $= v_2 - v_1$
Linear momentum of the two balls before collision
$= m_1u_1 + m_2u_2$ ..(1)
Linear momentum of the two balls after collision
$= m_1v_1 + m_2v_2$ ..(2)

As linear momentum is conserved in an elastic collision, therefore from equations (1) and (2)
$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$
or, $m_2 (v_2 - u_2) = m_1 (u_1 - v_1)$ ...(3)

Total K.E. of the two balls before collision

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \quad ...(4)$$

Total K.E. of the two balls after collision

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad ...(5)$$

As K.E. is also conserved in an elastic collision, therefore from equations (4) and (5),

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

Or, $\frac{1}{2}m_2\left(v_2^2 - u_2^2\right) = \frac{1}{2}m_1\left(u_1^2 - v_1^2\right)$

Or, $m_2\left(v_2^2 - u_2^2\right) = m_1\left(u_1^2 - v_1^2\right)$ ...(6)

Dividing, (6) by (3) we get

$$\frac{m_2\left(v_2^2 - u_2^2\right)}{m_2\left(v_2 - u_2\right)} = \frac{m_1\left(u_1^2 - v_1^2\right)}{m_1\left(u_1 - v_1\right)}$$

Or, $\frac{(v_2 + u_2)(v_2 - u_2)}{(v_2 - u_2)} = \frac{(u_1 + v_1)(u_1 - v_1)}{(u_1 - v_1)}$

Or, $v_2 + u_2 = u_1 + v_1$
Or, $v_2 - v_1 = u_1 - u_2$ ...(7)
General equation, $\boxed{\vec{v}_2 - \vec{v}_1 = \vec{u}_1 - \vec{u}_2}$

Hence, in one dimensional elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.

From (7), $\frac{v_2 - v_1}{u_1 - u_2} = 1$

By definition, $\frac{v_2 - v_1}{u_1 - u_2} = e = 1$ (For perfectly elastic collision)

Hence, the coefficient of restitution/resilience of a perfectly elastic collision in one dimension is unity.

4.3.1 Calculation of velocities after collision

Velocity of A:
From (7), $v_2 = u_1 - u_2 + v_1$
Putting in (3),
we get $m_1v_1 + m_2 (u_1 - u_2 + v_1) = m_1u_1 + m_2 u_2$
$\Rightarrow m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1 = m_1u_1 + m_2u_2$
$\Rightarrow v_1 (m_1 + m_2) = (m_1 - m_2) u_1 + 2 m_2 u_2$

$$\Rightarrow v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2} \quad ...(8)$$

General equation, $\vec{v}_1 = \frac{(m_1 - m_2)\vec{u}_1}{m_1 + m_2} + \frac{2m_2\vec{u}_2}{m_1 + m_2}$

Velocity of B :
Put this value of v_1 from (6) in (3),

$$v_2 = u_1 - u_2 + \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

$$= u_1\left[1 + \frac{m_1 - m_2}{m_1 + m_2}\right] + u_2\left[\frac{2m_2}{m_1 + m_2} - 1\right]$$

$$= u_1\left[\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right] + u_2\left[\frac{2m_2 - m_1 - m_2}{m_1 + m_2}\right]$$

$$\Rightarrow v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{m_1 + m_2} \quad ...(9)$$

General equation, $\vec{v}_2 = \frac{2m_1\vec{u}_1}{m_1 + m_2} + \frac{(m_2 - m_1)\vec{u}_2}{m_1 + m_2}$

NOTE:

The expression for v_2 can be obtained from the expression for v_1, by replacing m_1 by m_2 and u_1 by u_2. The reverse is also true, i.e., v_1 can also be obtained from v_2 similarly.

4.3.2 Special Cases

1. **When masses of two bodies are equal,**
   i.e., $m_1 = m_2 = m$, say
   From (8), $v_1 = \frac{2\,mu_2}{2m} = u_2$,
   i.e., velocity of A after collision = velocity of B before collision.

From (9), $v_2 = \frac{2m_1u_1}{2m_1} = u_1$,

i.e., velocity of B after collision = velocity of A before collision.

Hence, when two bodies of equal masses undergo a perfectly elastic collision in one dimension, their velocities are just interchanged.

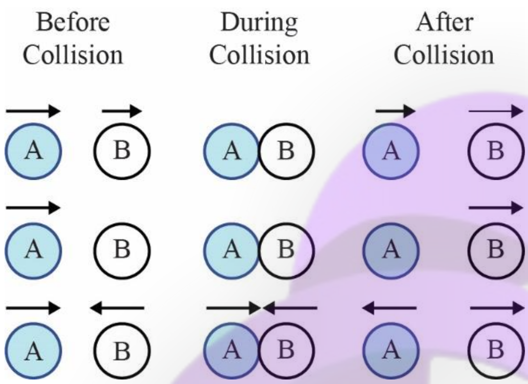


Fig 7.6

2. **When the target body B is initially at rest,** i.e., $u_2 = 0$

From (8), $v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2}$ ...(10)

From (9), $v_2 = \frac{2m_1u_1}{m_1 + m_2}$ ...(11)

Three cases arises further :

(a) **When masses of two bodies are equal,** i.e., $m_1 = m_2$
using equation (10),
From, $v_1 = 0$

From, $v_2 = \frac{2m_1u_1}{2m_1} = u_1$

i.e., body A comes to rest and body B starts moving with the initial velocity of A. Obviously, in such a collision, 100% K.E. of A is transferred to the body B. This is shown in figure above.

(b) **When body B at rest is very heavy,** i.e., $m_2 >> m_1$, i.e., m_1 can be ignored compared to m_2
Putting $m_1 = 0$ in equation (8), we obtain

$v_1 = -\frac{m_2}{m_2}u_1 = -u_1 ; v_2 = 0$

Hence, when a light body A collides against a heavy body B at rest; A rebounds with its own velocity and B continues to be at rest. This is what happens when a ball rebounds to the same height from which it was thrown, on striking a floor.

(c) **When body B at rest has negligible mass,**
i.e., $m_2 << m_1$; i.e., m_2 can be ignored compared to m_1
Putting $m_2 = 0$, in equation (8),

we get $v_1 = \frac{m_1}{m_1}u_1 = u_1 . ; v_2 = \frac{2m_1u_1}{m_1} = 2u_1$

Hence, when a heavy body A undergoes an elastic collision with a light body B at rest, the body A keeps on moving with the same velocity of its own and the body B starts moving with double the initial velocity of A.

4.4 Inelastic Collision in One Dimension

Figure below shows two bodies of masses m_1 and m_2 moving with velocities, u_1 and u_2 respectively, along a single axis. They collide involving some loss of kinetic energy. Therefore, the collision is inelastic. Let v_1 and v_2 be the velocities of the two bodies after collision.

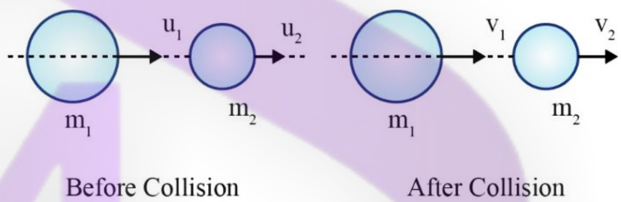


Fig 7.7

As the two bodies form one system, which is closed and isolated, we can write the law of conservation of linear momentum for the two body system as :
Total momentum before collision (P_i) = Total momentum after collision (P_f)
$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ ...(12)

(The overhead arrows for vectors have been avoided as the collision is one dimensional and velocity components along one axis are used.)

If we know masses m_1, m_2, initial velocities u_1, u_2 and one of the final velocities, we can calculate the other final velocity from the equation.

Figure shows perfectly inelastic collision between two bodies of masses m_1 and m_2. The body of mass m_2 happens to be initially at rest ($u_2 = 0$), we refer to this body as the target. The incoming body of mass m_1, moving with initial velocity u_1 is referred to as the projectile. After the collision, the two bodies move together with a common velocity V. The collision is perfectly inelastic.

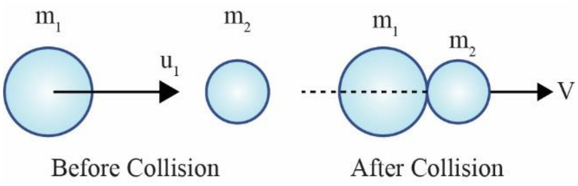


Fig 7.8

As the total linear momentum of the system remains constant, therefore $P_i = P_f$

i.e., $m_1u_2 + m_2u_2 = (m_1 + m_2) V$

or, $m_1u_1 = (m_1 + m_2)$ $(\because u_2 = 0)$

or, $V = \dfrac{m_1u_1}{m_1 + m_2}$

General equation, $\boxed{\vec{V} = \dfrac{m_1\vec{u}_1}{m_1 + m_2}}$

4.5 Oblique Collision

If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a **one-dimensional collision or head-on collision.**

In the case of small spherical bodies, this is possible if the direction of travel of body 1 passes through the centre of body 2.

When two bodies travelling initially along the same straight line collide without loss of kinetic energy and move along different directions in a plane after collision, the collision is said to be an elastic collision in two dimensions.

Suppose m_1, m_2 are the masses of two bodies A and B moving initially along the X–axis with velocities u_1 and u_2 respectively. When $u_1 > u_2$, the two bodies collide. After collision, let the body A move with a velocity v_1 at an angle θ with X–axis. Let the body B move with a velocity v_2 at an angle ϕ with X–axis as shown in figure.

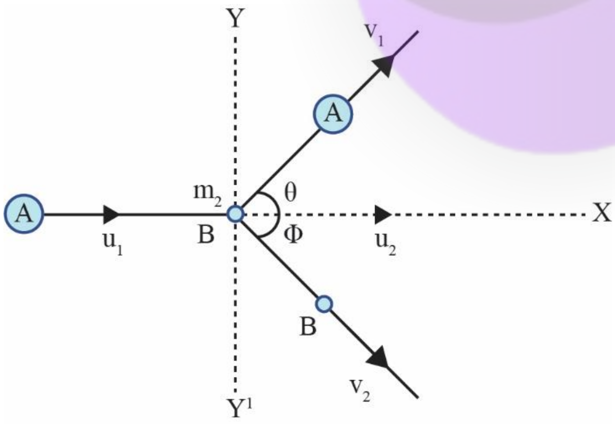


Fig 7.9

As the collision is elastic, kinetic energy is conserved.

∴ Total K.E. after collision = Total K.E. before collision

Or $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$ ...(13)

Or $m_1v_1^2 + m_2v_2^2 = m_1u_1^2 + m_2u_2^2$ ...(14)

As linear momentum is conserved in elastic collision, therefore, along the X–axis, total linear momentum after collision = total linear momentum before collision.

$m_1v_1 \cos θ + m_2v_2 \cos ϕ = m_1u_1 + m_2u_2$ ...(15)

Now, along Y–axis, linear momentum before collision is zero (as both the bodies are moving along X–axis). And after collision, total linear momentum along Y–axis is $(m_1v_1 \sin θ – m_2v_2 \sin ϕ)$

$\Rightarrow m_1v_1 \sin θ – m_2v_2 \sin ϕ = 0$ ...(16)

From three equations (14), (15) and (16), we have to calculate four variables v_1, v_2, θ and ϕ, which is not possible. We have, therefore, to measure experimentally any one parameter, i.e., final velocities v_1,v_2 of A, B or their direction θ and ϕ. The rest of the three parameters can then be calculated from the three equations.

When two bodies travelling initially along the same straight line collide involving some loss of kinetic energy, and move after collision, along different directions in a plane, the collision is said to be inelastic collision in two dimensions.

4.5.1 Perfectly inelastic collision in two dimensions

Figure shows perfectly inelastic collision between two bodies of masses m_1 and m_2. The body of mass m_2 is moving initially with velocity u_2 along X–axis. The body of mass m_1 is moving with velocity u_1 at an angle θ with X–axis as shown.

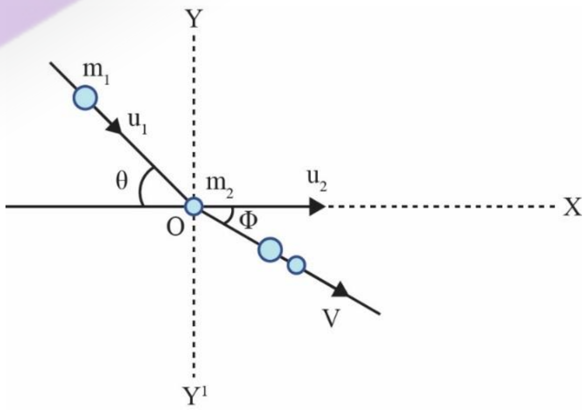


Fig 7.10

After the collision at O, the two bodies stick to each other and move with a common velocity V at an angle ϕ with X–axis as shown in figure.

As the system is closed and isolated, the total linear momentum of the system remains constant.

Referring to figure, and equating initial momentum along X–axis to final momentum along the same axis, we get

$m_1u_1 \cos\theta + m_2u_2 = (m_1 + m_2) V \cos\phi$ ...(17)

Again, applying the law of conservation of linear momentum along y–axis, we get.

$m_1u_1 \sin\theta + 0 = (m_1 + m_2) V \sin\phi$ ...(18)

Knowing m_1, m_2 ; u_1, u_2 and θ, we can calculate final velocity V and its direction, i.e., $\angle\phi$ from equations (17) and (18).

NCERT Corner

Important Points to Remember

1. Centre of Mass

Centre of mass of a system is the point that behaves as whole mass of the system is concentrated on it. For rigid bodies, centre of mass is independent of the state of the body, i.e. whether it is in rest or in accelerated motion, centre of mass will remain same.

- Centre of mass of two particles system,

$$\vec{x}_{CM} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1 + m_2}$$

Similarly, x_{CM} for n particle system.

$$\vec{x}_{CM} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 + ... + m_n\vec{x}_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

- Velocity of centre of mass (n-particles system).

$$\vec{V}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}{m_1 + m_2 + ... + m_n}$$

- Acceleration of centre of mass,

$$\vec{A}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n}{m_1 + m_2 + ... + m_n}$$

- Momentum of centre of mass.

$$\vec{P}_{CM} = \frac{m_1\vec{p}_1 + m_2\vec{p}_2 + ... + m_n\vec{p}_n}{m_1 + m_2 + ... + m_n}$$

2. Linear Momentum

The total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass. $\vec{P} = M\vec{v}_{cm}$

3. Impulse

Impulse of a force $\vec{F}$ acting on a body for the time interval $t = t_1$ to $t = t_2$ as: $I = \int_{t_1}^{t_2} F \cdot dt$

And also,

$$\vec{I} = m(\vec{v}_2 - \vec{v}_1) = \Delta\vec{P}$$

= Change in momentum due to force $\vec{F}$.

4. Conservation of Linear Momentum

According to law of conservation of linear momentum, total linear momentum of a system of particles remain constant or conserved in the absence of any external force.

i.e, When $\vec{F}_{ext} = 0$

$$\Rightarrow \frac{d\vec{p}}{dt} = 0$$

$$\Rightarrow \vec{p} = \text{constant}$$

i.e. $\vec{p}_{initial} = \vec{p}_{final}$

Also, for n number of particles

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...\vec{p}_n = \text{constant}$$

- For collision of two bodies, the total momentum before collision remains the same as the total momentum after the collision.
  i.e., $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$
- Recoil velocity of gun is calculated by, $v_2 = -\frac{m_1 v_1}{m_2}$
  where, m_2 = mass of the gun, m_1 = mass of bullet and v_1 = velocity of the bullet.

5. Collision

- It is an isolated event, in which two or more colliding bodies exert strong forces on each other for a short duration of time.
- It is mainly of two types: elastic and inelastic collision.
- For every type of collision, linear momentum of colliding body or system is conserved.
  i.e $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
  where, m_1 and m_2 = masses of the body which undergo collision.
  u_1 = initial velocity of the body of mass m_1,
  u_2 = initial velocity of the body of mass m_2,
  v_1 = final velocity of the body of mass m_1, and
  v_2 = final velocity of the body of mass m_2.
- But kinetic energy of the colliding body and system is conserved in elastic collision only.

6. Coefficient of Restitution (e)

It is the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is expressed as $e = \frac{|v_2 - v_1|}{|u_1 - u_2|}$, where

$0 \le e \le 1$.

(i) For perfectly inelastic collision, $e = 0$.
(ii) For perfectly elastic collision, $e = 1$ and for inelastic collision $0 < e < 1$.
(iii) For other collisions, it can be $0 < e < 1$.

7. Head-on Collision

- For bodies with masses m_1 and m_2 respectively following are the important relations for head-on collision**.**

  (i) When collision is elastic, final velocities for m_1 i.e,

  $$v_1 = \frac{(m_1 - m_2)}{m_1 + m_2} u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

  and for m_2, $v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}$

  (ii) When collision is inelastic

  Final velocities for m_1,

  $$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right) u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right) u_2$$

  and for m_2,

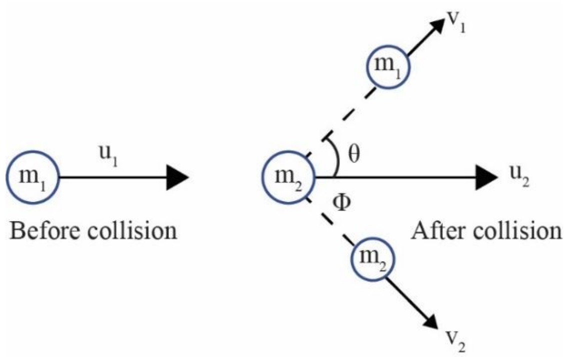
  $$v_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right) u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right) u_2$$

- If after collision, approaching bodies move with a common velocity, i.e. $e = 0$ (get stuck with one another). then collision is said to be **perfectly inelastic.**

8. For perfectly elastic oblique collision

Along X-axis, $m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi$

Along Y-axis, $0 = m_1v_2\sin\theta - m_2v_2\sin\phi$



- If two bodies of equal masses undergo perfect elastic oblique collision then scattering angle $\theta + \phi = \frac{\pi}{2}$ and $u_1^2 = v_1^2 + v_2^2$.

9. Rebounding of a Ball on collision with Floor.

- Speed of the ball after nth rebound, $v_n = e^n v_0 = e^n\sqrt{2gh}$
- Height covered by the ball after nth rebound, $h_n = e^{2n}h$
- Total distance s covered by the ball before it stops bouncing , $s = h\left(\frac{1+e^2}{1-e^2}\right)$

  where, h = height of the ball dropped from ground and e = coefficient of restitution

Physics

Class 11th NEET

08

ROTATIONAL MOTION

Rotational Motion

1. Kinematic of the System of Particles

System of particles can move in different ways as observed by us in daily life. To understand this, we need to understand few new parameters.

Rigid body: A body in which distance between any two particles remain same regardless of any external changes.

1.1 Kinematic of Rotational Motion

(i) Angular Displacement

Consider a particle moving from A to B in the following figures.

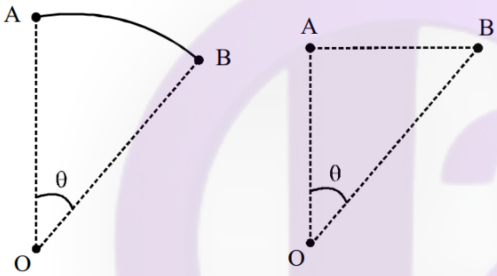


Fig. 8.1

Angle θ is the angular displacement of the particle about O.

Unit: radian (rad).

(ii) Angular Velocity

The rate of change of angular displacement is called as angular velocity.

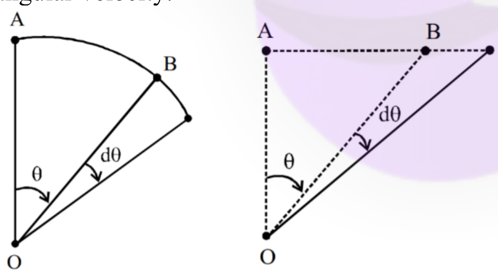


Fig. 8.2

Instantaneous Angular Velocity

$$\omega = \frac{d\theta}{dt}$$

Average Angular Velocity

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Unit $\rightarrow$ Rad/s.

Angular velocity is a vector quantity whose direction is given by right hand thumb rule.

According to right hand thumb rule, if we curl the fingers of right hand along the direction of angular displacement then the right-hand thumb gives us the direction of angular velocity. It is always along the axis of the rotation.

(iii) Angular Acceleration

Angular acceleration of an object about any point is rate of change of angular velocity about that point.

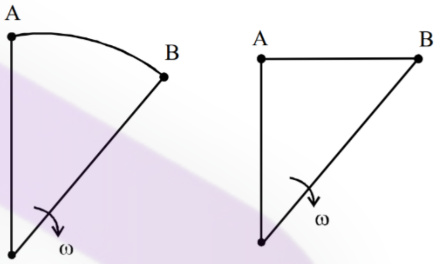


Fig.8.3

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d\omega}{dt}\cdot\frac{d\theta}{d\theta} = \omega\frac{d\omega}{d\theta}$$

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

Unit $\rightarrow$ Rad/s^2.

Angular acceleration is also a vector quantity.

If α is constant, then like equations of translatory motion we can also write relations between ω, α θ and t.

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\Delta\theta$$

Here, ω_0 is initial angular velocity and ω is final angular velocity.

1.2 Various Types of Motion

(i) Translational Motion

A system is said to be in translational motion, if all the particles within the system have same linear velocity

Example: Motion of a rod as shown below.

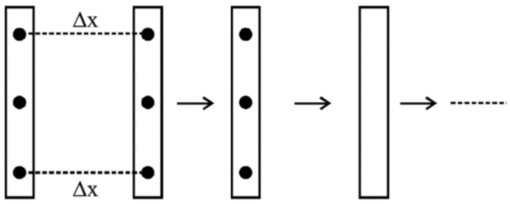


Fig. 8.4

Example: Motion of body of car on a straight rod.

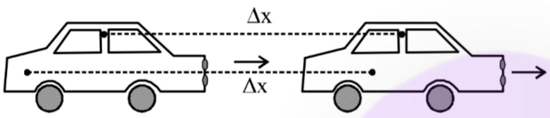


Fig.8.5

In both the above examples, velocity of all the particles is same as they all have equal displacements in equal intervals of time.

(ii) Rotational Motion

An object is said to be in pure rotational motion, when all the points lying on the system are in circular motion about one common fixed axis.

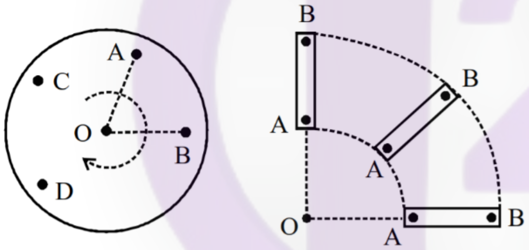


Fig.8.6

In pure rotational motion, angular velocity of all the points is same about the fixed axis.

(iii) Rotational + Translational motion

An object is said to be in rotational + translational motion, when the particle is rotating with some angular velocity about a movable axis.

For Example

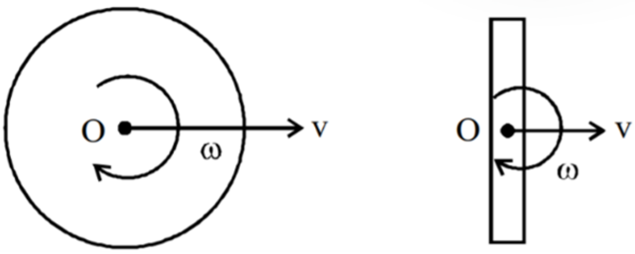


Fig.8.7

v = velocity of axis.

ω = Angular velocity of system about O.

1.3 Relationship Between Kinematics Variables

In general, if a body is rotating about any axis (fixed or movable), with angular velocity ω and angular acceleration α, then velocity of any point p with respect to axis is

$\vec{v}_p = \vec{\omega} \times \vec{r}$

$\vec{a} = \vec{a}_t + \vec{a}_r$

$\vec{a}_t = \vec{\alpha} \times \vec{r}$

$\vec{a}_r = \vec{\omega} \times \vec{v}$

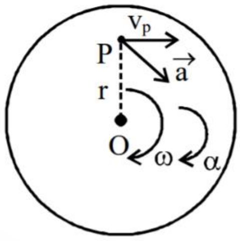


Fig.8.8

Example

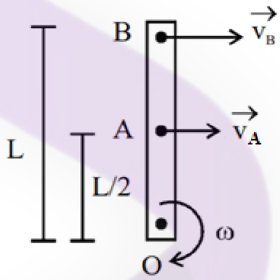


Fig.8.9

$v_B = \omega L$ and $v_A = \dfrac{\omega L}{2}$, with directions as shown in the figure above.

Now in rotational + translational motion, we just superimpose velocity and acceleration of axis on the velocity and acceleration of any point about the axis of rotation. (i.e.)

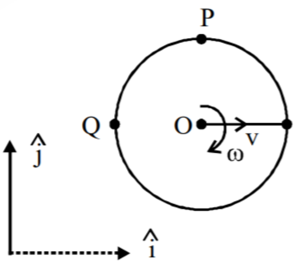


Fig.8.10

$\vec{v}_{PO} = \omega R\hat{i}$

$\vec{v}_O = v\hat{i}$

$\because \quad \vec{v}_P - \vec{v}_O = \vec{v}_{PO}$

$\Rightarrow \quad \vec{v}_P = \vec{v}_{PO} + \vec{v}_O = (\omega R + v)\hat{i}$

Similarly, $\vec{v}_{QO} = \omega R\hat{j}$

$\vec{v}_O = v\hat{i}$

$\therefore \quad \vec{v}_Q = v\hat{i} + \omega R\hat{j}$

2. Rotational Dynamics

2.1 Torque

Similar to force, the cause of rotational motion is a physical quantity called a torque/moment of force/angular force.
Torque incorporates the following factors.

- Amount of force.
- Point of application of force.
- Direction of application of force.

Combining all the above,
Torque about point O,

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = r.F\sin\theta$

Where,
r = distance from the point O to point of application of force.
F = force
θ = angle between $\vec{r}$ and $\vec{F}$

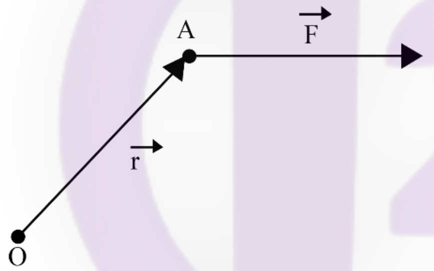


Fig. 8.11

Magnitude of torque can also be rewritten as

$\tau = rF_\perp$ or $\tau = r_\perp F$

Where,

$F_\perp$ = component of force in the direction perpendicular to $\vec{r}$.

$r_\perp$ = component of distance in the direction perpendicular to $\vec{F}$.

(i) Direction of Torque:
Direction of torque is given by right hand thumb rule. If we curl the fingers of right hand from first vector $(\vec{r})$ to the second vector $(\vec{F})$ then right-hand thumb gives us direction of their cross product, i.e., the torque.

(ii) Some Important Points about Torque:
Torque is always defined about a point or about an axis.

When there are multiple forces, the net torque needs to be calculated. i.e., all torque about same point/axis.

$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} + \ldots\vec{\tau}_{F_n}$

- If $\sum\tau = 0$, then the body is said to be in rotational equilibrium.
- If $\sum F = 0$ along with $\sum\tau = 0$, then body is said to be in mechanical equilibrium (Translation and rotational equilibrium).
- If two forces of equal magnitude, opposite direction and do not share a line of action act to produce same torque, then they constitute a couple. It does not produce any translation, only rotation.
- For calculating torque, it is very important to know the effective point of application of force.

2.2 Newton's Law in Rotation

$\sum\tau = I\alpha$

Where, I = moment of Inertia

α = Angular Acceleration

3. Moment of Inertia

Moment of inertia gives the measure of mass distribution about an axis.

$I = \sum m_i r_i^2$

Where r_i = Perpendicular distance of the i_{th} mass from the axis of rotation.
Moment of inertia is always defined about an axis.

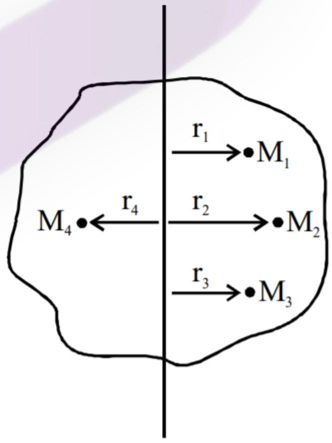


Fig.8.12

For example, moment of inertia for above case,

$I = M_1r_1^2 + M_2r_2^2 + M_3r_3^2 + M_4r_4^2$

- SI unit $\rightarrow$ kg-m^2
- Gives the measure of rotational inertia and is analogous to mass in linear motion.

3.1 Moment of Inertia of a Discreet Particle System :

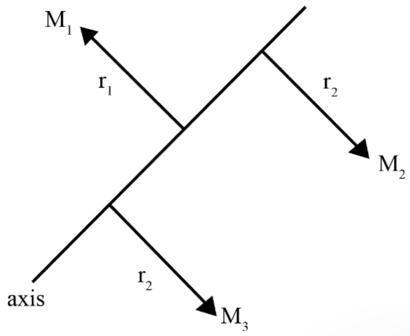


Fig.8.13

$I = M_1 r_1^2 + M_2 r_2^2 + M_3 r_3^2$

3.2 Moment of Inertia of Continuous Bodies

When the distribution of mass of a system of particle is continuous, the discrete sum $I = \sum m_1 r_1^2$ is replaced by an integral. The moment of inertia of the whole body takes the form

$I = \int r^2 dm$

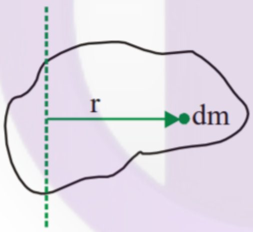


Fig.8.14

Keep in mind that here the quantity r is the perpendicular distance of point mass from axis of rotation, not the distance to the origin. To evaluate this integral, we must express m in terms of r.

3.5 Moment of Inertia of Some Important Bodies

(i) Circular Ring

Axis passing through the centre and perpendicular to the plane of ring.

$I = MR^2$

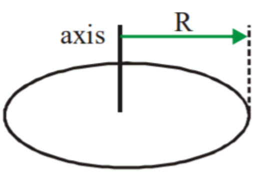


Fig.8.15

(ii) Hollow Cylinder

$I = MR^2$

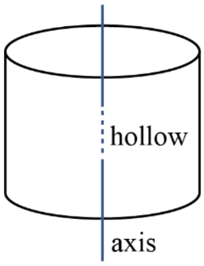


Fig.8.16

(iii) Solid Cylinder and a Disc

About its geometrical axis as shown below

$I = \frac{1}{2} MR^2$

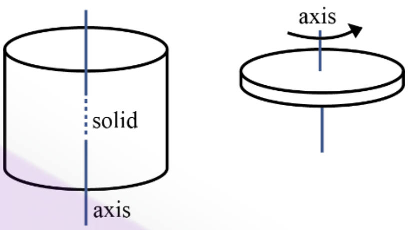


Fig.8.17

(iv) Sphere

Solid Sphere: Axis passing through the centre of mass

$I = \frac{2}{5} MR^2$

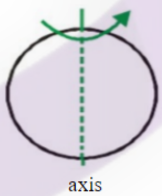


Fig.8.18

Hollow Sphere :

Axis passing through the centre of mass,

$I = \frac{2}{3} MR^2$

(v) Thin Rod of length *l* :

- Axis passing through midpoint and perpendicular to length :

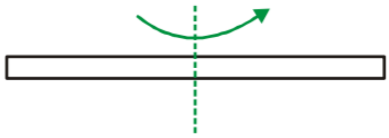


Fig.8.19

$I = \frac{M\ell^2}{12}$

- Axis passing through an end and perpendicular to the rod:

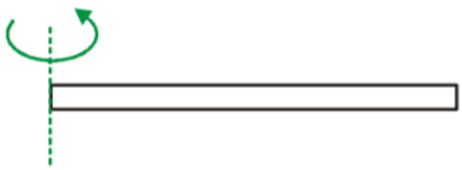


Fig.8.20

$$I = \frac{M\ell^2}{3}$$

3.4 Theorems on Moment of Inertia

(i) **Parallel Axis Theorem:** Let I_{CM} be the moment of inertia of a body about an axis through its centre of mass and let I_P be the moment of inertia of the same body about another axis which is parallel to the first one. If d is the distance between these two parallel axes and M is the mass of the body then according to the parallel axis theorem :

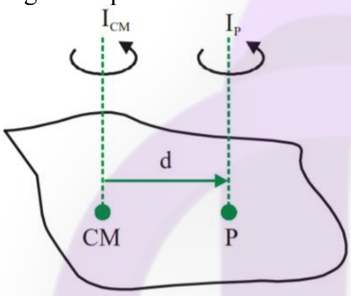


Fig.8.21

$$I_P = I_{cm} + Md^2$$

(ii) **Perpendicular Axis Theorem :**

Consider a planar body (i.e., a body of zero thickness) of mass M. Let X and Y axes be two mutually perpendicular lines in the plane of the body. The axes intersect at origin O.

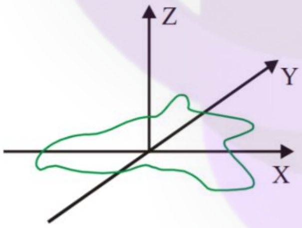


Fig.8.22

Let I_x = moment of inertia of the body about X–axis.

Let I_y = moment of inertia of the body about Y–axis.

Then the moment of inertia of the body about Z– axis (Passing through O and perpendicular to the plane of the body) is given by :

$$I_z = I_x + I_y$$

The above result is known as the perpendicular axis theorem.

3.5 Radius of Gyration

If M is the mass and I is the moment of inertia of a rigid body about a given axis then the radius of gyration (K) of the body about that axis is given by :

$$K = \sqrt{\frac{I}{M}}$$

e.g. $K_{ring} = R$, $K_{disc} = \frac{R}{\sqrt{2}}$ (About an axis passing through the com and perpendicular to the plane of body)

4. Angular Momentum and Angular Impulse

4.1 Angular Momentum

(i) **For a particle**

Angular momentum about a point (O) is given as

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times \left(m\vec{v}\right) = m\left(\vec{r} \times \vec{v}\right)$$

where $\vec{r}$ is position vector of the particle w.r.t. O and $\vec{v}$ is velocity of particle

(ii) **For a particle moving in a circle**

For a particle moving in a circle of radius r with a speed v, its linear momentum is mv, magnitude of angular momentum (L) is given as :

$$L = mvr_\perp = mvr$$

As θ being 90°, $\sin 90° = 1$

Direction of $\vec{L}$ is out of the plane of circle.

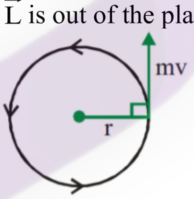


Fig.8.23

(iii) **For a rigid body (about a fixed axis)**

L = sum of angular momentum of all particles about that axis

$$\Rightarrow L = m_1v_1r_1 + m_2v_2r_2 + m_3v_3r_3 + ...$$

$$\Rightarrow L = m_1r_1^2\omega + m_2r_2^2\omega + m_3r_3^2\omega + ... \qquad (v = r\omega)$$

$$\Rightarrow L = \left(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + ...\right)\omega$$

$$\Rightarrow L = I\omega$$

Angular momentum is also a vector and its direction is same as that of ω

We know that,

$$\vec{L} = I\vec{\omega}$$

$$\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} = \vec{\tau}_{net} \text{ (Considering I constant)}$$

Similar to the definition of linear force in linear motion, Torque can be defined as the rate of change of angular momentum.

4.2 Conservation of Angular Momentum

If $\tau_{net} = 0$

$$\Rightarrow \frac{d\vec{L}}{dt} = 0$$

$$\Rightarrow \vec{L} = \text{constant}$$

$$\Rightarrow \vec{L}_f = \vec{L}_i$$

4.2 Angular Impulse

$$\vec{J} = \int \vec{\tau} dt = \Delta \vec{L}$$

5. Work and Energy

5.1 Work Done by a Torque

Consider a rigid body acted upon by a force F at perpendicular distance r from the axis of rotation. Suppose that under this force, the body rotates through an angle dθ.

Work done = force × displacement

$$dW = F(rd\theta)$$

$$\Rightarrow dW = \tau d\theta$$

⇒ Work done = (torque) × (angular displacement)

$$\Rightarrow W = \int \tau d\theta \ (\text{Where } \tau \text{ is function of } \theta)$$

$$\text{Power} = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

5.2 Kinetic Energy

Rotational kinetic energy of the system rotating about a fixed axis

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + ...$$

$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + ...$$

$$= \frac{1}{2}\left(m_1 r_2^2 + mr_2^2 + m_3 r_3^2 + ...\right)\omega^2$$

Hence rotational kinetic energy of the system $= \frac{1}{2} I\omega^2$

Where I = Moment of inertia about the axis.

NOTE:

Comparing the expression of rotational kinetic energy with 1/2 mv^2, we can say that the role of moment of inertia (I) is same in rotational motion as that of mass in linear motion. It is a measure of the resistance offered by a body to a change its rotational motion.

The total kinetic energy of a body which is translating as well as rotating is given by :

$$K = K_{translational} + K_{rotational}$$

$$K = \frac{1}{2} MV_{CM}^2 + \frac{1}{2} I_{CM}\omega^2$$

Where,

V_{CM} = speed of the centre of mass

I_{CM} = moment of inertia about axis passing through CM.

ω = angular velocity of rotation

6. Rolling

Rolling motion is a combination of rotation and translation

In case of rolling all point of a rigid body have same angular speed but different linear speed.

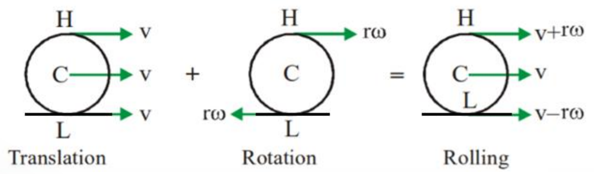


Fig.8.24

6.1 Pure Rolling (without Slipping)

For a rolling motion to be pure rolling the velocity of point of contact of body with platform should be equal for both rolling body and platform.

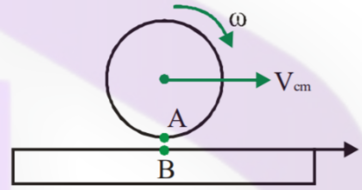


Fig.8.25

(i) General case (when surface is moving)

$$V_A = V_B$$

$$\Rightarrow V_{cm} - \omega R = V_B$$

In terms of acceleration: $a_{cm} - \alpha R = a_B$

(ii) special case (when $V_B = 0$)

$$V_{cm} - \omega R = 0$$

$$\Rightarrow V_{cm} = \omega R$$

6.2 Total KE of Rolling Body

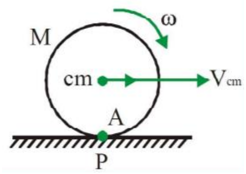


Fig. 8.26

(i) $K = \frac{1}{2} I_P \omega^2$

Or

(ii) $K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} MV_{cm}^2$

Here, (a) $I_P = I_{cm} + MR^2$ (parallel axes theorem)

(b) $V_{cm} = \omega R$ (condition for pure rolling)

NOTE:

Friction is responsible for the motion, but work done or dissipation of energy against friction is zero in pure rolling motion as point of application has zero velocity.

6.3 Forward Slipping

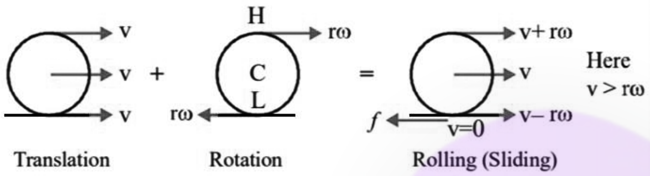


Fig.8.27

The bottom most point slides in the forward direction w.r.t. ground, so friction force acts opposite to velocity at lowest point i.e., opposite to direction of motion.

Example: When sudden brakes are applied to car its 'v' remain same while 'ωr' decreases so it slides on the ground.

6.4 Backward Slipping

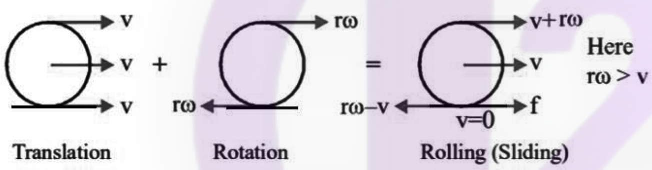


Fig.8.28

The bottom most point slides in the backward direction w.r.t. ground, so friction force acts opposite to velocity i.e., friction will act in the direction of motion.

Example: When car starts on a slippery ground, its wheels have small 'v' but large 'ωr' so wheels slips on the ground and friction acts against slipping.

6.5 Rolling and Sliding Motion on an Inclined Plane

(i) Pure rolling on an incline plane

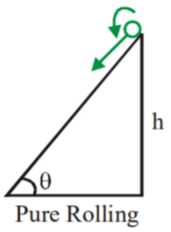


Fig.8.29

$$a_R = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

If initial velocity of body is zero then by using equation of motion,

$$V_R^2 - 0^2 = \frac{2g\sin\theta}{1+\frac{I}{MR^2}}\frac{h}{\sin\theta}$$

$$\Rightarrow V_R = \sqrt{\frac{2gh}{1+\frac{I}{MR^2}}}$$

$$\text{Also, } t_R = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{I}{MR^2}\right)}$$

Where, V_R = Final velocity of com of rolling body,
t_R = Time taken by body to reach the ground,

(ii) Sliding on an incline plane

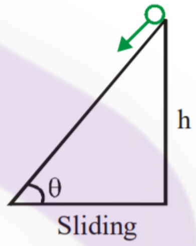


Fig.8.30

Using energy conservation,

$$\frac{1}{2}mV_s^2 = mgh$$

$$\Rightarrow V_s = \sqrt{2gh}$$

Component of acceleration along incline is g sinθ.

Time taken by body to reach ground by sliding:

$$\frac{h}{\sin\theta} = \frac{1}{2}g\sin\theta\, t_s^2$$

$$\Rightarrow t_s = \frac{1}{\sin\theta}.\sqrt{\frac{2h}{g}}$$

NCERT Corner

Important Points to Remember

1. Ideally a rigid body is one for which the distances between different particles of the body do not change, even though there are forces acting on them.

2. A rigid body fixed at one point or along a line can have only rotational motion. A rigid body not fixed in some way can have either pure translational motion or a combination of translational and rotational motions.

3. In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every point in the rotating rigid body has the same angular velocity at any instant of time.

4. In pure translation, every particle of the body moves with the same velocity at any instant of time.

5. Angular velocity is a vector quantity. Its magnitude is $\omega = \frac{d\theta}{dt}$ and it is directed along the axis of rotation. For rotation about a fixed axis this vector $\vec{\omega}$ has a fixed direction.

6. The linear velocity of a particle of a rigid body rotating about a fixed axis is given by $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{r}$ is the position vector of the particle with respect to an origin along the fixed axis. The relation applies even to more general rotation of a rigid body with one point fixed. In that case $\vec{r}$ is the position vector of the particle with respect to the fixed point taken as the origin.

7. The angular momentum of a system of n particles about the origin is

$$\vec{L} = \sum_{t=1}^{n} \vec{r}_t \times \vec{p}_t$$

8. The torque or moment of force on a system of n particles about the origin is

$$\vec{\tau} = \sum_{t=1}^{n} \vec{r}_t \times \vec{F}_t$$

9. A rigid body is in mechanical equilibrium if

(1) It is in translational equilibrium, i.e., the total external force on it is zero: $\sum \vec{F}_t = 0$, and

(2) It is in rotational equilibrium, i.e., the total external torque on it is zero: $\sum \vec{\tau}_t = \sum \vec{r}_t \times \vec{F}_t = 0$.

10. The centre of gravity of an extended body is that point about which the total gravitational torque on the body is zero.

11. The moment of inertia of a rigid body about an axis is defined by the formula $I = \sum m_i r_i^2$ where r_i is the perpendicular distance of the i^{th} point of the body from the axis.

12. The theorem of parallel axes: $I' = I_{com} + Ma^2$, allows us to determine the moment of inertia of a rigid body about an axis as the sum of the moment of inertia of the body about a parallel axis through its centre of mass and the product of mass and square of the perpendicular distance between these two axes.

13. Rotation about a fixed axis is directly analogous to linear motion in respect of kinematics and dynamics.

14. The kinetic energy of rotation about an axis is $K = \frac{1}{2} I\omega^2$.

15. For a rigid body rotating about a fixed axis of rotation, $L = I\omega$, where I is the moment of inertia about that axis.

16. The angular acceleration of a rigid body rotating about a fixed axis is given by $I\alpha = \tau$.

17. If the external torque τ acting on the body about the axis is zero, then angular momentum about the axis ($L = I\omega$) of such a rotating body is constant.

18. For rolling motion without slipping on ground v_{cm} = $R\omega$, where v_{cm} is the velocity of translation (i.e., of the centre of mass), R is the radius and m is the mass of the body. The kinetic energy of such a rolling body is the sum of kinetic energies of translation and rotation:

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

Physics

Class11th NEET

09

GRAVITATION

Gravitation

Introduction

Gravity is the force of attraction exerted by earth towards is centre on a body lying on or near the surface of earth.
Gravity
is merely a special case of gravitation and is also called earth's gravitational pull.
Weight of a body is defined as the force of attraction exerted by the earth on the body towards its centre.
The units and dimensions of gravity pull or weight are the same as those of force.

1. Newton's Law of Gravitation

1.1 Definition

Every particle attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

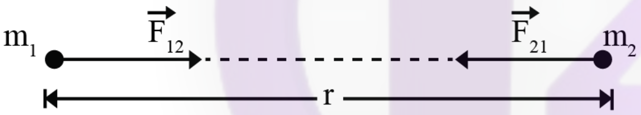


Fig. 9.1

1.2 Mathematical Form

If m_1 and m_2 are the masses of the particles and r is the distance between them, the force of attraction F between the particles is given by

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\therefore F = G\frac{m_1 m_2}{r^2}$$

Where G is the universal constant of gravitation.
Universal gravitational constant is measured in $N\,m^2/kg^2$

The dimensional formula is $\left[L^3M^{-1}T^{-2}\right]$ universal gravitational constant

The value of G is: $6.67408\times10^{-11}\ Nm^2/kg^2$

1.3 Vector Form

In vector form, Newton's law of gravitation is represented in the following manner. The force $\left(\vec{F}_{21}\right)$ exerted on particle m_2 by particle m_1 is given by,

$$\vec{F}_{21} = G\frac{m_1 m_2}{r^2}(\hat{r}_{12}) \ ...(i)$$

Where $(\hat{r}_{12})$ is a unit vector drawn in the direction of vector from particle m_2 to
particle m_1. Similarly, the force $\left(\vec{F}_{12}\right)$ exerted on particle m_1 by particle m_2 is given by

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}(\hat{r}_{12}) \ ...(ii)$$

From (i) and (ii)

$$\therefore \vec{F}_{12} = -\vec{F}_{21}$$

2. Acceleration due to Gravity

2.1 Definition

Acceleration due to gravity is the acceleration gained by a an object due to gravitational force. It SI unit is m/s^2. It has both magnitude direction, hence, it is a vector quantity.
Acceleration due to gravity is represented by g.
The stranded value of g on the surface of earth at sea level is $9.8\,m/s^2$.

2.2 The Acceleration due to Gravity at a Height h above the Earth's Surface

Let M and R be the mass and radius of the earth and g be the acceleration due to gravity at the earth's surface.
Suppose that a body of mass m is placed on the surface of the earth.

$$\therefore mg = \frac{GMm}{R^2}$$

$$\therefore g = \frac{GM}{R^2} \ ...(i)$$

Now suppose that the body is raised to a height h, above the earth's surface,

$$Mg_n = \frac{GMm}{(R+h)^2} \ ....(ii)$$

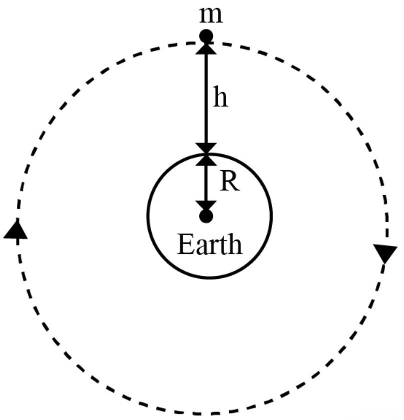


Fig. 9.2

Dividing eq (ii) by eq (i), we get,

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$\therefore g_h = \left[\frac{R^2}{(R+h)^2}\right] g$$

2.3 Acceleration due to Gravity at a Very Small Height

$$g_h = g\left(\frac{R+h}{R}\right)^{-2}$$

$$g_h = g\left(1+\frac{h}{R}\right)^{-2}$$

If h << R, then neglecting high power's of 'h' we get,

$$g_h = g\left(1-\frac{2h}{R}\right)$$

2.4 Effect of Depth on a Acceleration due to Gravity

Also g in terms of ρ

$$g = \frac{GM}{R^2}$$

If ρ is density of the material of earth, then

$$M = \frac{4}{3}\pi R^3 \rho$$

$$\therefore g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$\therefore g = \frac{4}{3}\pi G R \rho$$

Let g_d be acceleration due to gravity at the point B at a depth d below the surface of earth. A body at the point B will experience force only due to the portion of the earth of radius OB (R – d). The outer spherical shell, whose thickness is d, will not exert any force on body at point B. Because it will acts as a shell and point is inside.

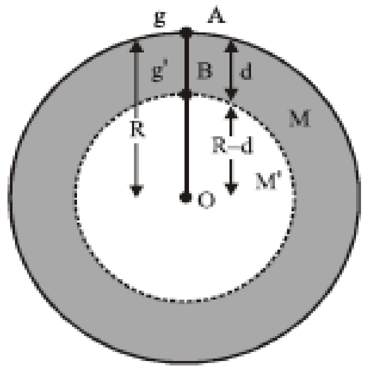


Fig. 9.3

Now, $M' = \frac{4}{3}\pi (R-d)^3 \rho$

$$g_d = \frac{GM^1}{(R-d)^2}$$

or $g_d = \frac{4}{3}\pi G(R-d)\rho$ ...(ii)

Dividing the equation (ii) by (i), we have

$$\frac{g_d}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi G R\rho} = \frac{R-d}{R} \text{ or } g_d = g\left(1-\frac{d}{R}\right) \quad ..(iii)$$

Therefore, the value of acceleration due to gravity decreases with depth.

2.5 Variation of 'g' with Latitude due to Rotational Motion of Earth

Due to the rotational of the earth the force $mr\omega^2 \cos\lambda$ radially outwards. Hence the net force of attraction exerted by the earth of the particle and directed towards the centre of the earth is given by

$$mg' = mg - mr\omega^2 \cos\lambda$$

where g' is the value of the acceleration due to gravity at the point P.

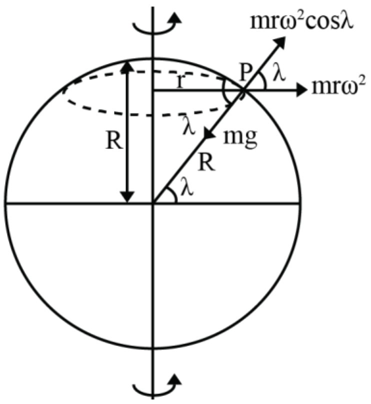


Fig. 9.4

$\therefore g' = g - r\omega^2 \cos\lambda$

Now, $r = R\cos\lambda$ (where R is the radius of the earth)

Then $g' = g - (R\cos\lambda)\omega^2 \cos\lambda$

$\therefore g' = g - R\omega^2 \cos^2\lambda$

The effective acceleration due to gravity at a point 'P' is given by,

$g' = g - R\omega^2 \cos^2\lambda$

Thus value of 'g' changes with λ and ω

1. At poles,

$\lambda = 90°$

$g' = g - R\omega^2 \cos^2 90°$

$g' = g$

This is maximum acceleration due to gravity.

2. At equator

$\lambda = 0$

$g' = g - R\omega^2 \cos^2 0$

$g' = g - R\omega^2$

This is minimum acceleration due to gravity.

Variation due to shape.

NOTE:

The variation of acceleration due to gravity according to the depth and the height from the earth's surface can be expressed with help of following graph.

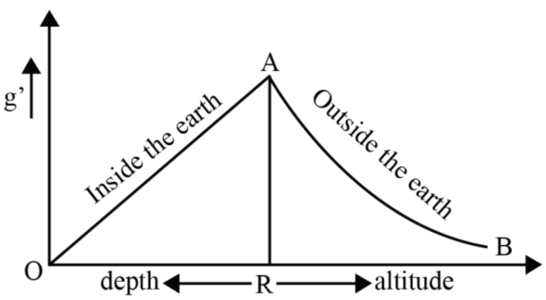


Fig. 9.5

3. Gravitation Field and Gravitation Potential

3.1 Gravitational Field

The space surrounding the body within which its gravitational force of attraction is experienced by other bodies is called gravitational field. Gravitational field is very similar to electric field in electrostatics where charge 'q' is replaced by mass 'm' and electric constant 'K' is replaced by gravitational constant 'G'. The intensity of gravitational field at a points is defined as the force experienced by a unit mass placed at that point.

$$\vec{E} = \frac{\vec{F}}{m}$$

The unit of the intensity of gravitational field is N kg^{-1}.

Intensity of gravitational field due to point mass:

The force due to test mass m_0 placed at point P is given b:

$$F = \frac{GMm_0}{r^2}$$

Hence $E = \frac{F}{m_0} \Rightarrow E = \frac{GM}{r^2}$

In vector form $\vec{E} = -\frac{GM}{r^2}\hat{r}$

Dimensional formula of intensity of gravitational field

$$= \frac{F}{m} = \frac{[MLT^{-2}]}{[M]} = [M^0LT^{-2}]$$

3.2 Gravitational Potential

The **gravitational potential** at any point in a gravitational field is defined as the work done to bring a unit mass slowly from
infinity to that point.

1. The gravitational potential (V) at a point at distance r from a point mass M is given by,

$V = -\frac{GM}{r}$ (Where G is the constant of gravitation)

2. The work done on a unit mass is converted into its potential energy. Thus, the gravitational potential at any point is equal to the potential energy of a unit mass placed at that point.

3. If a small point mass m is placed in a gravitational field at a point where the gravitational potential is V, the gravitational

potential energy (P.E.) of the mass m is given by.
P.E. = mass × gravitational potential
P.E = mV

$$P.E = -\frac{GMm}{r}$$

3.3 Gravitational Potential Energy

Gravitational potential energy of a body at a point is defined as the work done in slowly bringing the body from infinity to that point.

Let a body of mass m is displaced through a distance 'dr' towards the mass M, then work done given by,

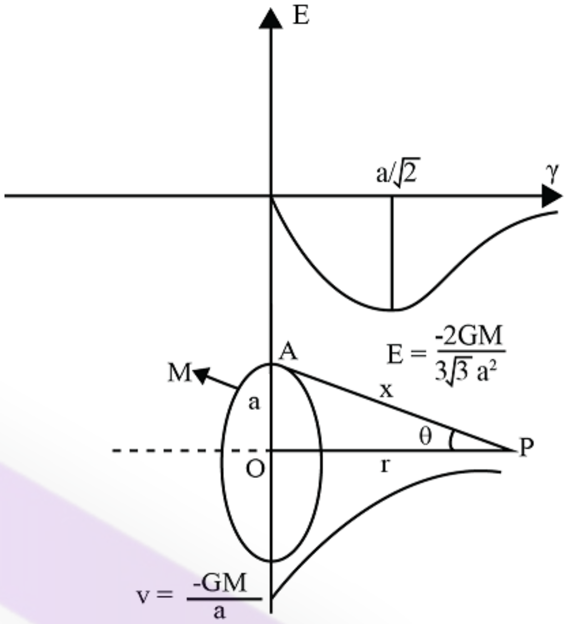
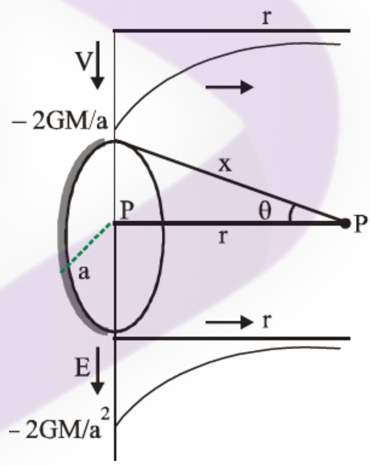
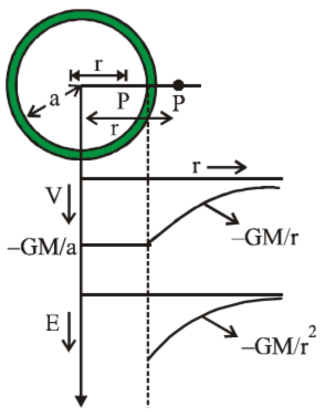
$$dW = Fdr = \frac{GMm}{r^2}dr \Rightarrow \int dW = \int_{\infty}^{r} \frac{GMm}{r^2}dr$$

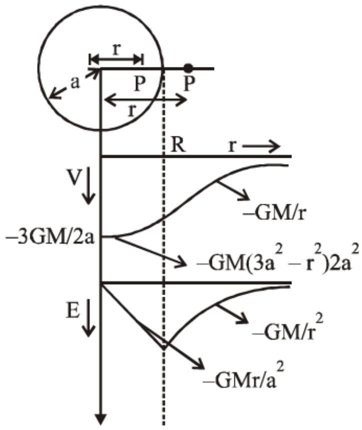
Gravitational potential energy,

$$U = -\frac{GMm}{r}$$

(i) From above equation, it is clear that gravitational potential energy increases with increase in distance (r) (i.e. it becomes less negative).

(ii) Gravitational P.E. becomes maximum (or zero) at $r = \infty$

Object	Potential (V)	Electric field ($\vec{E}$)	Figure
Ring	$V = \frac{-GM}{(a^2+r^2)^{1/2}}$	$\vec{E} = \frac{-GMr}{(a^2+r^2)^{3/2}}\hat{r}$	
Thin circular	$V = \frac{-2GM}{a^2}\left[\sqrt{a^2+r^2}-r\right]$	$\vec{E} = -\frac{2GM}{a^2}\left[1-\frac{r}{\sqrt{r^2+a^2}}\right]\hat{r}$	
Uniform Thin spherical shell (a) Point P inside the shell (r < a) (b) Point P outside the shell (r > a)	$V = -\frac{GM}{a}$ $V = -\frac{GM}{r}$	$E = 0$ $\vec{E} = \frac{-GM}{r^2}\hat{r}$	

Uniform Solid sphere (a) Point P inside the sphere ($r \le a$) (b) Point P outside the sphere ($r \ge a$)	$V = -\frac{GM}{2a^3}\left(3a^2 - r^2\right)$ $V = -\frac{GM}{r}$	$\vec{E} = \frac{-GMr}{a^3}\hat{r}$ $\vec{E} = \frac{-GM}{r^2}\hat{r}$	

4. Escape Velocity of a Body

4.1 Expression for the Escape Velocity of A Body at Rest on the Earth's Surface

The minimum velocity with which a body should be projected from the surface of the earth, so that it escapes from the earth's gravitational field, is called the escape velocity. Thus, if a body or a satellite is given the escape velocity, its kinetic energy of projection will be equal to its binding energy.
Kinetic Energy of projection = Binding Energy.

$$\therefore \frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$\therefore v_e = \sqrt{\frac{2GM}{R}}$$

4.2 Expression for 'V_e' in Terms's of 'g'

The escape velocity for any object on the earth's surface is given by.

$$v_e = \sqrt{\frac{2GM}{R}}$$

If m is the mass of the object, its weight mg is equal to the gravitational force acting on it.

$$\therefore mg = \frac{GMm}{R^2}$$

$$\therefore GM = gR^2$$

Substituting this value in the expression for v_e we get,

$$v_e = \sqrt{2gR}$$

4.3 Expression for 'V_e' in Terms's of Density

We have,

$$v_e = \sqrt{\frac{2GM}{R}}$$

Let ρ be the mean density of the planet. Then,

$$M = \frac{4}{3}\pi R^3\rho$$

$$v_e = \sqrt{\frac{2G}{R}\times\frac{4}{3}\pi R^3\rho}$$

$$v_e = 2R\sqrt{\frac{2\pi G\rho}{3}}$$

5. Satellite

5.1 Definition

Any smaller body which revolves around another larer body under the influence of its gravitation is called a **satellite.** The satellite may be natural or artificial.

1. The moon which revolves around the earth, is a satellite of the earth. There are sixteen satellites revolving around the planet Jupiter. These satellite are called natural satellites.

2. A satellite made and launched into circular orbit by man is called an artificial satellite. The first satellite was launched by USSR named SPUTNIK–I and the first Indian satellite was 'ARYABHATTA'.
There are two types of satellites:
1. GEO stationary satellite
2. SPY satellite
Let's discuss GEO stationary satellite

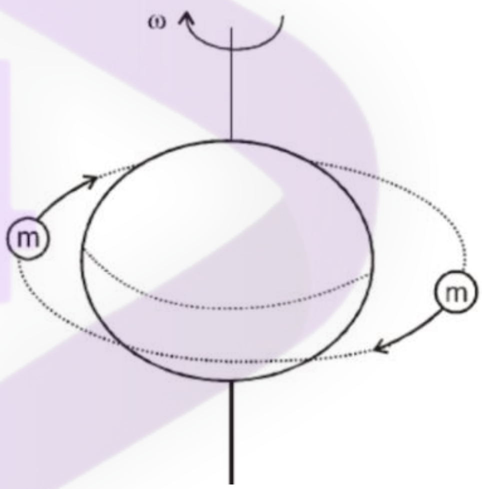


Fig 9.6

We know that the earth rotates about tis axis with angular velocity ω_{earth} and time period $T_{earth} = 24\,\text{hours}$. Suppose a satellites is set in an orbit which is in plane of the equator, whose ω is equal to ω_{earth}, (or its T is equal to $T_{earth} = 24\,\text{hours}$) and direction is also same as that of earth. Then as seen from earth, it will appear to be stationery. This type of satellite is called geo-stationery satellite.

$$\omega_{satellite} = \omega_{earth}$$

$$\Rightarrow T_{satellite} = T_{earth} = 24\,\text{hr}.$$

So time period of a geo-stationery satellite must be 24 hours.
To achieve T = 24 hour, the orbital radius geo-stationery satellite.

$$T^2 = \left(\frac{4\pi^2}{GM_e}\right) r^3$$

Putting the values, we get orbital radius of geo-stationery satellite $r = 6.6R_e$ (here Re = Radius of the earth) height from the surface $h = 5.6R_e$.

6. Period of Revolution of a Satellite

The time taken by a satellite to complete one revolution round the earth is called its **period or periodic time** (T). Consider a satellite of mass m revolving in a circular orbit with a orbital velocity v_0 at a height h above the surface of the earth. Let M and R be the mass and the radius of the earth respectively. The radius (r) of the circular orbit of the satellite is r = R + h. For the circular motion,

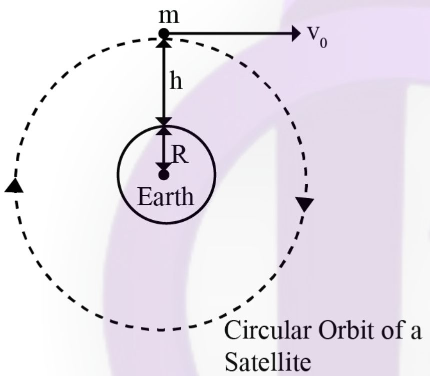


Fig. 9.7

$$\therefore v_0 = \sqrt{\frac{GM}{r}} \quad ...(i)$$

If T is the period of revolution of the satellite,

$$\text{Period (T)} = \frac{\text{circumference of orbit}}{\text{critical velocity}} = \frac{2\pi r}{v_0}$$

$$R = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} \quad ...\text{(From i)}$$

$$\therefore T = 2\pi\sqrt{\frac{r^3}{GM}}$$

This expression gives the periodic time of the satellite. Squaring the expression, we get

$$\therefore T^2 = \frac{4\pi^2 r^3}{GM}$$

$\therefore T^2 \propto r^3$ ...(since G and M are constants)

Thus, the square of the time period of revolution of a satellite is directly proportional to the cube of the radius of its orbit

7. Orbital Velocity

7.1 Definition

The horizontal velocity with which a satellite must be projected from a point above the earth's surface, so that it revolves in a circular orbit round the earth, is called the **orbital velocity** of the satellite.

7.2 An Expression for the Orbital Velocity of a Satellite Revolving Round the Earth

Suppose that a satellite of mass m is raised to a height h above the earth's surface and then projected in a horizontal direction with the orbital velocity v_0. The satellite begins to move round the earth in a circular orbit of radius, R + h, where R is the radius of the earth.

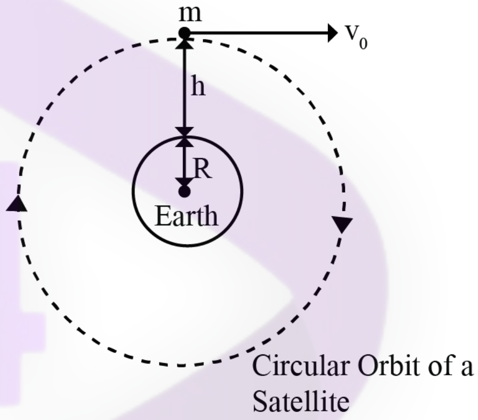


Fig.9.8

The gravitational force acting on the satellite is $\frac{GMm}{(R+h)^2}$, where M is the mass of the earth and G is the constant of gravitation.

For circular motion,

$$\frac{mv_0^2}{(R+h)} = \frac{GMm}{(R+h)^2},$$

$$\therefore v_0 = \sqrt{\frac{GM}{(R+h)}}$$

This expression gives the orbital velocity of the satellite. From the expression, it is clear that the orbital velocity depends upon.

1. Mass of the earth
2. Radius of earth and
3. Height of the satellite above the surface of the earth.

7.3 The Escape Velocity of a Body from the Surface of the Earth is $\sqrt{2}$ Times its Critical Velocity When it Revolves Close to the Earth's Surface

Let M and R be the mass and radius of the earth and m be the mass of the body. When orbiting close to the earth's surface, the radius of the orbit is almost equal to R. If v_e is the critical velocity of the body, then for a circular orbit.
Centripetal force = Gravitational force

$$\therefore mv_c^2 = \frac{GMm}{R^2}$$

$$\therefore v_c = \sqrt{\frac{GM}{R}} \quad ...(i)$$

If v_e is the escape velocity from the earth's surface,
K.E. of projection = Binding energy

$$\therefore \frac{1}{2}mv_e^2 = \frac{GMm}{2}$$

$$\therefore v_e = \sqrt{\frac{2GM}{R}} \quad ...(ii)$$

From Eq (i) and Eq. (ii), we get,

$$v_e = \sqrt{2}v_c$$

7.4 Different Cases of Projection

When a satellite is taken to some height above the earth and then projected in the horizontal direction, the following four cases may occur, depending upon the magnitude of the horizontal velocity.

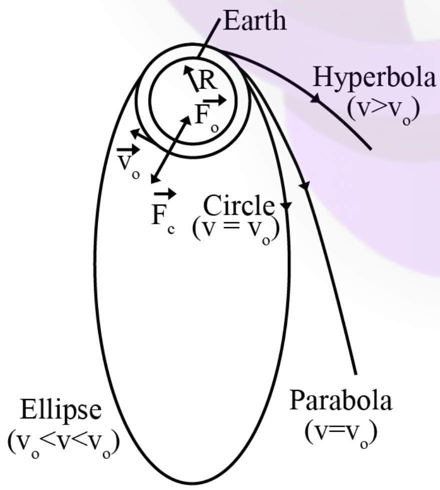


Fig. 9.9

1. If the velocity of the projection is less than the orbital velocity then the satellite moves in elliptical orbit, but the point of projection is apogee and in the orbit, the satellite comes closer to the earth with its perigee point lying at 180°. If it enters the atmosphere while coming towards perigee it will loose energy and spirally comes down. If it does not enters the atmosphere it will continue to move in elliptical orbit.

2. If the velocity of the projection is equal to the orbital velocity then the satellite moves in circular orbit round the earth.

3. If the velocity of the projection is greater than the orbital velocity but less than the escape velocity, then the satellite moves in elliptical orbit and its apogee, or point of greatest distance from the earth, will be greater than projection height.

4. If the velocity of the projection is equals to the escape velocity, then the satellite moves in parabolic path.

5. It the velocity of the projection is greater than the escape velocity, then orbit will hyperbolic and will escape the gravitational pull of the earth and continue to travel infinitely.

NOTE:

Launching Of An Artificial Satellite Around Earth

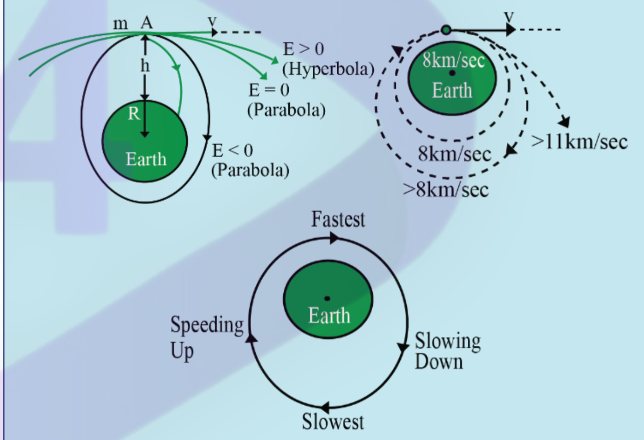


Fig. 9.10

The satellite is placed upon the rocket which is launched from the earth. After the rocket reaches its maximum vertical height h, a spherical mechanism gives a thrust to the satellite at point A (figure) producing a horizontal velocity v. The total energy of the satellite at A is thus,

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R+h}$$

The orbit will be an ellipse (closed path), a parabola, or an hyperbola depending on whether E is negative, zero, or positive. In all cases the centre of the earth is at one focus of the path. If the energy is too low, the elliptical orbit will intersect the earth and the satellite will fall back.

Otherwise, it will keep moving in a closed orbit, or will escape from the earth, depending on the values of v and h. Hence a satellite carried to a height h (<< R) and given a horizontal velocity of 8 km/sec will be placed almost in a circular orbit around the earth (figure). If launched at less than 8 km/sec, it would get closer and closer to earth until it hits the ground. Thus, 8 km/sec is the critical (minimum) velocity.

8. Communication Satellite

An artificial satellite revolving in a circular obrit round the earth in the same sense of the rotational of the earth and having same period of revolution as the period of rotation of the earth (i.e. 1 day = 24 hours = 86400 seconds) is called as geo-stationary or communication satellite.
As relative velocity of the satellite with respective to the earth is zero it appears stationary from the earth's surface. Therefore it is know as geo-stationary satellite or geosynchronous satellite.

1. The height of the communication satellite above the earth's surface is about 36000 km and its period of revolution is 24 hours or 24 × 60 × 60 seconds.

8.1 Uses of the Communication Satellite

1.F or sending TV signals over large distances on the earth's surface.

2.T elecommunication.

3.W eather forescasting.

4.F or taking photographs of astronomical objects.

5.F or studying of solar and cosmic radiations.

9. Weightlessness

1.T he gravitational force with which a body is attracted towards the centre of earth is called the weight of body.

2.W hen an astronaut is on the surface of earth, gravitational force acts on him. This gravitational force is the weight of astronaut and astronant exerts this force on the surface of earth. The surface of earth exerts an equal and opposte reaction and due to this reaction he feels his weight on the earth.

3.F or an astronaut in an orbiting satellite, the satellite and astronaut both have same acceleration towards the centre of earth and this acceleration is equal to the acceleration due to gravity at the place.

4. Therefore astronaut does not produce any action on the floor of the satellite. Naturally the floor does not exert any force of reaction on the astronaut. As there is no reaction, the astronaut has a feeling of weightlessness. (i.e. no sense of his own weight).

NOTE:

1. sensation of weightlessness experienced by an astronaut is not the result of there being zero gravitational acceleration, but of there being zero difference between the acceleration of the spacecraft and the acceleration of the astronaut.
2. The most common problem experienced by astronauts in the initial hours of weightlessness is known as space adaptation syndrome (space sickness).

10. Kepler's Laws

10.1 Law of Orbit

Each Planet moves around the sun in an elliptical orbit with the sun at one of the foci as shown in figure. The eccentricity of an ellipse is defined as the ratio of the

distance SO and AO i.e. $e = \frac{SO}{AO}$

$\therefore e = \frac{SO}{a}$, SO = ea

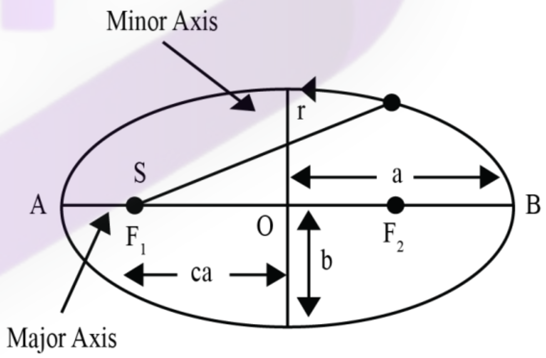


Fig. 9.11

The distance of closest approach with sun at F_1 is AS. This position is called perigee. The greatest distance (BS) of the planet from the sun is at position B apogee.
At, Perigee (AS) = AO – OS = a – ea = a (1 – e)
At, apogee (BS) = OB + OS = a + ea = a (1 + e)

10.2 Law of Area

The line joining the sun and a planet sweeps out equal areas in equal intervals of time. A planet takes the same time to travel from A to B as from C to D as shown in figure.

(The shaded areas are equal). Naturally the planet has to move faster between C to D.

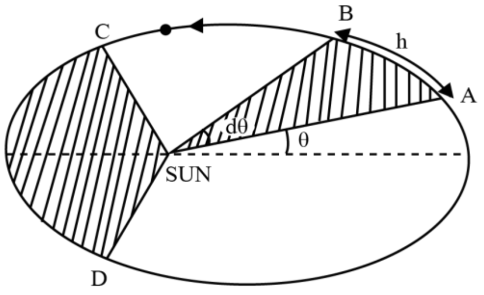


Fig. 9.12

Areal velocity $= \frac{\text{area swept}}{\text{time}}$

$$= \frac{\frac{1}{2}r(rd\theta)}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}\frac{mr^2\omega}{m} = \frac{L}{2m}$$

Hence $\frac{L}{2m}$ = constant. [As L = constant]

10.3 Law of Periods

The square of the time for the planet to complete a revolution about the sun is proportional to the cube of semimajor axis of the elliptical orbit.

$T^2 \propto a^3$

Astronomical Data

Body	Sun	Earth	Moon
Mean radius,	6.95×10^{8}	6.37×10^{6}	1.74×10^{6}
Mass, kg	1.97×10^{30}	5.96×10^{24}	7.30×10^{22}
Mean density, 10^3 kg/m^3	1.41	5.52	3.30
Period of rotation about axis, days	25.4	1.00	27.3

Inertial Mass

NOTE:

Inertial mass of a body is related to its inertia in linear motion; and is defined by Newton's second law of motion. Let a body of mass m_G move with acceleration a under the action of an external force F. According to Newton's second law of motion, $F = m_i a$ or $m_i = F/a$

Thus, inertial mass of a body is equal to the magnitude of external force required to produce unit acceleration in the body.

Gravitational Mass

NOTE:

Gravitational mass of a body is related to gravitational pull on the body and is defined by Newton's law of gravitational.

$$F = \frac{GMm_G}{R^2} \text{ or } m_G = \frac{F}{(GM/R^2)} = \frac{F}{I}$$

The mass m_G of the body in this sence is the gravitational mass of the body. The inertia of the body has no effect on the gravitational mass of the body. $m_G = F$

Thus, **Gravitational mass** of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.

11. Binary Star System

11.1 Double Star System

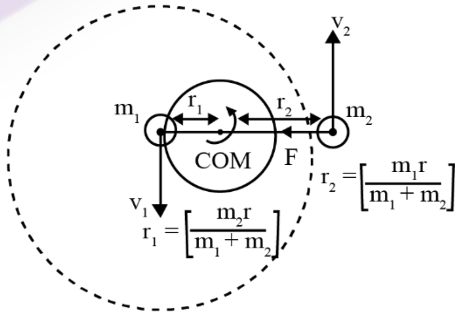


Fig. 9.13

From mass M_2

$$\frac{Gm_1m_2}{r^2} = m_2\omega^2 r_2$$

$$\frac{Gm_1}{r^2} = \frac{\omega^2 m_1 r}{m_1 + m_2}$$

$$\boxed{\omega^2 = \frac{G(m_1 + m_2)}{r^3}}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{G(m_1 + m_2)}{r^3}$$

$$T^2 = \frac{4\pi^2 r^3}{(m_1 + m_2)G}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{(m_1 + m_2)G}}$$

In reality all the planets arel satellite revolves around their COM but if the mass of planet is very large as compare to the mass of satellite, then we can assume that planet is stationary and satellite is removing around it in circular path (because $r_1 \to 0$)

NCERT Corner

Important Points to Remember

- **Gravitational force:** The constitutents of the universe are galaxy, stars, planets, comets, asteriods, meteroids. The force which keeps them bounded together is called gravitational force.
- Gravitation is a natural phenomenon by which particles get attracted towards one another.
- NEWTON'S LAW OF GRAVITATION: Every particle attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- Variation of 'g' with latitude due to Rotational motion of Earth:
  Due to the rotational of the earth the force mrw2 cos l acts radially outwards. Hence the net force of attraction exerted by the earth of the particle and directed towards the centre of the earth is given by mg' = mg – mrw2 cosl where g' is the value of the acceleration due to gravity:
  1.A t poles, l = 90, g' = g – R w2 cos2 90. g' = g This is maximum acceleration due to gravity.
  2.A t equator l = 0, g' = g – Rw2 cos2 0 g' = g – Rw2 This is minimum acceleration due to gravity
- Any smaller body which revolves around another larger body under the influence of its gravitation is called a satellite. The satellite may be natural or artificial.
- The horizontal velocity with which a satellite must be projected from a point above the earth's surface, so that it revolves in a circular orbit round the earth, is called the orbital velocity of the satellite.
- Escape Velocity: The minimum velocity with which a body should be protected from the surface of the earth, so that it escapes from the earth's gravitational field, is called the escape velocity.
- The gravitational potential at any point in a gravitational field is defined as the work done to bring a unit mass from slowly infinity to that point.
- Binding Energy: The minimum energy which must be supplied to a satellite, so that it can escape from the earth's gravitation field, is called the binding energy of a satellite.
- Kepler's laws of planetary motion

Kepler First law – The Law of Orbits

According to Kepler's first law," All the planets revolve around the sun in elliptical orbits having the sun at one of the foci". The point at which the planet is close to the sun is known as perihelion and the point at which the planet is farther from the sun is known as aphelion.

Kepler's Second Law – The Law of Equal Areas

Kepler's second law states" The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time"

Kepler's Third Law – The Law of Periods

According to Kepler's law of periods," The square of the time period of revolution of a planet around the sun in an elliptical orbit is directly proportional to the cube of its semi-major axis".

$T^2 \propto a^3$

Shorter the orbit of the planet around the sun, shorter the time taken to complete one revolution.

- An artificial satellite revolving in a circular orbit orbit round the earth in the same sense of the rotational of the earth and having same period of revolution as the period of rotation of the earth (i.e. 1 day = 24 hours = 86400 seconds) is called as geo-stationary or communication satellite.

10

MECHANICAL PROPERTIES OF SOLIDS

Mechanical Properties of Solids

1. Elastic Behaviour of Solids

1.1 Rigid Body

A body whose size and shape cannot be changed is called rigid body however if the applied force is large enough then it can change its shape and size.

1.2 Deforming Force and Restoring Force

- Deforming force is the external force applied on a body which tends to change the natural size or shape of the body.
- Under the action of deforming force, a body opposes any change in its shape & size due to the net effect of internal (molecular) forces. The resulting force which opposes the deformation is knows as **Restoring Force.**

1.3 Elasticity

- The property of a body due to which it opposes the action of the deforming forces is called Elasticity.
- A material is said to be elastic if it returns back to its original shape or size, when the deforming forces are removed.
- Plastic materials on the other hand, remains permanently distorted when the deforming forces are removed. The property is called Plasticity.
- Some of the examples of elastic materials are rubber band, steel, etc.

2. Stress & Strain

2.1 Stress

- The deforming action is measured (described) in terms of a physical quantity, known as stress, that is developed in the body.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area of cross section}}$$

- Unit N/m^2 or pascal.

In general there are three types of stress as given below:

- **Longitudinal Stress** When the restoring force is normal to the area of cross section, then it is known as Longitudinal stress. It is of two types.
  - **Tensile Stress** If the stress produced in an object is due to increase in its length, then it is called Tensile stress.
  - **Compressive Stress** If the stress produced in an object due to decrease in its length, then it is called Compressive stress.
- **Volumetric Stress** The restoring force acting per unit area inside the object opposing change in volume is called Volumetric stress.
- **Shearing Stress** When a force applied on an object along the tangential direction of the surface of the object, then stress produced in the object is called Shearing stress.

2.1.1 Breaking Stress

- The minimum stress after which the object breaks is called Breaking stress.
- A wire of length ℓ will break due to its own weight, when $\ell = \frac{\text{Breaking stress}}{dg}$

  where, d = density of material of wire
  and g = acceleration due to gravity.

2.2 Strain

- The deformation of the solid is described in terms of a physical quantity strain, that is created in the body as a result of deformation force.

$$\text{Strain} = \frac{\text{Change in dimsnion}}{\text{Orignal dimension}}$$

- It has no unit.

There are three types of stresses results in three types of strain as given below:

- **Longitudinal Strain** (S_L): It is the change in length per unit length of the body on application of force, i.e.

$$S_L = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L}$$

- **Bulk or Volume Strain** (S_V): It is the change in volume per unit volume of the object on application of force, i.e., $S_V = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$.
- **Shearing Strain** (S_S): It is the ratio of displacement of the upper surface to the distance between two layers, i.e.,

$$S_S = \frac{\text{Relative displacement of layer}}{\text{Distance between two layers}} = \tan\theta$$

where, θ shearing angle.

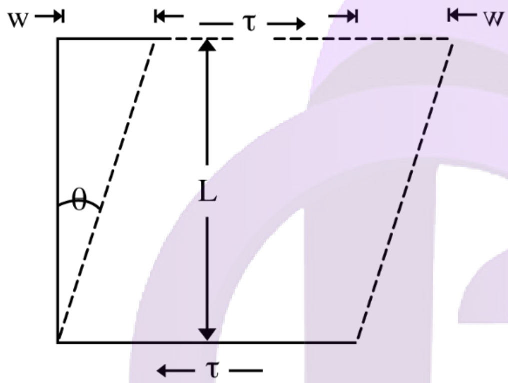


Fig. 10.1

NOTE:

> Shearing strain $= 2 \times$ longitudinal strain.
> and volume strain $= 3 \times$ longitudinal strain.

2.3 Hooke's Law

- If the deforming forces are within a limit (known as elastic limit), the stress created in the body is proportional to the resulting strain. i.e., stress $\propto$ strain

  The ratio $\frac{\text{stress}}{\text{strain}}$ is known as Modulus of Elasticity.
- According to various types of stresses, we have corresponding types of moduli of elasticity.
- Units of modulus of elasticity is same as the unit of stress.

2.4 Stress-Strain Graph

If by gradually increasing the load on a vertically suspended metal wire, a graph is plotted between stress (or load) and longitudinal strain (or elongation) we get the curve as shown in figure. From this curve it is clear that:

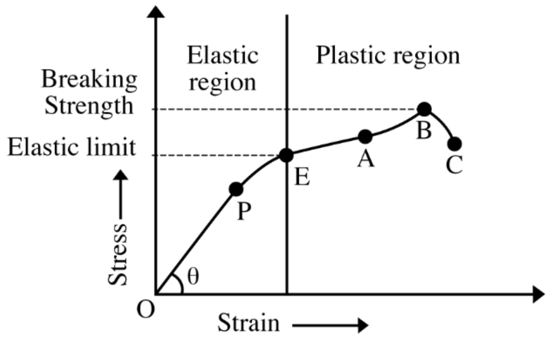


Fig. 10.2

a) When the strain is small (<2%) (i.e., in the region OP)
- Stress is proportional to strain.
- Hooke's law is obeyed.
- The point P is called limit of proportionality and
- Slope of line OP gives the Young's modulus of the material of the wire. $Y = \tan\theta$.

NOTE:

> **Elastic limit**
> The maximum value of the stress within which the body regains its original shape and size.

b) If the strain is increased a little bit (i.e., in the region PE.)
- The stress is not proportional to strain.
- The wire still regains its original length after the removal of stretching force.
- Point E is known as elastic limit or yield-point.
- The region OPE represents the elastic behaviour of the material of wire.
- Yield point is the stress beyond which the material becomes plastic.

c) If the wire is stretched beyond the elastic limit E (i.e., between EA)
- The strain increases much more rapidly
- If the stretching force is removed the wire does not come back to its natural length. Some permanent increase in length takes place.

d) If the stress is increased further, then
- A very small increase in stress produces a very large increase in strain (region AB).
- After reaching point B, the strain increases even if the wire is unloaded and it ruptures at C.
- In the region BC the wire literally flows. The maximum stress corresponding to B after which the wire begins to flow and breaks is called breaking or tensile strength.

- The region EABC represents the plastic behavior of the material of wire.
- Stress-strain curve is different for different materials.

2.5 Elastic Hysteresis

- The strain persists even when the stress is removed. This lagging behind of strain is called elastic hysteresis. This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.

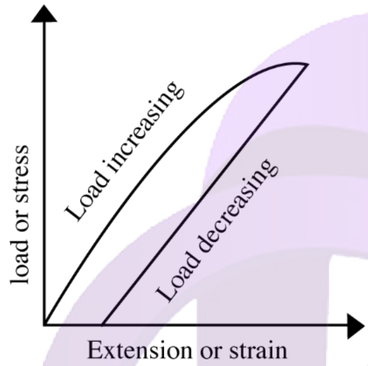


Fig. 10.3

- **Brittle material**
  The plastic region between E and C is small for brittle material and it will break soon after the elastic limit is crossed.
- **Ductile material**
  The material of the wire have a good plastic range and such materials can be easily changed into different shaped and can be drawn into this wires.
- **Elastomers**
  Stress strain curve is not a straight line within the elastic limit for elastomers and strain produced is much larger than the stress applied. Such materials have no plastic range and the breaking point lies very close to elastic limit, eg, rubber.

3. Moduli of Elasticity

- **Young's Modulus** It is the ratio of longitudinal stress to the corresponding strain for small change in length and it is expressed as $Y = \frac{FL}{A\ell}$.

where,
A = area of the body
ℓ = change in the length due to the strain and
L = Natural length of the body.

- **Bulk Modulus** It is the ratio of volume stress to the corresponding volume strain in the material for small value of strain and it is given by

$$B = \frac{F/A}{-\Delta V/V} = -\frac{VdP}{dV}.$$

where, dP = change in pressure
& dV = change in volume

$$\text{Compressibility} = \frac{1}{\text{Bulk modulus}(B)} = \frac{-dV}{VdP}$$

Gases can have following types of bulk modulus

a) **Isothermal Bulk Modulus :** Under isothermal condition, Bulk modulus is equal to pressure of gas, i.e., $B_i = P$ (pressure of gas).

b) **Adiabatic Bulk Modulus :** Under adiabatic condition, Bulk modulus is equal to γ times pressure, i.e.,

$$B_a = \gamma P \qquad \left(\gamma = \frac{C_P}{C_V}\right).$$

where, γ = heat capacity ratio.

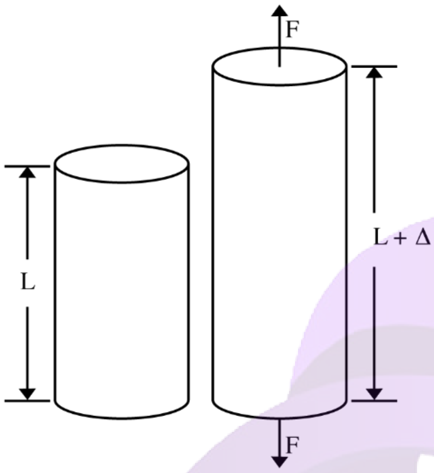
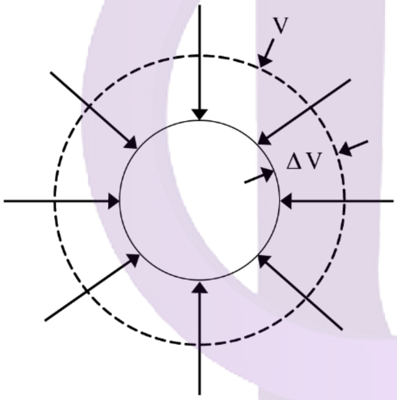
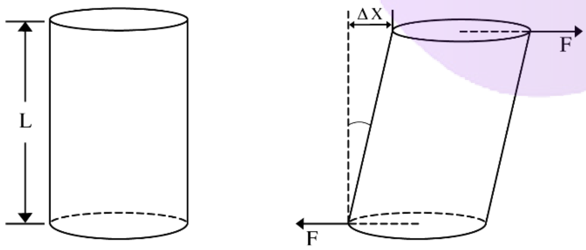
Also, $B_a = \gamma B_i$.

- **Modulus of Rigidity :** It is the ratio of shearing stress to the shearing strain for small strain in a body, which is given as : $G = \frac{F}{A\theta}$, where, θ = shearing angle.

NOTE:

- Young's modulus and modulus of rigidity are defined for solids only while Bulk modulus is defined for all solids, liquids and gases.
- For perfectly rigid body, we have $Y = \infty, B = \infty$ and $G = \infty$.

3.1 Stress, Strain & Various Elastic Moduli

Types of Stress	Stress	Strain	Modulus of Elasticity	Name of Modulus	State of Matter
Longitudinal (Tensile or compressive)  Fig. 10.4	Two equal and opposite forces perpendicular to opposite faces $(\sigma = F/A)$.	Elongation or compression $(\Delta L/L)$.	$Y = \frac{FL}{A\Delta L}$	Young's Modulus	Solid
Shearing  Fig. 10.5	Two equal and opposite forces parallel to opposite surfaces $(\sigma = F/A)$.	$\tan\theta = \frac{\Delta x}{L}$	$G = \frac{FL}{A\Delta x}$	Shear Modulus	Solid
Hydraulic/Volume  Fig. 10.6	Forces perpendicular everywhere to the surface, force per unit area (pressure) same everywhere.	Volume change $(\Delta V/V)$	$B = \frac{-PV}{\Delta V}$	Bulk Modulus	Solid, liquid and gas.

3.2 Poisson's Ratio

When a rod or bar is subjected to a longitudinal stress, not only its length changed but its transverse dimensions also change and thus giving rise to transverse or lateral strain in additional to longitudinal strain.

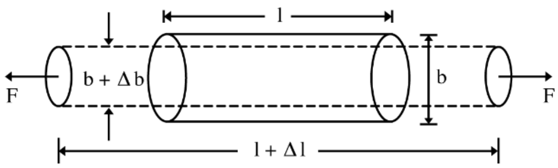


Fig. 10.7

Transverse or lateral strain is $\frac{\Delta b}{b}$.

The ratio of transverse to longitudinal strain is termed as Poisson's ratio, σ.

$\sigma = \frac{-\Delta b / b}{\Delta \ell / \ell}$.

Since, $\sigma = \frac{\text{Transverse strain}}{\text{Longitudinal strain}}$.

Negative sign is introduced to make σ positive quantity. Since, an increase in length always results in decrease of transverse dimensions and vice-versa.

NOTE:

- σ has no units, as it's a ratio.
- $0 < \sigma \le 0.5$. [For Most Materials]

3.3 Derivation for Relations Between Elastic Constants

Individually Young's Modulus, Bulk Modulus and Modulus of Rigidity are related as-

Relations	Formula	SI units
The relation between modulus of elasticity and modulus of rigidity.	$Y = 2G(1+\sigma)$	N/m^2 or pascal (Pa)
The relation between Young's modulus and bulk modulus.	$Y = 3B(1-2\sigma)$	N/m^2 or pascal (Pa)
Relation between Young's Modulus and Bulk Modulus B and Modulus of rigidity as.	$E = \frac{9BG}{G+3B}$	No unit

Where,

- B is Bulk modulus.
- G is shear modulus or modulus of rigidity.
- Y is Young's modulus or modulus of Elasticity.

3.4 Applications of Modulus of Elasticity

- If a beam is fixed at its ends and loaded with weight at its middle, then depression at the centre is given as

  $\delta = \frac{Mg\ell^3}{4bd^3Y}$

  where,

  Y = Young's modulus, ℓ = length of beam,

  b = breadth of bean and d = thickness of beam

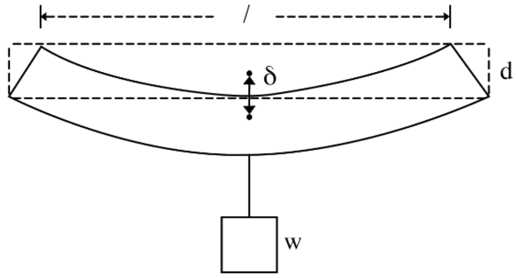


Fig. 10.8

- If a rubber ball of volume V is taken to a depth h in water, then decrease in its volume is given as

  $dV = \frac{-ndgV}{B}$.

  Where B is Bulk modulus.

3.5 Factors Affecting Elasticity

Elasticity of any material is affected due to many factors as given below:

- Due to **hammering** and rolling, the elasticity of material increases.
- Due to **annealing** (heating and slow cooling), the elasticity of material decreases.
- With rise in **temperature for almost all materials** the elasticity decreases.
- Due to **impurity,** the elasticity can be increased or decreased.

4. Elastic Potential Energy

- When an elastic body is deformed, work is done by the applied force. This work is stored as elastic potential energy and is released when the body returns back to its original size.

  Elastic energy stored per unit volume,

  $= \frac{1}{2}(\text{stress})(\text{strain})$

  $= \frac{1}{2}(\text{modulus of elasticity})(\text{strain})^2$

  $= \frac{1}{2}\frac{(\text{stress})^2}{\text{modulus of elasticity}}$.

- In case of longitudinal stress (compressive or tensile)

  $\frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}Y\left(\frac{\Delta\ell}{\ell}\right)^2$

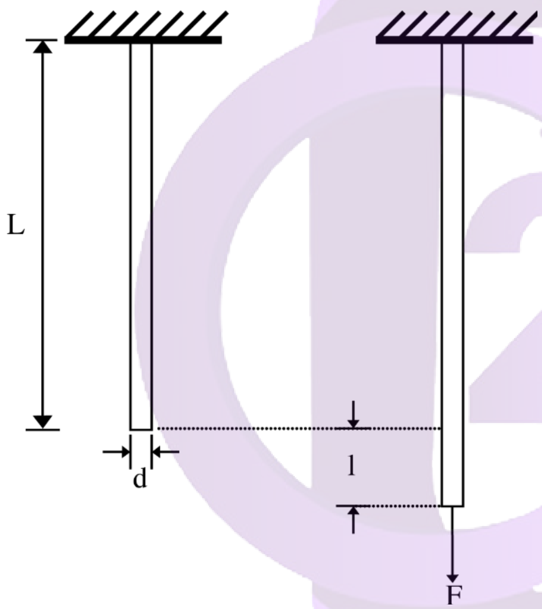
  Total energy $= \frac{1}{2}\frac{YA}{\ell}\cdot(\Delta\ell)^2 = \frac{1}{2}F\Delta\ell$

5. Miscellaneous Cases in Elasticity

5.1 Determination of Young's Modulus by Searle's Method

Consider a wire of length L and diameter d. Let its length L increases by an amount ℓ when the wire is pulled by a longitudinal external force F. Young's modulus of the material of the wire is the ratio of longitudinal stress to the longitudinal strain i.e., $Y = \frac{F/A}{\ell/L} = \frac{4FL}{\pi d^2 \ell}$.

The units of Young's modulus are the same as that of stress (note that strain is dimensionless) which is same as the units of pressure i.e., Pa or N/m^2.



Wire Extension Due to Pulling force $F = k\theta$

Fig. 10.9

- As Young's modulus is independent of the shape of the material, we can utilize any shape for its calculation. in particular, a thin circular wire fulfills our requirement.
- In this method, the length L of the wire is measured by a scale, diameter d of the wire is measured by a screw gauge, length ℓ of the wire is measured by a Micrometer or Vernier scale, and F is specified external force.
- Differentiate the expression for Y to get the relative error in the measured value of Y

$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + 2\frac{\Delta d}{d} + \frac{\Delta \ell}{\ell}$$

where, $\Delta L, \Delta d$ and $\Delta \ell$ are the errors in the measurement of L, d and ℓ, respectively. Generally, accuracy of these errors measurements depends on the least count of the measuring instruments.

5.2 Thermal Stress & Strain

- The strain and stress produced by heating a material is called the thermal strain and stress, respectively which are gives as

  Thermal strain $= \frac{\Delta \ell}{\ell} = \Delta t[As, \alpha \Delta t << 1]$

  where, α = thermal coefficient of linear expansion

  and Δt = change in temperature.

  Thermal stress $= \frac{F}{A} = Y\alpha\Delta t$.

- If a metal cube is heated, then pressure applied on the cube to prevent its expansion is

  $$P = B\gamma_V dt$$

  where, γ_V = coefficient of volume expansion

  and dt = rise in temperature.

5.3 Stress Developed due to Rotation of Objects : Torsion Constant of a Wire

The torsion constant is a geometrical property of a bar's cross-section which is involved in the relationship between angle of twist and applied torque along the axis of the bar, for a homogeneous linear-elastic bar. The torsion constant, together with material properties and length, describes a bar's torsional stiffness.

For a beam of uniform cross-section along its length

$$\theta = \frac{TL}{GJ}$$

Where,

θ is the angle of twist in radians

T is applied torque

L is the beam length

G is the Modulus of rigidity (shear modulus) of the material

J is the torsional constant

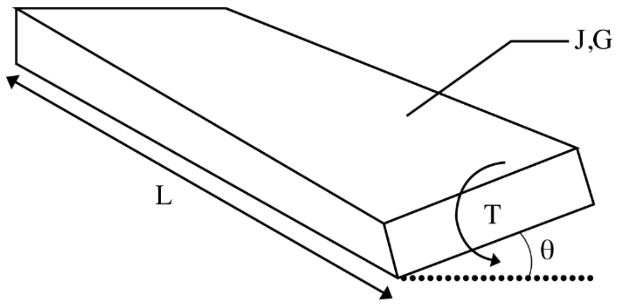


Fig. 10.1

- The SI unit for torsion constant is m^4.
- Inverting the previous relation, we can define two quantities: the torsional rigidity $GJ = \frac{TL}{\theta}$ with SI units Nm^2/rad
- And the torsional stiffness:
  $\frac{GJ}{L} = \frac{T}{\theta}$ with SI units N.m/rad.

5.4 Interatomic Force Constant

- It is the ratio of interatomic force to that of change in interatomic distance, i.e., $k = \frac{F}{\Delta r}$.

  Also, $k = Yr_0$

  where, Y = Young's Modulus,

  r_0 = distance between two atoms.

NCERT Corner

(Some important points to remember)

1. Elastic and Plastic behavior of Materials

- Whenever a force is applied on a body, then it tends to change the size or shape of the body.
- The property of a body by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as E**lasticity** and the deformation caused is known as E**lastic Deformation.**
- Those substances which do not have a tendency to regain their shape and hence gets permanently deformed are called P**lastic** and the property is called P**lasticity**.

2. Stress

- The restoring force per unit cross-sectional are set up within the body is called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area of cross-section}} = \frac{F}{A}$$

- In SI system, unit of stress is N/m^2 or pascal (Pa).
- In general there are three types of stresses

  (a) Longitudinal Stress – Tensile stress (associated with stretching or compressive stress (associated with compression).

  (b) Shearing Stress.

  (c) Bulk Stress.

3. Breaking Stress

- The minimum stress after which the wire breaks is called breaking stress.
- A wire of length ℓ will break due to its own weight,

  when $\ell = \dfrac{\text{Breaking stress}}{dg}$

  where, d = density of material of wire

  and g = acceleration due to gravity.

4. Strain

- Strain is defined as the ratio of change in dimension of an object to the original dimension.

$$\text{Strain} = \frac{\text{Change in configuration of the object}}{\text{Original configuration of the object}}$$

- It is a pure number and has no unit.

5. Hooke's Law

- This law states that, for small deformations, the stress and strain are proportional to each other.

  Thus, $\text{stress} \propto \text{strain}$

  $\text{stress} = k \times \text{strain}$

- The SI unit of modulus of elasticity is Nm^{-2}.
- A class of solids called elastomers do not obey Hooke's law.

6. Moduli of Elasticity

- Three elastic moduli viz. Young's Modulus, shear modulus and bulk modulus are used to describe the elastic behavior of objects as they respond to deforming forces that act on them.
- When an object is under tension or compression, the Hooke's law takes the form:

  $F / A = Y\Delta L / L$

  where $\Delta L / L$ is the tensile or compressive strain of the object, F is the magnitude of the applied fore causing the strain. A is the cross-sectional area over which F is applied (perpendicular to A) and Y is the Young's modulus for the object.
- A pair of forces when applied parallel to the upper and lower faces, the solid deforms so that the upper face moves sideways with respect to the lower. The horizontal displacement ΔL of the upper face is perpendicular to the vertical height L. This type of deformation is called shear and the corresponding stress is the shearing stress. This type of stress is possible only in solids.

  In this kind of deformation the Hooke's law takes the form :

  $F / A = G \times \Delta L / L$ where ΔL is the displacement of one end of object in the direction of the applied force F, and G is the shear modulus.
- When an object undergoes hydraulic compression due to a stress exerted by a surrounding fluid, the Hooke's law takes the form $P = -B(\Delta V / V)$. where p is the pressure (hydraulic stress) on the object due to the fluid, $\Delta V / V$ (the volume strain) is the absolute fractional change in the object's volume due to that pressure and B is the bulk modulus of the object.
- The Young's modulus and shear modulus are relevant only for solids since only solids have lengths and shapes.
- Bulk modulus is relevant for solids, liquid and gases. it refers to the change in volume when every part of the

body is under the uniform stress so that the shape of the body remains unchanged.

- Metal shave larger values of Young's modulus than alloys and elastomers. A material with large value of Young's modulus require a large force to produce small changes in its length.

7. Poisson's Ratio

- It is the ratio of transverse or lateral strain to longitudinal strain in the direction of stretching force. It is expressed as

$$\sigma = \frac{\text{-Lateral contraction strain}}{\text{Longitudinal contraction strain}} = \frac{-d/D}{\ell / L}$$

where, d = change in diameter & D = original diameter,

ℓ = change in length & L = original length.

- Theoretical limits of Poisson's ratio are -1 and 0.5, while the practical limits are 0 and 0.5.
- Relation between Y, K, G and σ as given below

(a) $L = \frac{Y}{3(1-2\sigma)}$

(b) $G = \frac{Y}{2(1+\sigma)}$

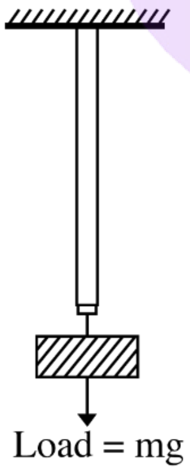
(c) $\sigma = \frac{(3B-2G)}{2(3B+G)}$

(d) $Y = \frac{9GB}{3B+G}$

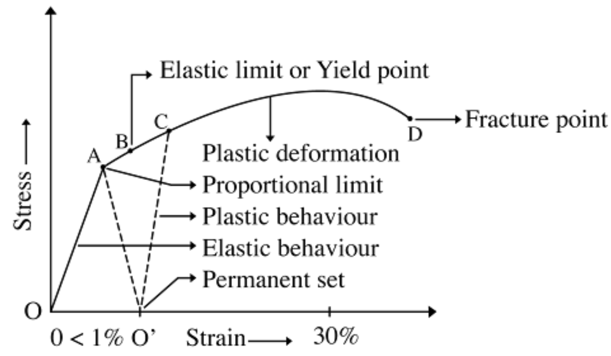
(e) $\frac{9}{Y} = \frac{3}{G} + \frac{1}{B}$

8. Stress-strain Curve

- A typical stress-strain curve for a metal is shown in figure



(a)



(b)

- In the region O to A, stress is found proportional to strain. Thus, Hooke's law is fully obeyed in this region and body regain its original shape.
- Point A is known as point of proportional limit.
- In the region from A to B, stress and strain are not proportional, but the body still returns to its original shape and size.
- The point B is yield point (also called elastic limit) and corresponding stress is yield stress (σ_y).
- If stress increases beyond point B, the strain further increases, but on removing the strain wire does not regain its original length.
- Beyond point C, for a small stress, the strain produced is large upto point D. The wire will break at point D called fracture point of wire.

9. Elastic Potential energy Stored in a Stretched Wire

- The work done in stretching a wire against the interatomic forces is stored as the elastic potential energy.
- Potential energy of work done per unit is given by

$$U = \frac{W}{V} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times Y \times (\text{Strain})^2 = \frac{1}{2Y} \times (\text{Stress})^2 \left[\because Y = \frac{\text{Stress}}{\text{Strain}}\right]$$

- Potential energy stored in stretching wire by restoring force is expressed as

$$W = \frac{1}{2} \times F \times \ell$$

where, F = restoring force

and ℓ = elongation produced.

- If the force acting on the body is increased from F_1 to F_2 within the elastic limit, then

$$W = \frac{(F_1 + F_2)}{2} \times \text{extension}.$$

11

FLUID MECHANICS

Fluid Mechanics

1. Introduction to Fluids

- The liquids and gases together are termed as fluids, in other words, we can say that the substances which can flow are termed as fluids.
- We assume fluid to be incompressible (i.e., the density of liquid is independent of variation in pressure and remains constant) and non-viscous (i.e. the two liquid surfaces in contact are not exerting any tangential force on each other).

1.1 Fluid Pressure

Pressure p at any point is defined as the normal force per unit area.

$$P = \frac{dF_1}{dA}$$

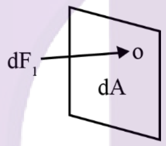


Fig. 11.1

The SI unit of pressure is the Pascal and 1 Pascal = 1 N/m^2

- Fluid force acts perpendicular to any surface in the fluid, no matter how that surface is oriented. Hence pressure, has no intrinsic direction of its own, it is a scalar quantity.

1.2 Relative Density and Specific Gravity

Relative density or specific gravity is the ratio of the density (mass of a unit volume) of a substance to the density of a given reference material. If the relative density is exactly 1 then the densities are equal i.e., equal volumes of the two substances have the same mass.

$$RD = \frac{\rho_{substance}}{\rho_{reference}}$$

2. Hydrostatic Pressure

2.1 Variation of Pressure

(a) Pressure at two points in a horizontal plane or at same level when the fluid is at rest or moving with constant velocity is same.

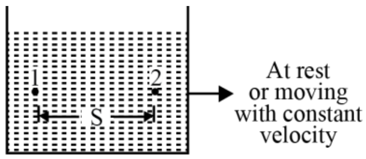


Fig.11.2

(b) Pressure at two points which are at a depth separation of h when fluid is at rest or moving with constant velocity is related by the expression

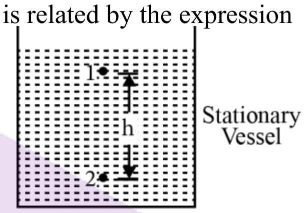


Fig. 11.3

$p_2 - p_1 = \rho gh$, where ρ is the density of liquid.

(c) Pressure at two points in a horizontal plane when fluid container is having some constant horizontal acceleration are related by the expression

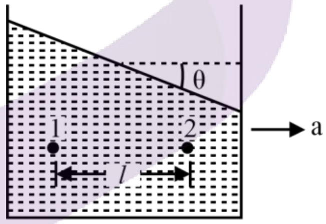


Fig.11.4

$p_2 - p_1 = \rho la$

and $\tan\theta = \frac{a}{g}$, where θ is the angle which the liquid's free surface is making with horizontal.

2.2 Atmospheric Pressure, Gauge Pressure and Absolute Pressure

- **Atmospheric Pressure:** It is the pressure exerted by earth's atmosphere. Normal atmospheric pressure at sea level (an average value) is 1 atmosphere (atm) that is equal to 1.013×10^5 Pa.
- **Gauge Pressure**: It is the difference between absolute pressure and atmospheric pressure. If the gauge pressure is above the atmospheric pressure, it is positive. If the gauge pressure is below the atmospheric pressure, it is negative.

- **Absolute Pressure:** Absolute pressure is gauge pressure plus atmospheric pressure. An absolute pressure reading of zero can only be achieved in a perfect vacuum and only naturally occurs in outer space.
- **Barometer**: It is a device used to measure atmospheric pressure while U–tube manometer or simply manometer is a device used to measure the gauge pressure.

3. Force Exerted by Fluids on the Walls of the Container

3.1 Force on the Sidewall of a Container

Force on the side wall of the container cannot be directly determined as at different depths pressures are different. To find this we consider a strip of width dx at a depth x from the surface of the liquid as shown in figure,

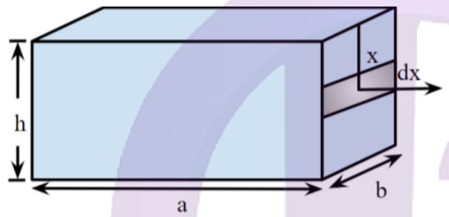


Fig.11.5

On this strip the force due to the liquid is given as:

$$dF = P_x A$$

$$dF = \rho g x (b dx)$$

$$F = \int dF = \rho g b \int_0^h x dx$$

$$F = \frac{\rho g b h^2}{2}$$

This force is acting in the direction normal to the side wall.

3.2 Average Pressure on the Sidewall:

The absolute pressure on the side wall cannot be evaluated because at different depths on this wall pressure is different. The average pressure on the wall can be given as:

$$P_{avg} = \frac{F}{bh}$$

$$P_{avg} == \frac{\rho g h}{2}$$

Above equation shows that the average pressure on side vertical wall is half of the net pressure at the bottom of the container.

4. Pascal's Law

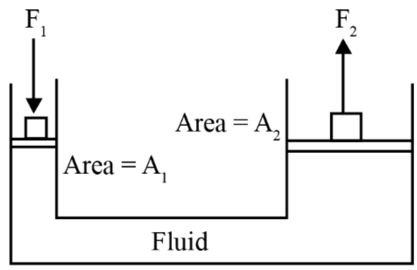
- A change in the pressure applied to an enclosed fluid is transmitted equally to every portion of the fluid in all direction of the walls of the containing vessel. **Hydraulic lift**: Hydraulic lift is a practical applications of Pascal's law
- 

Fig. 11.6

According to principle of hydraulics

$$P_1 = P_2 \Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = \frac{A_2}{A_1} F_1$$

5. Buoyancy

Buoyancy is the tendency of an object to float in a fluid. All liquids and gases in the presence of gravity exert an upward force known as the buoyant force on any object immersed in them. Buoyancy results from the differences in pressure acting on opposite sides of an object immersed in a static fluid.

5.1 Buoyant Force

The buoyant force is the upward force exerted on an object wholly or partly immersed in a fluid. This upward force is also called Upthrust. Due to the buoyant force, a body submerged partially or fully in a fluid appears to lose its weight, i.e.. appears to be lighter.
Following factors affect buoyant force:
(i)the density of the fluid
(ii)the volume of the fluid displaced
(iii)the local acceleration due to gravity
An object whose density is greater than that of the fluid in which it is submerged tends to sink. If the object is either less dense than the liquid or is shaped appropriately (as in a boat), the force can keep the object afloat.

5.2 Archimedes' Principle

Archimedes' principle states that:
"The upward buoyant force that is exerted on a body immersed in a fluid, whether partially or fully submerged, is equal to the weight of the fluid that the body displaces and acts in the upward direction at the center of mass of the displaced fluid".

$$F_B = V\sigma g$$

Where, F_B=Upthrust of Buoyant force
V = volume of liquid displaced
σ = density of liquid.

Apparent decrease in weight of body = Upthrust – weight of liquid displaced by the body
$W_{app} = F_B - W$

6. Types of Fluid Flows

6.1 Steady Flow (Streamline Flow)

When a body is partially or fully dipped into a fluid, the fluid exerts contact force on the body. The resultant of all these contact forces is called buoyant force (upthrust).
Line of flow: It is the path taken by a particle in flowing liquid. In case of a steady flow, it is called streamline. Two streamlines can never intersect each other.

6.2 Turbulent Flow

It is irregular flow in which particles move in zig zag way

6.3 Reynold's Number

Reynolds defined a dimensionless number whose value gives one an approximate idea, whether the flow rate would be turbulent or laminar.
This number, called the Reynolds number R_e is defined as,

$$R_e = \frac{\rho v D}{\eta}$$

where, ρ = the density of the fluid flowing with a speed v.
D = the diameter of the tube.
η = the coefficient of viscosity of the fluid.
It is found that flow is streamline or laminar for Re less than 1000. The flow is turbulent for $R_e > 2000$. The flow becomes unsteady for R_e between 1000 and 2000.

NOTE:

For lower density and higher viscosity fluids laminar flow is more probable.

7. Equation of Continuity

In a tube of varying cross section as shown in diagram:

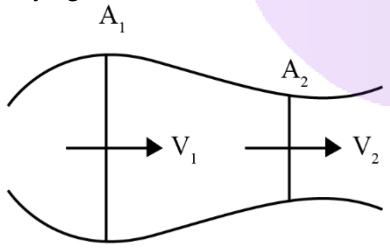


Fig. 11.7

The fluid flows for a short interval of time in the tube. So, assume that short interval of time as Δt. In this time Δt, the volume of liquid entering the tube of flow in a steady flow is $A_1 v_1 \Delta t$.
Where v_1 is velocity of fluid at cross section A_1
The same volume must flow out as the liquid is incompressible. The volume flowing out in Δt is $A_2 v_2 \Delta t$.
Where v_2 is velocity of fluid at cross section A_2

Now from conservation of mass
$A_1 v_1 \Delta t = A_2 v_2 \Delta t$
$\Rightarrow A_1 v_1 = A_2 v_2$
This equation is known as continuity equation

- Mass flows rate = $\rho A v$
  (Where ρ is the density of the liquid.)

8. Bernoulli's Theorem

In a streamline flow of an ideal fluid, the sum of pressure energy per unit volume, potential energy per unit volume and kinetic energy per unit volume is always constant at all cross section of the liquid.

$$P + \rho g h + \frac{\rho v^2}{2} = \text{Constant}$$

- It is a mathematical consequence of low of conservation of energy and fluid dynamics.

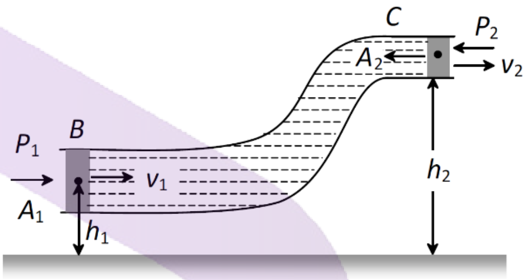


Fig. 11.8

$$P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

- Bernoulli's equation is valid only for incompressible steady flow of a fluid with no viscosity.

8.1 Application of Bernoulli's Theorem

(a) Velocity of Efflux

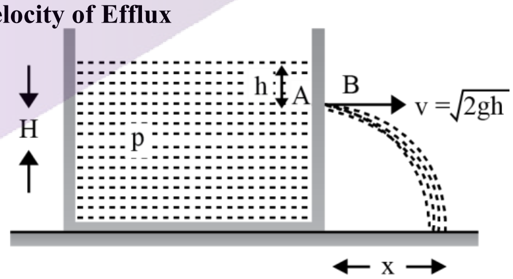


Fig. 11.9

Let us find the velocity with which liquid comes out of a hole at a depth h below the liquid surface. Using Bernoulli's theorem,

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

$$\Rightarrow \quad P_{atm} + \frac{1}{2}\rho v_A^2 + \rho g h = P_{atm} + \frac{1}{2}\rho v^2 + 0$$

[$P_B = P_{atm}$, because we have opened the liquid to atmosphere]

$$\Rightarrow \quad v^2 = v_A^2 + 2gh$$

Using equation of continuity

$Av_A = av$

A: area of cross-section of vessel, a: area of hole

$$\Rightarrow \quad v^2 = \frac{a^2}{A^2}v^2 + 2gh$$

$$\Rightarrow \quad v = \frac{\sqrt{2gh}}{\sqrt{1-\frac{a^2}{A^2}}}$$

If the hole is very small

$$\Rightarrow \quad v \approx \sqrt{2gh}$$

(b) Venturi Meter

Venturi meter is an instrument for measuring the rate of flow of fluids.

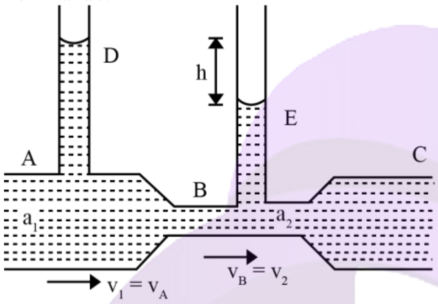


Fig. 11.10

If P_A is pressure at A and P_B is pressure at B,

$P_A - P_B = h\rho g$

[h : difference of heights of liquids of density ρ in vertical tubes]

If v_1 is velocity at A and v_2 is velocity at B

$Q = a_1v_1 = a_2v_2$ [equation of continuity]

$$P_A + \rho\frac{v_1^2}{2} = P_B + \rho\frac{v_2^2}{2} \quad \text{[Bernoulli's Theorem]}$$

$$\Rightarrow \quad v_2^2 - v_1^2 = \frac{2}{\rho}(P_A - P_B) = \frac{2}{\rho}h\rho g$$

$$\Rightarrow \quad \frac{Q^2}{a_2^2} - \frac{Q^2}{a_1^2} = 2gh$$

$$\Rightarrow \quad Q = a_1a_2\sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

Here Q is the rate of fluid flow

9. Viscosity

The property of a fluid by virtue of which it opposes the relative motion between its different layers is known as viscosity and the force that is into play is called the viscous force.

Newton's Law of Viscosity: According to Newton, the frictional force F (or viscous force) between two layers depends upon the following factors,

(i) Force F is directly proportional to the area (A) of the layers in contact, i.e. $F \propto A$

(ii) Force F is directly proportional to the velocity gradient $\frac{dv}{dx}$ between the layers.

Combining these two, we have viscous force

$$F = -\eta A\frac{dv}{dx}$$

Where η is a constant depending upon the nature of the liquid and is called the coefficient of viscosity. Its value depends on the nature of the fluid.

The negative sign in the above equation shows that the direction of viscous force F is opposite to the direction of relative velocity of the layer.

S.I. unit of coefficient of viscosity is Pa-s or N-s/m^2 or decapoise.

CGS unit of viscosity is poise. (1 decapoise = 10 poise).

9.1 Stoke's Law

When a solid moves through a viscous medium, its motion is opposed by a viscous force depending on the velocity and shape and size of the body.

The viscous drag on a spherical body of radius r, moving with velocity v, in a viscous medium of viscosity η is given by

$$F_{viscous} = 6\pi r\eta v$$

This relation is called Stoke's law.

Importance of Stoke's Law

- This law is used in the determination of electronic charge with the help of Millikan's experiment.
- This law accounts the formation of clouds.
- This law accounts why the speed of raindrops is less than that of a body falling freely with a constant velocity from the height of clouds.
- This law helps a man coming down with the help of a parachute.

9.2 Terminal Velocity

It is maximum constant velocity acquired by the body while falling freely in a viscous medium.

$$v_r = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Where r is radius of body, ρ is density of body, σ is density of liquid and η is the coefficient of viscosity.

9.3 Poiseuille's Formula

This law states that the flow of liquid depends on variables such as length of tube (L) radius(r), pressure difference (P) and coefficient of viscosity η.

According to this law volume of liquid coming out of tube per second in given by

$$V = \frac{\pi Pr^4}{8\eta L}$$

10. Surface Tension

The surface tension of a liquid is defined as the force per unit length in the plane of the liquid surface at right angles to either side of an imaginary line drawn on that surface.

So, $S = \frac{F}{\ell}$ where S = surface tension of liquid

The surface tension can be defined as the property of a liquid at rest by virtue of which its free surface behaves like a stretched membrane under tension and tries to occupy as small area as possible.

Unit of surface tension in MKS system : N/m, J/m^2

CGS system : Dyne/cm, erg/cm

10.1 Surface Energy

In order to increase the surface area, the work has to be done over the surface of the liquid. This work done is stored in the liquid surface as its potential energy. Hence the surface energy of a liquid can be defined as the excess potential energy per unit area of the liquid surface.

$W = S\Delta A$, where ΔA = increase in surface area.

NOTE:

1. Work done in formation of drop of radius r = surface tension × $\Delta A = 4\pi r^2 S$
2. Work done in formation of soap bubble = 2 × surface tension × $\Delta A = 8\pi r^2 S$

10.2 Excess Pressure

- Excess pressure in a liquid drop or bubble in a liquid is $P = \frac{2S}{R}$, S is surface tension
- Excess pressure in a soap bubble is $P = \frac{4S}{R}$

  (Because it has two free surfaces)

11. Cohesive and Adhesive Forces

The force of attraction between the molecules of the same substance is called cohesion.

For solids, the force of cohesion is very large and due to this solids have definite shape and size.

But in case of liquids, the force of cohesion is weaker than that of solids. Hence, liquids do not have definite shape but have definite volume. The force of cohesion is negligible in for gases so, gases don't have fixed shape and volume.

Some Important points:

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Due to force of adhesion, water wets the glass plate.

11.1 Angle of Contact

When a liquid surface touches a solid surface, the shape of the liquid surface near the contact is generally curved.

The angle between the tangent planes at the solid surface and the liquid at the contact is called the contact angle.

11.2 Shape of Liquid Meniscus

Concave upwards: When adhesive force between solid and liquid molecules is more than the cohesive force between liquid-liquid molecules in this case liquids wet the walls of the container (For example water and glass) have meniscus concave upwards and their values of angle of contact is less than 90°.

Convex upwards: When adhesive force between solid and liquid molecules is less than the cohesive force between liquid-liquid molecules in this case liquids don't wet the walls of the container (For example mercury and glass) and have meniscus convex upwards and their value of angle of contact is greater than 90°.

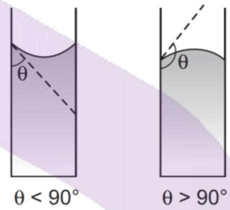


Fig. 11.11

11.3 Capillary Tube and Capillarity Action

A very narrow glass tube with fine bore and open at both ends is known as capillary tube. When a capillary tube in dipped in a liquid, then liquid will rise or fall in the tube, this action is termed as capillarity.

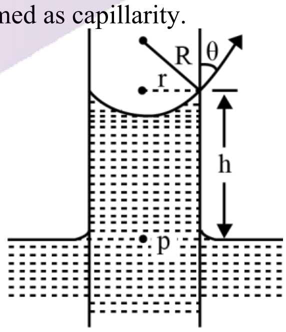


Fig. 11.12

$$h = \frac{2S\cos\theta}{r\rho g} = \frac{2S}{R\rho g}$$

where, S = surface tension,

θ = angle of contact,

r = radius of capillary tube,

R = radius of meniscus, and

ρ = density of liquid.

- **Capillary rise in a tube of insufficient length :**

If the actual height to which a liquid will rise in a capillary tube is 'h' then a capillary tube of length less than 'h' can be called a tube of "insufficient length".
In such a case, liquid rises to the top of the capillary tube of length *l* (*l* < h) and adjusts the radius of curvature of its meniscus until the excess pressure is equalized by the pressure of liquid column of length *l*. (Note liquid does not overflow).

$\Rightarrow \frac{2\sigma}{r'} = \ell\rho g \quad \ldots(i)$

If r were the actual radius of curvature,

$\Rightarrow \frac{2\sigma}{r} = h\rho g \quad \ldots(ii)$

Comparing (i) and (ii)

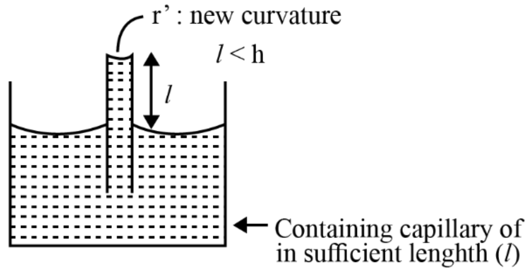


Fig. 11.13

$\frac{2\sigma}{\rho g} = \ell r' = hr$

$r' = \frac{hr}{\ell}$ i.e. radius of curvature r' can be calculated.

11.3 Important Points to Remember

Adhesion > Cohesion	Adhesion = Cohesion	Adhesion < Cohesion
1. Liquid will wet the solid 2. Meniscus is concave. 3. Angle of contact is acute ($\theta < 90°$). 4. Pressure below the meniscus is lesser than above it by (2T/r), i.e. $P = P_0 - \frac{2T}{r}$. 5. In capillary there will be rise	1. Critical 2. Meniscus is plane. 3.Angle of contact is 90° 4. Pressure below the meniscus is same as above it, i.e. $P = P_0$ 5. No capillarity	1. Liquid will not wet the solid. 2. Meniscus is convex. 3. Angle of contact is obtuse (θ > 90°) 4. Pressure below the meniscus more then above it by (2T/r), i.e, $P = P_0 + \frac{2T}{r}$. 5. In capillary there will be fall

NCERT Corner

1. The basic property of a fluid is that it can flow. The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container

2. A liquid is incompressible and has a free surface of its own. A gas is compressible, and it expands to occupy all the space available to it.

3. If F is the normal force exerted by a fluid on an area A then the average pressure P_{av} is defined as the ratio of the force to area

   $$P_{av} = \frac{F}{A}$$

4. The unit of the pressure is the pascal (Pa). It is the same as N m^{-2}. Other common units of pressure are

   1 atm = 1.01×10^5 Pa

   1 bar = 10^5 Pa 1 torr = 133 Pa = 0.133 kPa

   1 mm of Hg = 1 torr = 133 Pa

5. Pascal's law states that: Pressure in a fluid at rest is same at all points which are at the same height. A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.

6. The pressure in a fluid varies with depth h according to the expression

   $P = P_0 + \rho gh$

   where ρ is the density of the fluid, assumed uniform.

7. The volume of an incompressible fluid passing any point every second in a pipe of non-uniform cross section is the same in the steady flow.

   vA = constant (v is the velocity and A is the area of cross section)

   The equation is due to mass conservation in incompressible fluid flow.

8. Bernoulli's principle states that as we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume ($\rho v^2/2$) and the potential energy per unit volume (ρgh) remains a constant.

   $P + \rho v^2/2 + \rho gh$ = constant

   The equation is basically the conservation of energy applied to non viscuss fluid motion in steady state. There is no fluid which have zero viscosity, so the above statement is true only approximately. The viscosity is like friction and converts the kinetic energy to heat energy.

9. Though shear strain in a fluid does not require shear stress, when a shear stress is applied to a fluid, the motion is generated which causes a shear strain growing with time. The ratio of the shear stress to the time rate of shearing strain is known as coefficient of viscosity, η.

10. Stokes' law states that the viscous drag force F on a sphere of radius r moving with velocity v through a fluid of viscosity is, $F = 6\pi r\eta v$.

11. Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of interface between the liquid and the bounding surface. It is the extra energy that the molecules at the interface have as compared to the interior.

12

SIMPLE HARMONIC MOTION

Simple Harmonic Motion

1. Periodic & Oscillatory Motion

- A motion which repeats itself over and over again after a regular interval of time is called **periodic motion.**

- Oscillatory or vibratory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time.

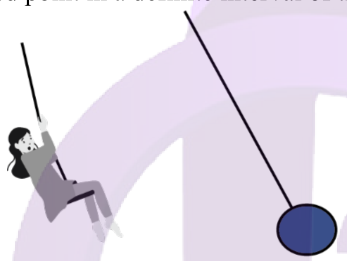


Fig. 12.1

The above two are the examples of both oscillatory and periodic motions.

2. Simple Harmonic Motion

Simple harmonic motion is a specific type of oscillatory motion, in which:

- particle moves in one dimension,
- particle moves to and fro about a fixed mean position (where $F_{net} = 0$),
- net force on the particle is always directed towards mean position, and
- magnitude of net force is always proportional to the displacement of particle from the mean position at that instant.

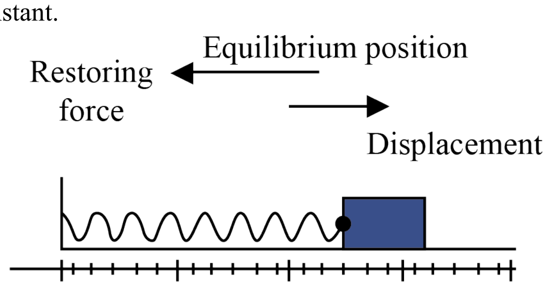


Fig. 12.2

2.1 Restoring Force in SHM

Simple Harmonic Motion (SHM) is that type of oscillatory motion in which the particle moves to and fro about a fixed point under a restoring force whose magnitude is directly proportional to its displacement and directed towards means position, i.e.

$F \propto -x$

So, $F_{net} = -kx$

where, k is known as force constant

Also, $ma = -kx$

$\Rightarrow a = \frac{-k}{m}x$

or, $a = -\omega^2 x \quad ...(i)$

where, ω is known as angular frequency.

2.2 Equation of SHM

From (i), we can write

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

This equation is called as the differential equation of S.H.M. The general expression for x(t) satisfying the above equation is:

$$x(t) = A\sin(\omega t + \phi).$$

Mathematically a SHM can be expressed as

$$x = A\sin\omega t = A\sin\frac{2\pi}{T}t$$

or, $x = A\cos\omega t = A\cos\frac{2\pi}{T}t$

where,

x = displacement from mean position at time t,

A = amplitude or maximum displacement,

ω = angular frequency, and

T = time period.

ϕ = initial phase.

2.3 Some Important Terms

a) Amplitude

The amplitude of particle executing S.H.M. is its magnitude of maximum displacement on either side of the means position.

b) Time period

Time period of a particle executing S.H.M. is the time taken to complete one cycle and is denoted by T.

Time period $(T)=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{k}}\left(\text{as } \omega=\sqrt{\frac{m}{k}}\right)$.

c) Frequency

The frequency of a particle executing S.H.M. is equal to the number of oscillations completed in one second.

$$\upsilon=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

d) Phase

The phase of particle executing S.H.M. at any instant is its state as regard to its position and direction of motion at that instant. It is measured as argument (angle) of sine in the equation of S.H.M.

Phase $=(\omega t+\phi)$

- At $t=0$, phase $=\phi$; the constant ϕ is called initial phase of the particle or phase constant.

2.4 Important Relations

a) Position (For $\phi=2n\pi$)

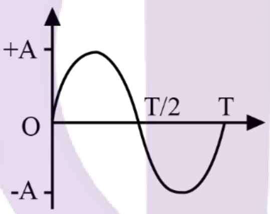


Fig. 12.3

If mean position is at origin the position (X coordinate depends on time in general as:

$$x(t)=\sin(\omega t+\phi)$$

- At mean position, $x=0$
- At extremes, $x=\pm A$.

b) Velocity (For $\phi=2n\pi$)

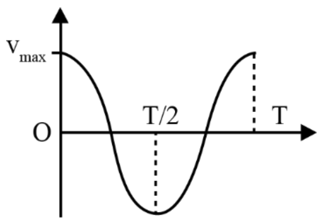


Fig. 12.4

- At any time instant t, $v(t)=A\omega\cos(\omega t+\phi)$.
- At any position x, $v(x)=\pm\omega\sqrt{A^2-x^2}$.
- Velocity is minimum at extremes because the particles is at rest. i.e., $v=0$ at extreme position.
- Velocity has maximum magnitude at mean position. $|v|_{max}=\omega A$ at mean position.

c) Acceleration (For $\phi=2n\pi$)

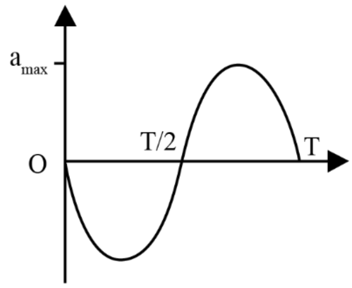


Fig. 12.5

- At any instant t, $a(t)=-\omega^2 A\sin(\omega t+\phi)$.
- At any position x, $a(x)=-\omega^2 x$.
- Acceleration is always directed towards mean position.
- The magnitude of acceleration is minimum at mean position and maximum at extremes.

i.e., $|a|_{min}=0$ at mean position.

$|a|_{max}=\omega^2 A$ at extremes

2.5 Time Period of S.H.M.

To find whether a motion in S.H.M. or not and to find its time period, follow these steps:

a) Locate the mean (equilibrium) position mathematically by balancing all the forces on it.

b) Displace the particle by a displacement 'x' from the mean position in the probable direction of oscillation.

c) Find the net force on it and check if it is towards mean position.

d) Try to express net force as a proportional function of its displacement 'x'.

NOTE:

If step **c)** and step **d)** are proved, then it is a simple harmonic motion

e) Find k from expression of net force $(F=-kx)$ and find time period using $T=2\pi\sqrt{\frac{m}{k}}$.

2.6 Graphs in S.H.M.

2.6.1 Displacement, Velocity and Acceleration of a Body Executing SHM

a) Displacement $y(t) = A\cos\omega t$

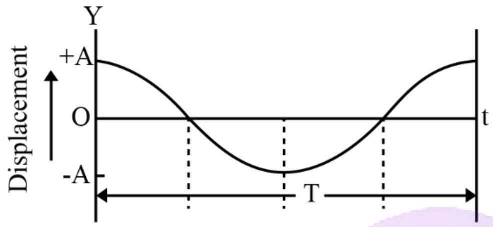


Fig. 12.6

It is Sinusoidal in nature, y varies from -A to A. It has zero phase difference.

b) Velocity $v(t) = -\omega A\sin\omega t$

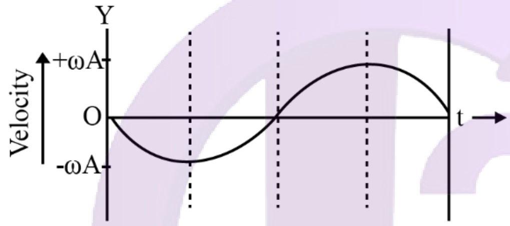


Fig. 12.7

It is Sinusoidal in nature, v(t) varies from $-\omega A$ to ωA. It has a phase difference of $\frac{\pi}{2}$ w.r.t. y(t).

(It is leading y(t) in the phase by $\frac{\pi}{2}$.)

c) Acceleration $a(t) = -\omega^2 A\cos\omega t$

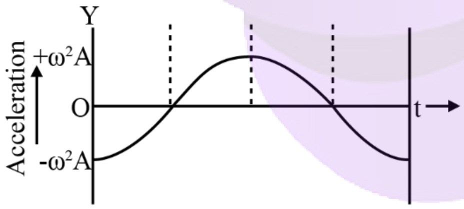


Fig. 12.8

It is Sinusoidal in nature, a(t) varies from $-\omega^2 A$ to $\omega^2 A$.

It has a phase difference of π w.r.t. y(t).

(It is π phase ahead of y(t))

3. Phasor in S.H.M.'s

3.1 SHM Projection for Circular Motion

Uniform Circular Motion:

a) Particle moves on a circular path of radius A with angular velocity ω.

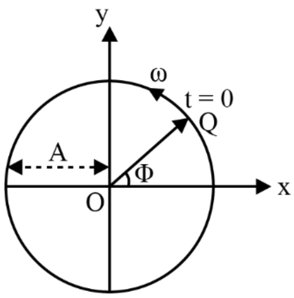


Fig. 12.9

At t = 0,

- Particle is at point Q on the circle
- Position vector is $\overrightarrow{OQ}$.
- Angle with X-axis $= \phi$.

b)

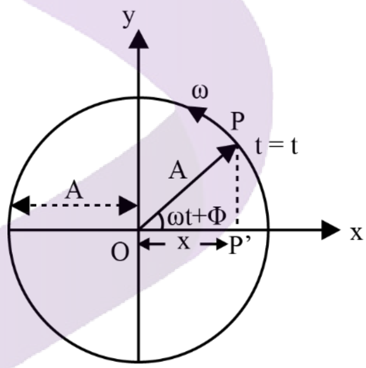


Fig. 12.10

At any given time t

- Particle is at point P
- Position vector $\overrightarrow{OP}$.
- Angle with X-axis $= \phi + \omega t$.

$\therefore y = A\sin(\omega t + \phi)$

So, its projection on y-axis does SHM.

3.2 Phase in S.H.M.

We can write an expression for the displacement of the mass as follows:

$x(t) = A\cos\omega t$

where, A is the amplitude of the oscillation, and ω is the angular frequency in units of rad/s.

- That means that A is the value of largest deviation (plus or minus) of the mass from its equilibrium position, and the value of ω governs the time it takes for the mass to complete one oscillation (recall that the period of the oscillation, $T = \frac{2\pi}{\omega}$,so a small ω means a long period while a large ω means a short period).

- Together, A and ω completely define the oscillation.
- We can represent these two quantities and therefore the oscillation itself with a phasor diagram. A phasor is nothing more than a vector with magnitude is A that rotates with angular velocity ω.
- For the oscillation defined above $x(0) = A$, since $\cos 0° = 1$. We can represent the oscillation at this time with a phasor of length A that lies on the horizontal axis.

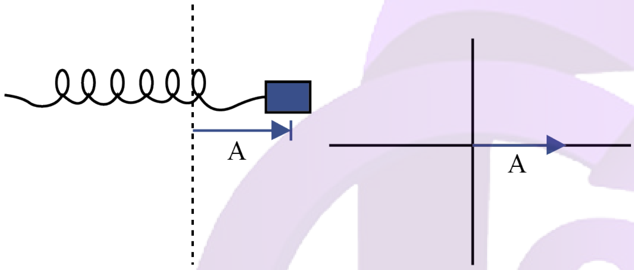


Fig. 12.11

- As time passes, the mass moves back toward its equilibrium position. At the same time, the phasor rotates in the counterclockwise direction. The phasor's magnitude doesn't change, but the projection of the phasor onto the horizontal axis gets smaller.

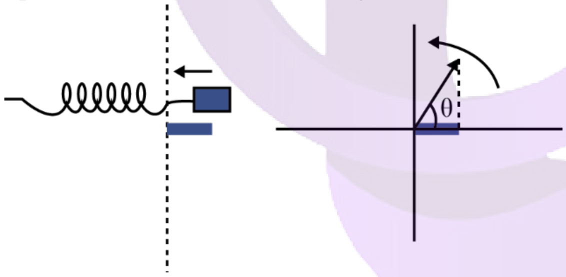


Fig. 12.12

- Now, if $\theta = \omega t$ in the phasor diagram, then projection of the phasor on the horizontal axis is $A\cos\theta = A\cos\omega t$, and that's precisely the expression for the displacement of the mass at time t. This means that if the phasor rotates with angular velocity ω, its projection on the horizontal axis will always describe the displacement of the oscillation.

Phasor Diagram: -

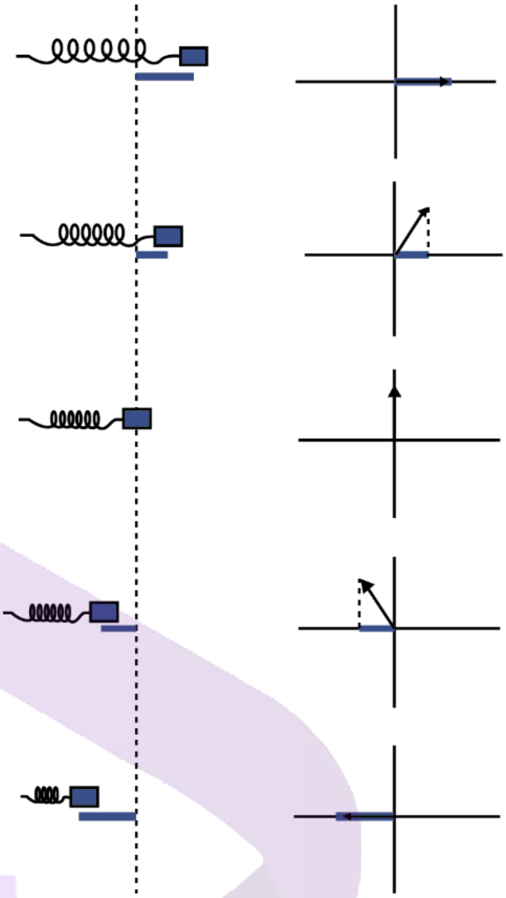


Fig. 12.13

NOTE:

If particle start from

- mean towards positive direction
  $\phi = 0, x = A\sin\omega t$
- mean towards negative direction
  $\phi = \pi, x = A\sin(\omega t + \pi)$
- from positive extreme
  $\phi = \frac{\pi}{2}, x = A\sin\left(\omega t + \frac{\pi}{2}\right)$
  $x = A\cos(\omega t)$
- negative extreme
  $\phi = \frac{3\pi}{2}, x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$
  $x = -A\cos\omega t$.

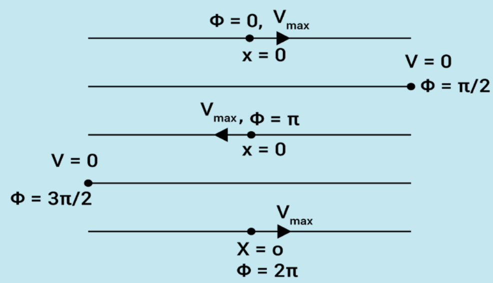


Fig. 12.14

4. Energy in S.H.M.

4.1 Kinetic Energy

- $K = \frac{1}{2}mv^2$

$\Rightarrow K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)\left(\text{as } v = \omega\sqrt{A^2 - x^2}\right)$

$= \frac{1}{2}m\omega^2 A^2 \cos^2\left(\omega t + \phi\right)$

For $\phi = 0°$

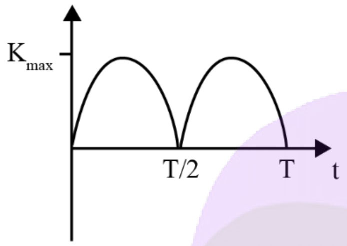


Fig. 12.15

- K is maximum at mean position and minimum at extremes.

and: $K_{max} = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$ at mean position

$K_{min} = 0$ at extremes.

4.2 Potential Energy

If potential energy is taken as zero at mean position, then at any position x,

$U(x) = \frac{1}{2}kx^2 = \frac{1}{2}mA^2\omega^2 \sin^2\left(\omega t + \phi\right)$

For $\phi = 0°$

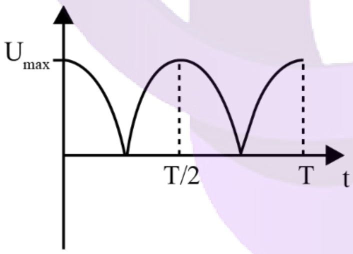


Fig. 12.16

- U is maximum at extremes, $U_{max} = \frac{1}{2}kA^2$.
- U is minimum at mean position

4.3 Total Energy

T.E = K.E + P.E

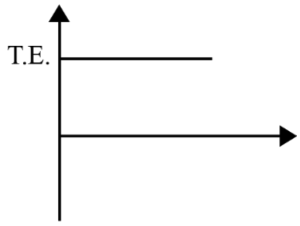


Fig. 12.17

$T.E. = \frac{1}{2}kA^2 = \frac{1}{2}mA^2\omega^2$

NOTE:

It is constant at all time instant and at all positions.

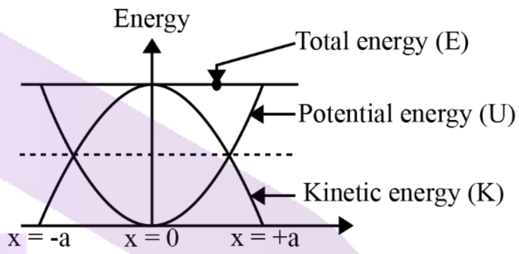


Fig. 12.18 Energy Position Graph

5. Spring Block System

5.1 Horizontal Spring

- Let a block of mass m be placed on a smooth horizontal surface and rigidly connected to spring of force constant K whose other end is permanently fixed.

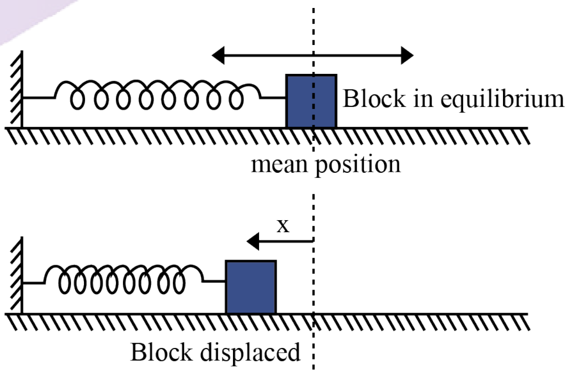


Fig. 12.19

- Mean position: when spring is at its natural length.
- Time period: $T = 2\pi\sqrt{\frac{m}{k}}$.

5.2 Vertical Spring

- If the spring is suspended vertically from a fixed point and carries the block at its other end as shown, the block will oscillate along the vertical line.

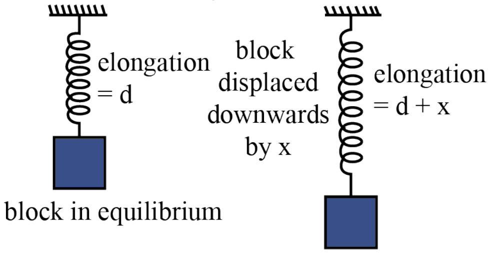


Fig. 12.20

- Mean position: spring is elongated by $d = \frac{mg}{k}$.
- Time period : $T = 2\pi\sqrt{\frac{m}{k}}$.

5.3 Combination of Spring

a) Spring in series

When two spring of force constant K_1 and K_2 are connected in series as shown, they are equivalent to a single spring of force constant K, which is given by

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K = \frac{K_1 K_2}{K_1 + K_2}$$

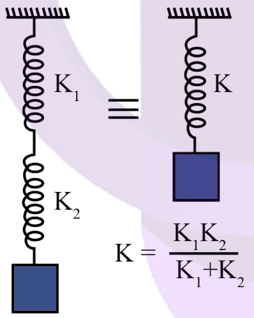


Fig. 12.21

b) Spring in parallel

For a parallel combination as shown, the effective spring constant is $K = K_1 + K_2$

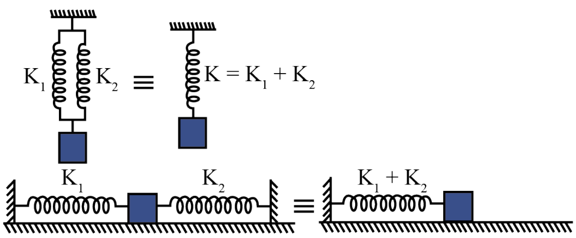


Fig. 12.22

5.4 Important Points Regarding Spring Block System

- If the length of the spring is made n times, then the effective force constant becomes 1/n times and the time period becomes $\sqrt{n}$ times.
- If a spring of spring constant k is divided into n equal parts, then the spring constant of each part becomes nk and time period becomes $\frac{1}{\sqrt{n}}$ times.
- The force constant of a stiffer spring is higher than that of soft spring.

6. Angular S.H.M.

Instead of straight-line motion, if a particle or centre of mass of body is oscillating on a small arc of circular path, then it is called angular S.H.M.

For angular S.H.M., $\tau = -k\theta$

$\Rightarrow I\alpha = -k\theta$

$\Rightarrow$ Time period, $T = 2\pi\sqrt{\frac{I}{k}}$.

where I is the moment of inertia of the body along the given axis.

6.1 Simple Pendulum

- If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum

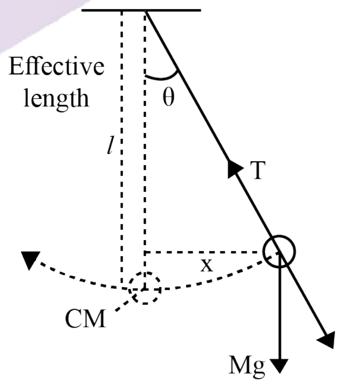


Fig. 12.23

- Time period : $T = 2\pi\sqrt{\frac{\ell}{g}}$

Special Cases:

a) If reference system is lift

- If velocity of lift v = constant, Acceleration a = 0 and $g_{eff} = g$

  $\therefore T = 2\pi\sqrt{\frac{\ell}{g}}$.

- If lift is moving upwards with acceleration a

  $g_{eff} = g + a$

  $T = 2\pi\sqrt{\frac{\ell}{g+a}} \Rightarrow T$ decreases.

- If lift is moving downwards with acceleration a

  $g_{eff} = g - a$

  $\therefore T = 2\pi\sqrt{\frac{\ell}{g-a}} \Rightarrow T$ decreases.

- If lift falls downwards freely

  $g_{eff} = g - g = 0 \Rightarrow T = \infty$.

b) A simple pendulum is mounted on a moving truck

- If truck is moving with constant velocity, no pseudo force acts on the pendulum and time period remains same $T = 2\pi\sqrt{\frac{\ell}{g}}$.

- If truck accelerates with acceleration a, then a pseudo force acts in opposite direction.
  So effective acceleration,

  $g_{eff} = \sqrt{g^2 + a^2}$ & $T' = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$

  Time period $T' = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2+a^2}}} \Rightarrow T'$ decreases.

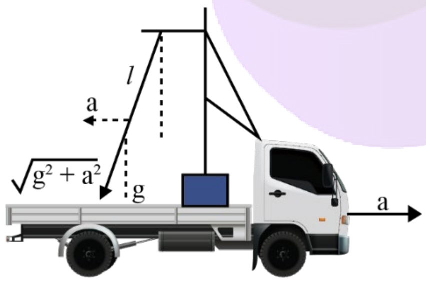


Fig. 12.24

6.2 Physical Pendulum

Consider a body of irregular shape and mass (m) is free to oscillate in a vertical plane about a horizontal axis passing through a point, weight mg acts downwards at the centre of gravity (C).

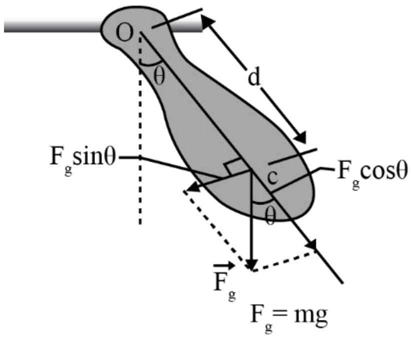


Fig. 12.25

If the body displaced through a small angle 'θ' and released from this position a torque is exerted by the weight of the body to restore to its equilibrium,

$\tau = -(F_g \sin\theta)d \quad [\text{as } F_g = mg]$

$\tau = -mgd\sin\theta$

$k = mgd$

$T = 2\pi\sqrt{\frac{I_{hinge}}{k}} \quad (I_{hinge} = \text{moment of inertia about hinge})$

$\Rightarrow T = 2\pi\sqrt{\frac{I_{Hinge}}{mgd}}$

6.2.1 Condition for Minimum Time period

$T = 2\pi\sqrt{\frac{I}{mgd}}$

$I = I_c + md^2$

$I = mK^2 + md^2$

where I is Moment of inertia of the given body along the given axis of rotation.

I_c is the moment of inertia of the given body along its centre of mass.

K is radius of gyration.

$T = 2\pi\left(\frac{mK^2 + md^2}{mgd}\right)^{1/2} \quad \left(\text{as, } K = \sqrt{\frac{I_c}{m}}\right)$

For minimum time period,

d = K

6.3 Torsional Pendulum

A torsional pendulum consists of a disk (or some other object) suspended from a wire, which is then twisted and released, resulting in an oscillatory motion.

- The oscillatory motion is caused by a restoring torque which is proportional to the angular displacement.

  $\tau_R = I\frac{d^2\theta}{dt^2} = -\kappa\theta$

where, I is the rotational inertia of the disk about the twisting axis. k (kappa) is the torsional constant (equivalent to the spring constant).

- This equation is exactly the same as SHM we have already discussed.
- By direct comparison the period of the torsional pendulum is given by.

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

and we can write, $\left[\theta(t) = A\cos(\omega t + \phi)\right]$

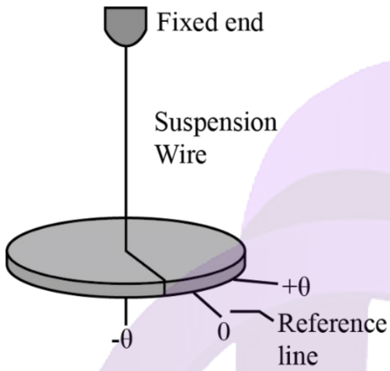


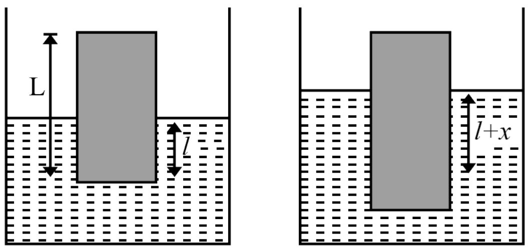
Fig. 12.26

- Similar to the simple pendulum, so long as the angular displacement is small (which means the motion in SHM) the period is independent of the displacement.
- Torsional pendulums are also used as a time keeping devices, as in for example, the mechanical wristwatch.

7. Miscellaneous Examples of S.H.M.

7.1 Oscillation of a Cylinder Floating in a Liquid

Let a cylinder of mass m and density d be floating on the surface of a liquid of density ρ. The total length of cylinder is L.



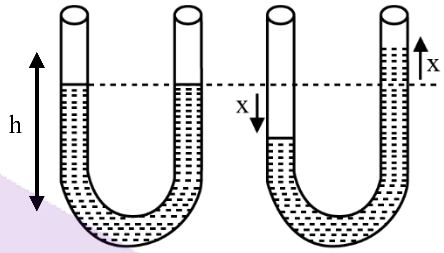
cylinder in equilibrium   cylinder displaced downwards by a distance x

Fig. 12.27

- Mean position : cylinder is immersed upto $\ell = \frac{Ld}{\rho}$.
- Time period : $T = 2\pi\sqrt{\frac{Ld}{\rho g}} = 2\pi\sqrt{\frac{\ell}{g}}$.

7.2 Liquid Oscillating in a U-tube

Consider a liquid column of mass m and density ρ in a U-tube of area of cross section A. Let L = 2H,



In equilibrium   liquid displaced by x

Fig. 12.28

- Mean position : when height of liquid is same in both limbs.
- Time period :

$$T = 2\pi\sqrt{\frac{m}{2A\rho g}} = 2\pi\sqrt{\frac{2L\cdot A}{2A\rho g}} = 2\pi\sqrt{\frac{L}{2g}}$$

where, L is length of liquid column.

7.3 Body Oscillation in Tunnel Along Any Chord of the Earth

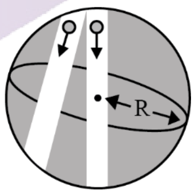


Fig. 12.29

- Mean position : At the centre of the chord.
- Time period :

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6\,\text{minutes}$$

where, R is radius of earth = 6400 Km.

7.4 Pulley Spring Block System

A system is consisting of massless pulley, a spring of spring constant k and a block of mass m. If the block is slightly displaced vertically down from its equilibrium position and released.

Let's find the frequency of its vertical oscillation in given cases:

Case (A):

As the pulley is fixed and string in inextensible, if mass m is displaced by y the spring will stretch by y

$F = T = ky$ i.e.,

restoring force is linear and so motion of mass m will be linear simple harmonic with frequency

$$f_A = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

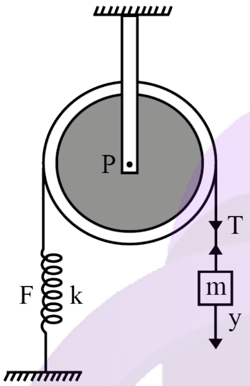


Fig. 12.30

Case (B):

The pulley is moveable and string inextensible, so if mass m moves down a distance y, the pulley will move down by $\left(\frac{y}{2}\right)$.

So the force in the spring $F = \frac{ky}{2}$.

Now as pulley is massless $F = 2T$,

$$\Rightarrow T = \frac{F}{2} = \frac{ky}{4}.$$

So the restoring force on the mass m,

$$T = \frac{1}{4}ky = k'y \Rightarrow k' = \frac{1}{4}k$$

$$\Rightarrow f_B = \frac{1}{2\pi}\sqrt{\frac{k'}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{4m}} = \frac{f_A}{2}$$

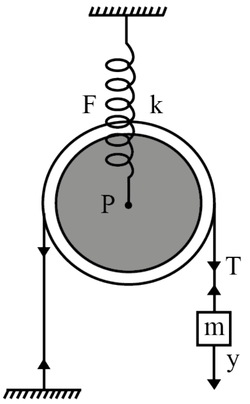


Fig. 12.31

Case (C):

In this situation if the mass m moved by the pulley will also move by y and so the spring will stretch by 2 (as string is inextensible) and so, $T' = F = 2ky$.

Now as pulley is massless so $T = F + T' = 4ky$

i.e., the restoring force on the mass m

So, $T = 4ky = k'y \Rightarrow k' = 4k$

$$\Rightarrow f_C = \frac{1}{2\pi}\sqrt{\frac{k'}{m}} = \frac{1}{2\pi}\sqrt{\frac{4k}{m}} = 2f_A$$

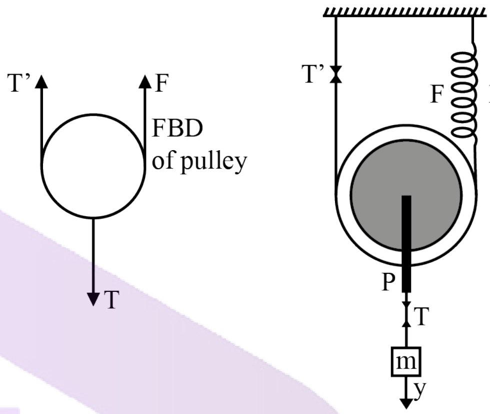


Fig. 12.32

8. Superposition of SHM

A simple harmonic motion is produced is produced when a force (called restoring force) proportional to the displacement acts on a particle. If a particle is acted upon by two such forces the resultant motion of the particle is combination of two simple harmonic motions.

8.1 In Same Direction

a) Having Same Frequencies:

Suppose the two individual motions are represented by,

$$x_1 = A_1 \sin\omega t \;\&\; x_2 = A_2 \sin(\omega t + \phi)$$

Both the simple harmonic motions have same angular frequency ω.

$$x = x_1 + x_2$$

$$= A_1 \sin\omega t + A_2 \sin(\omega t + \phi)$$

$$= A\sin(\omega t + \alpha).$$

Here, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

and $\tan\alpha = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$.

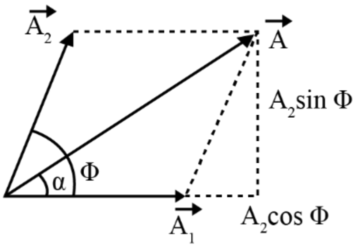


Fig. 12.33

Thus, we can see that this is similar to the vector addition. The same method of vector addition can be applied to the combination of more than two simple harmonic motions.

b) Having Different Frequencies:

$x_1 = A_1 \sin \omega_1 t$

$x_2 = A_2 \sin \omega_2 t$

Then resultant displacement

$x = x_1 + x_2$

$= A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$

This resultant motion is not SHM.

8.2 In Two Perpendicular Directions

Let the motion be in x as well as y directions. They are given as:

$x = A_1 \sin \omega t$ ....(1)

$y = A_2 \sin(\omega t + \phi)$ .....(2)

The amplitude A_1 and A_2 may be different and phase difference ϕ and ω is same. So equation of the path may be obtained by eliminating t from (1) & (2)

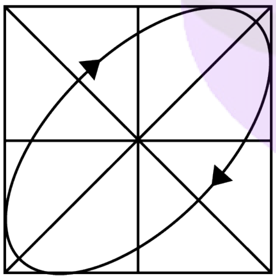


Fig. 12.34

On rearranging we get

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy\cos\phi}{A_1 A_2} = \sin^2\phi \quad ....(3)$$

Special Case:

a) If $\phi = 0°$

$$\therefore \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1 A_2} = 0$$

$$\therefore y = \frac{A_2}{A_1} \cdot x \quad \text{(eq. of straight line)}$$

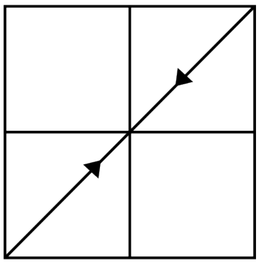


Fig. 12.35

b) If $\phi = 90°$

$$\Rightarrow \frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} = 1 \quad \text{(eq. of ellipse)}$$

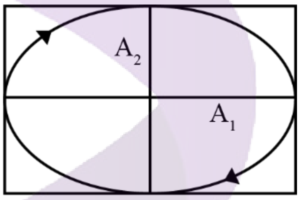


Fig. 12.36

c) If $\phi = 90°$ & $A_1 = A_2 = A$ then

$x^2 + y^2 = A^2$ (eq. of circle)

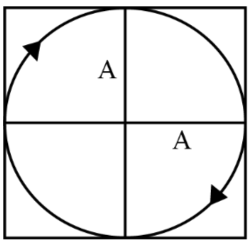


Fig. 12.37

9. Damped & Forced Oscillations

9.1 Damped Oscillation

- The oscillation of a body whose amplitude goes on decreasing with time is defined as **damped oscillation**.
- In this oscillation the amplitude of oscillation decreases exponentially due to damping forces like fractional force, viscous force etc.

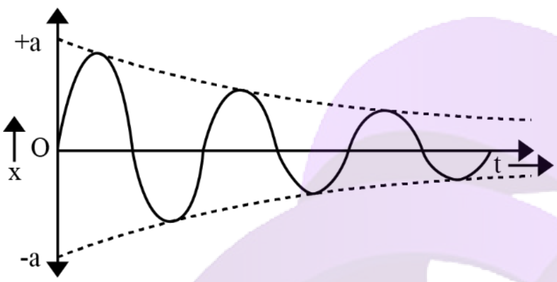


Fig. 12.38

- Due to decrease in amplitude of the oscillator, energy goes on decreasing exponentially.
- The damping force can be expressed as F = -bv (where b is a constant called the damping coefficient) and restoring force on the system is -kx, we can write Newton's second law as

$F_{net} = -kx - bv = ma$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

This is the differential equation of damped oscillation.

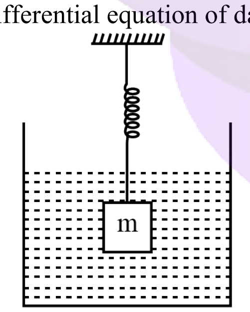


Fig. 12.39

The solution of this equation is given by

$$x = A\left(e^{-\frac{bt}{2m}}\right)\cos(\omega' t + \phi)$$

Where angular frequency of oscillation is

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}$$

Where $\omega = \sqrt{\frac{k}{m}}$ represents the angular frequency in the absence of retarding force (the undamped oscillator) & is called natural frequency.

9.2 Forced Oscillation

- The oscillation in which a body oscillates under the influence of an external periodic force are known as **forced oscillation**.
- Resonance: When the frequency of external force is equal to the natural frequency of the oscillator, then this state is known as the state of resonance. And this frequency is known as resonant frequency.
- Suppose an external force F(t) of amplitude F_0 that varies periodically with time is applied to a damped oscillator.

  Such a force is $F(t) = F_0 \cos\omega_d t$.

  The equation of particle under combined force is

  $ma(t) = -kx - bv + F_0 \cos\omega_d t$

  $$\frac{md^2x}{dt^2} + \frac{bdx}{dt} + kx = F_0 \cos\omega_d t$$

  After solving,

  $x = A'\cos(\omega_d t + \phi)$

  where, $$A' = \frac{F}{m\sqrt{\left(\omega_d^2 - \omega^2\right) + \left(\frac{b\omega_d}{m}\right)^2}}$$

  (ω is natural frequency & $\omega = \sqrt{\frac{k}{m}}$)

  Graph of amplitude versus frequency for a damped oscillator when a periodic driving force is present. When the frequency of the driving force equals the natural frequency ω, resonance occurs.

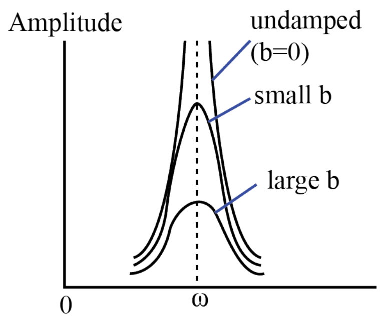


Fig. 12.40

NCERT Corner

(Some Important Points to Remember)

1. Periodic Motion

- A motion which repeats itself over and over again after a regular interval of time is called a Periodic Motion. E.g., revolution of planets around the sun, rotation of the earth about its polar axis etc.
- The function which are used to represent periodic motion are called Periodic Functions.
- One of the simplest periodic function is given by $f(t) = A\cos\omega t$.

2. Oscillatory Motion

A motion in which a body moves back and forth repeatedly about a fixed point (called mean position) is called **Oscillatory** or **Vibratory Motion.**

3. Simple Harmonic Motion

- In simple harmonic motion (SHM), the displacement x(t) of a particle from its equilibrium position is given by,
  $x(t) = A\cos(\omega t + \phi)(\text{displacement})$
  in which A is the amplitude of the displacement, the quantity $(\omega t + \phi)$ is the phase of the motion, and ϕ is the phase constant. The angular frequency ω is related to the period and frequency of the motion by,
  $\omega = \frac{2\pi}{T} = 2\pi\upsilon(\text{angular frequency})$.
- Simple harmonic motion can also be viewed as the projection of uniform circular motion on the diameter of the circle in which the latter motion occurs.
- The particle velocity and acceleration during SHM as function of time are given by,
  Velocity: $v(t) = -\omega A\sin(\omega t + \phi)$,
  Acceleration:
  $a(t) = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t)$
  Thus, we see that both velocity and acceleration of a body executing simple harmonic motion are periodic functions, having the velocity amplitude $v_m = \omega A$ and acceleration amplitude $a_m = \omega^2 A$, respectively.

4. The force acting in a simple harmonic motion is proportional to the displacement and is always directed towards the centre of motion.

5. A particle of mass m oscillating under the influence of Hooke's law restoring force given by F = -kx exhibits simple harmonic motion with

$$\omega = \sqrt{\frac{k}{m}} \ (\text{angular frequency})$$

$$T = 2\pi\sqrt{\frac{m}{k}} \ (\text{period})$$

Such a system is also called a linear oscillator.
The period T is the time required for one complete oscillation, or cycle. It is related to the frequency υ by.

$$T = \frac{1}{\upsilon}.$$

6. Energy in SHM

- If a particle of mass m executes SHM, then at a displacement y from mean position, the particle possesses potential and kinetic energy.
- At any displacement y,

  a) Potential energy, $U = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}ky^2$.

  b) Kinetic energy,

  $$K = \frac{1}{2}m\omega^2\left(A^2 - y^2\right) = \frac{1}{2}k\left(A^2 - y^2\right).$$

  c) Total energy,

  $$E = U + K = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 m\upsilon^2 A^2.$$

- At mean position, kinetic energy is maximum and potential energy is zero.
- At extreme position, potential is maximum and kinetic energy is zero.
- The time period of potential and kinetic energy is T/2.
- The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration.

7. Simple Pendulum

A simple pendulum, in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string.

- The motion of a simple pendulum is simple harmonic whose time period and frequency are given by

$$T = 2\pi\sqrt{\frac{\ell}{g}} \ \& \ \upsilon = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

- If a pendulum of length ℓ at temperature θ°C has a time period T, then on increasing the temperature by $\Delta\theta$°C its time period changes to $T + \Delta T$.

  where, $\frac{\Delta T}{T} = \frac{1}{2}\alpha \cdot \Delta\theta$.

- If the length of a simple pendulum is increased to such an extent that $\ell \to \infty$, i.e., $\ell >> R$, then its time period is

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$$

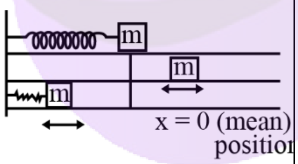
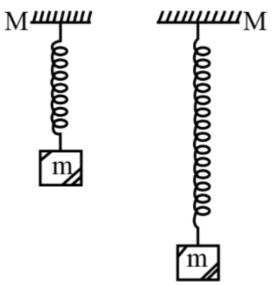
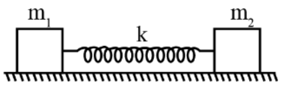
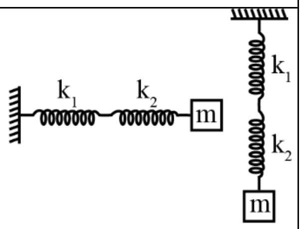
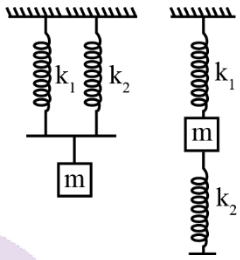
  where, R = radius of the earth.

8. Spring Mass system

If the mass is once pulled so as to stretch the spring and released, the spring pendulum oscillates simple harmonically having time period and frequency.

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ and } \upsilon = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Angular frequency, $\omega = \sqrt{\frac{k}{m}}$

Various Cases	Figures
(i) Spring is light $T = 2\pi\sqrt{\frac{m}{k}}$	
(ii) Spring is not light but has m_s $T = 2\pi\sqrt{\frac{m + \frac{1}{3}m_s}{k}}$	
(iii) Spring connected with two masses $(m_1 \text{ and } m_2)$ $T = 2\sqrt{\frac{\mu}{k}}$	
(iv) Spring with spring constants (k_1 and k_2) connected in series $T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$	
(v) Spring connected in parallel $T = 2\pi\sqrt{\frac{m}{k_p}}$ $= 2\pi\sqrt{\frac{m}{k_1 + k_2}}$	

where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass of the system,

$k_s = \frac{k_1 k_2}{k_1 + k_2}$ and $k_p = k_1 + k_2$.

9. Simple Harmonic Motion in Special Cases

- The time period of SHM due to the motion of incompressible and non-viscous liquid in U-tube is given as $T = 2\pi\sqrt{\frac{h}{g}}$

  where, h = height of undisturbed liquid in each limb

  and L = 2h = total length of liquid column.

- The time period of a ball performing SHM in hemispherical bowl is expressed as $T = 2\pi\sqrt{\frac{R - r}{g}}$

  where, R and r are radii of bowl and ball.

- The time period of a ball executing SHM in a tunnel through the earth is expressed as

  $T = 2\pi\sqrt{\frac{R}{g}}$ where, R = radius of earth.

10. Undamped and Damped Oscillations

When a simple harmonic system oscillates with a constant amplitude which does not change with time, its oscillations are called **undamped oscillations.**

When a simple harmonic system oscillates with a decreasing amplitude with time, its oscillations are called **damped oscillations.**

11. Free and Forces Oscillations

A body capable of oscillating is said to be executing **free oscillations,** if it vibrates with its own natural frequency without the help of any external periodic force.

When a body oscillates with the help of an external periodic force with a frequency different from the natural frequency of the body, these oscillations are called **forced oscillations.**

12. Resonance

If an external force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω, the system oscillates with angular frequency ω_d. The amplitude of oscillations is the greatest when $\omega_d = \omega$ a condition called resonance.

13. Superposition of SHM

A simple harmonic motion is produced when a force called restoring force proportional to the displacement acts on a particle. If a particle is acted upon by two such forces, the resultant motion of the particle is a combination of two simple harmonic motions.

Suppose the two individual motions are represented by $y_1 = A_1 \sin \omega t$ and $y_2 = A_2 \sin(\omega t + \phi)$

Both the simple harmonic motions have same angular frequency ω.

Resultant amplitude is expressed as

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$.

13

Waves

Waves

1. Introduction and Classification of Waves

Introduction of Waves:

When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (One way of transport energy from one place to another place).

There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

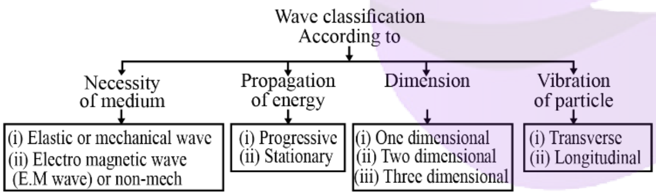
NOTE:

> A **wave** is a disturbance that propagates in space, transport energy and momentum from one point to another without the transport of matter.

Few Examples of waves:

The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

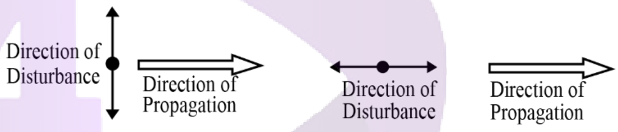
1.1 Classification of Waves



- **Based on medium necessity: -** A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves.**

Example: Sound waves in water, seismic waves in earth's crust.

- **Based on energy propagation: -** Waves can be divided into two parts on the basis of energy propagation (i) Progressive wave (ii) Stationary waves. The progressive wave propagates with constant velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.
- **Based on direction of propagation:-** Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional, while sound or light waves from a point source are three dimensional.
- **Based on the motion of particles of medium:**



Transverse wave Longitudinal wave

Waves are of two types on the basis of motion of particles of the medium.

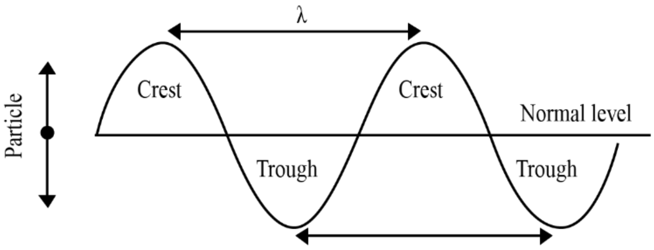
(i) Longitudinal waves

(ii) Transverse waves

In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

1.2 Transverse Wave Motion

Mechanical transverse waves are produced in such type of medium which have shearing property, so they are known as shear wave or S-wave



NOTE:

Shearing is the property of a body by which it changes its shape on application of force.

⇒ Mechanical transverse waves are generated only in solids and surface of liquid.

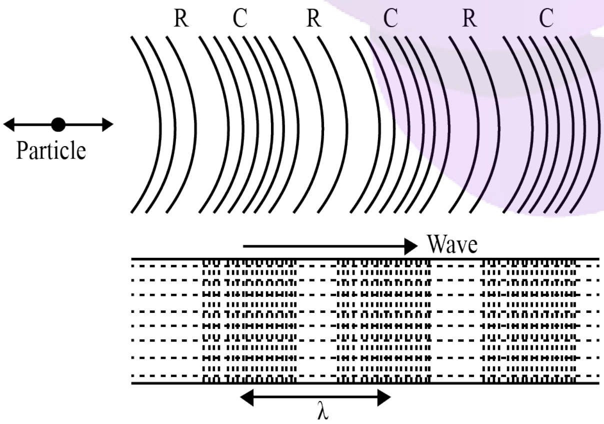
Individual particles of the medium execute SHM about their mean position in direction perpendicular to the direction of propagation of wave.

A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium, when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium, when a transverse wave passes.

1.3 Longitudinal Wave Motion

In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).



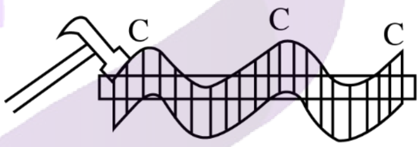
NOTE:

The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

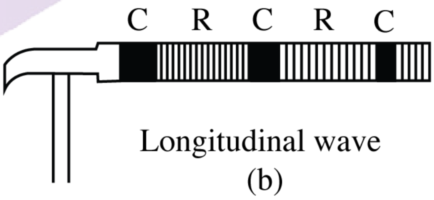
The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

1.4 Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depending on the nature ot medium and mode of excitation.
- In strings, mechanical waves are always transverse when string is under a tension. In the bulk of gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. (A) the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. (B) or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.



Transverse wave
(a)

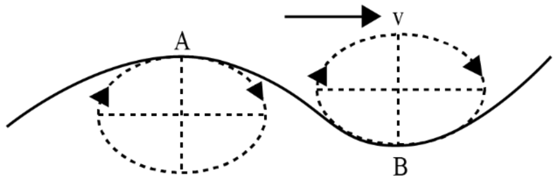


Longitudinal wave
(b)

Furthermore in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds

$\left[v_s \cong 5km/s \text{ while } v_p \cong 9km/s\right]$ S-waves are transverse while P – waves are longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane.



Ripple

1.5 Characteristics of Wave Motion

- In wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- The energy is transferred from one place to another without any actual transfer of the particles of the medium.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.
- The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at mean position and zero at extreme position.
- For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction among its particles.

2. Equation of Plane Progressive Wave

2.1 Important Terms Connected with Wave Motion

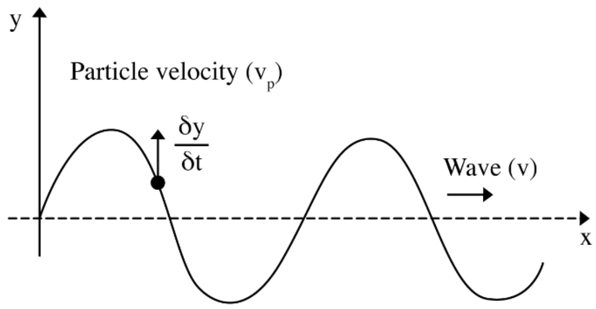
- **Wavelength (λ) [length of one wave]:** Distance travelled by the wave during the time interval in which any one particle of the medium completes one cycle about its mean position. We may also define wavelength as the distance between any two nearest particle of the medium, vibrating in the same phase.
- **Frequency (f):** Number of cycle (number of complete wavelengths) completed by a particle in unit time.
- **Time period (T):** Time taken by wave to travel a distance equal to one wavelength.
- **Amplitude (A):** Maximum displacement of vibrating particle from its equilibrium position.
- **Angular frequency (ω) :** It is defined as $\omega = \frac{2\pi}{T} = 2\pi f$
- **Phase:** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation $y = A\sin(\omega t - kx); (\omega t - kx) = \text{phase}$.
- **Angular wave number or propagation constant (k):** It is defined as $k = \frac{2\pi}{\lambda}$
- **Wave number $(\bar{v})$:** it is defined as $\bar{v} = \frac{1}{\lambda} = \frac{k}{2\pi} =$ number of waves in unit length of the wave pattern.
- **Particle velocity, wave velocity and particle's acceleration:** In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae that we studied in SHM apply to the particles here also. For example, maximum particle speed is ωA at mean position and it is zero at extreme positions. Similarly maximum particle acceleration is $\omega^2 A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between $+A\omega$ and $-A\omega$) the wave velocity is constant for given characteristics of the medium.
- **Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave:** The acceleration of the particle is the second partial derivative of $y(x,t)$ with respect to t,

$$\therefore a_p = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\sin(\omega t - kx) = -\omega^2 y(x,t)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the same result we obtained for SHM. Thus, $a_p = -\omega^2$ (displacement)

2.2 Equation of a Plane Progressive Wave

- **Particle velocity in wave motion:** The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "particle velocity".



- **Wave velocity:** The velocity with which the disturbance, or planes of equal (wave front), travel through the medium is called wave (or phase) velocity.
  Wave velocity (v) = Frequency of waves (f) × wavelength of waves (λ)
- **Relation between particle velocity and wave velocity:** Wave equation: - $y = A\sin(\omega t - kx)$,

Particle velocity $v_P = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$. Wave velocity

$$\Rightarrow V = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{k}, \frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx)$$

$$\frac{\partial y}{\partial x} = -\frac{1}{V}\frac{\partial y}{\partial t} \Rightarrow \boxed{\frac{\partial y}{\partial x} = -\frac{1}{V}\frac{\partial y}{\partial t}}$$

Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

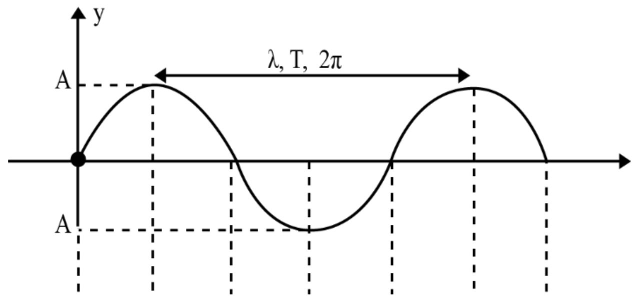
2.3 Differential Equation of Harmonic Progressive Waves:

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - kx)$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(\omega t - kx)$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2}\frac{\partial y}{\partial t}$$

2.4 Relation Between Phase Difference and Path Difference



Phase Difference $(\Delta\phi)$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Phase Difference $(\Delta\lambda)$	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3\lambda}{2}$
Phase Difference (Δt)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

Path difference $= \frac{2\pi}{\lambda} \times$ phase difference

3. Transverse Waves on a String

A transverse wave is a moving wave whose oscillations are perpendicular to the direction of the wave. A simple demonstration of the wave can be created on a horizontal length of the string by securing one end of the string and moving the other up and down. Light is another example of a transverse wave, where the oscillations are electric and magnetic fields that are at right angles to the ideal light rays that describe the direction of propagation.

Transverse waves commonly occur in elastic solids, oscillations, in this case, are the displacement of solid particles from their relaxed position, in the direction perpendicular to the propagation of the wave.

For example: - The ripples on the surface of the water, Electromagnetic waves, Ocean waves, etc.

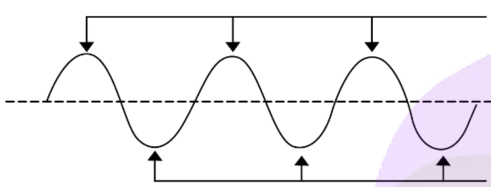
The speed of a wave on a string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where T is tension in the string (in Newtons) and μ is mass per unit length of the string (kg/m).

It should be noted that v is speed of the wave w.r.t. the medium (string).

In case the tension is not uniform in the string or string has nonuniform linear mass density then v is speed at a given point and T and μ are corresponding values at that point.



The velocity of a wave is calculated by dividing the distance traveled by the time it took to travel that distance. For waves, this is calculated by dividing the wavelength by the period as follows: $v = \frac{\lambda}{T}$

We can take the inverse proportionality to period and frequency and apply it to this situation as follows:

$$v = \frac{\lambda}{T} v = \lambda \frac{1}{T} v = \lambda f$$

4. Energy Transfer in a String Wave

4.1 Rate of Energy, Power and Intensity of Wave

- Energy Transferred $= \int_0^t P_{av} dt$

  Energy transferred in one time period $= P_{av} T$

  This is also equal to the energy stored in one wavelength.

- When a travelling wave is established on a string, energy is transmitted along the direction of propagation of the wave, in form of potential energy and kinetic energy

  Average Power $(P) = 2\pi^2 f^2 A^2 \mu v$

- **Intensity:** Energy transferred per second per unit cross sectional area is called intensity of the wave.

$$I = \frac{\text{Power}}{\text{Cross sectional area}} = \frac{P}{s} \Rightarrow I = \frac{1}{2}\rho\omega^2 A^2 v$$

This is average intensity of the wave.

Energy density: Energy per unit volume of the wave

Energy density per unit volume $= \frac{Pdt}{svdt} = \frac{I}{v} = \frac{1}{2}\rho\omega^2 A^2$

4.2 Relation Between Amplitude and Intensity of Wave

For light waves, the energy of the light wave is proportional to the intensity.

$E \propto I$, where E is the energy of the wave and I is the intensity.

$E \propto (\text{Amplitude})^2$ ......(1)

Also, the intensity of a wave is power transferred per unit area.

We know that power is energy expended per unit time.

Therefore,

$I = \frac{E}{At}$, where A is the area of cross section and t is the time.

Therefore, we can say that.

$I \propto E$ ...(2)

From expression (1) and (2) we can say that.

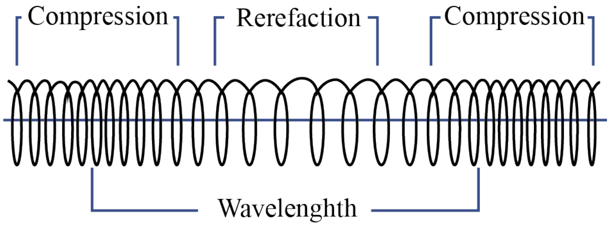
$I \propto (\text{Amplitude})^2$ or Amplitude $\propto \sqrt{\text{Intensity}}$

5. Longitudinal Waves

5.1 Longitudinal Waves and Equation of Longitudinal Waves

Longitudinal waves are the waves where the displacement of the medium is in the same direction as the direction of the travel of the wave.

The distance between the centres of two consecutive regions of compression or the rarefaction is defined by wavelength λ.



A compression in a longitudinal wave is a region where the particles are the closer together while rarefaction in a longitudinal wave rarefaction is a region where the particle are spread out.

5.2. Sound as a Pressure Wave

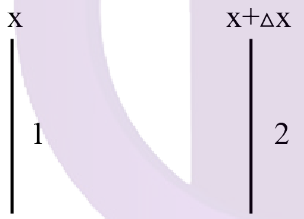
We can describe sound waves either in terms of excess pressure or in terms of the longitudinal displacement suffered by the particles of the medium w.r.t. mean position.

$s = s_0 \sin \omega(t - x/v)$ represents a sound wave where,

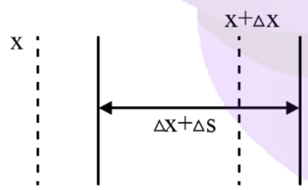
s = displacement of medium particle from its mean position at x,

$s = s_0 \sin(\omega t - kx) \quad .....(1)$

When sound is not propagating particles are at mean position 1 and 2



When particles are displaced from mean position.



Change in volume $= \Delta V = (\Delta x + \Delta s)A - \Delta x A = \Delta s A$

$$\frac{\Delta V}{V} = \frac{\Delta s A}{\Delta x A} = \frac{\Delta s}{\Delta x}$$

$$\Delta P = -\frac{B \Delta V}{V}$$

$$\Delta P = -\frac{B \Delta s}{\Delta x}$$

$$dp = -\frac{Bds}{dx}$$

$$dp = -B(-ks_0)\cos(\omega t - kx)$$

$$dp = Bks_0 \cos(\omega t - kx)$$

$$dp = (dp)_{max} \cos(\omega t - kx)$$

$$p = p_0 \sin\left(\omega t - kx + \frac{\pi}{2}\right) \quad .....(2)$$

Where $p = dp =$ variation in pressure at position x and

$p_0 = Bks_0 =$ maximum pressure variation

Equation 3.2 represents that same sound wave where, p is excess pressure at position x, over and above the average atmospheric pressure and pressure amplitude p_0 is given by

$P_0 = Bks_0 \quad .....(3)$

(B = Bulk modulus of the medium, k = angular wave number)

Note from equation (1) and (2) that the displacement of a medium particle and excess pressure at any position are out of phase by $\frac{\pi}{2}$. Hence a displacement maximum corresponds to a pressure minima and vice-versa.

5.3. Speed of Sound and Laplace's Correction

Velocity of sound waves in a linear solid medium is given by

$$v = \sqrt{\frac{Y}{\rho}} \quad .....(1)$$

Where $Y =$ Young's modulus of elasticity and $\rho =$ density.

Velocity of sound waves in a fluid medium (liquid or gas) is given by

$$v = \sqrt{\frac{B}{\rho}} \quad .....(2)$$

Where $\rho =$ density of the medium and $B =$ Bulk modulus of the medium given by,

$$B = -V\frac{dP}{dV} \quad .....(3)$$

Newton's formula: Newton assumed propagation of sound through a gaseous medium to be an isothermal process.

$PV =$ constant

$$\Rightarrow \frac{dP}{dV} = \frac{-P}{V}$$

and hence B = P using equation (3) and thus he obtained for velocity of sound in a gas,

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}} \text{ where M = molar mass in kg}$$

The density of air at 0° at pressure 76 cm of Hg column is $\rho = 1.293 kg/m^3$. This temperature and pressure is called standard temperature and pressure at STP. Speed of sound in air is 280m/s. This value is somewhat less than measured speed of sound in air 332m/s. Then Laplace suggested the correction.

Laplace's Correction: Later Laplace established that propagation of sound in a gas is not an isothermal but an adiabatic process and hence $PV^\gamma = \text{constant}$

$$\Rightarrow \frac{dP}{dV} = -\gamma\frac{P}{V}$$

Where, $B = -V\frac{dP}{dV} = \gamma P$ and hence speed of sound in a gas,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \quad .....(4)$$

Factors Affecting Speed of Sound in Atmosphere.

(a) Effect of Temperature:

As temperature (T) increases velocity (v) increases.

$v \propto \sqrt{T}$

For small change in temperature above room temperature v increases linearly by 0.6m/s for every $1°C$ rise in temperature.

$$v = \sqrt{\frac{\gamma R}{M}} \times T^{1/2}$$

$$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\Delta v = \left(\frac{1}{2}\frac{v}{T}\right)\Delta T$$

$$(\Delta v)_T = (0.6)\Delta T,\ \Delta T = (T - 273)\text{kelvin}$$

(b) Effect of Pressure:

The speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

So, at constant temperature, if P changes then ρ also changes in such a way that P/ρ remains constant. Hence pressure does not have any effect on velocity of sound as long as temperature is constant.

(c) Effect of Humidity:

With increase in humidity density decreases. This is because the molar mass of water vapour is less than the molar mass of air. The speed of sound increases in humid air compared to dry air as. The density of water vapour is about $\frac{5}{8}$th times the density of dry air at ordinary temperature, therefore the increase of moisture in air tends to decrease the density of air.

6. Sound Intensity and Loudness

Intensity of Sound Waves: Like any other progressive wave, sound waves also carry energy from one point of space to the other. This energy can be used to do work, for example, forcing the eardrums to vibrate or in the extreme case of a sonic boom created by a supersonic jet, can even cause glass panes of windows to crack. The amount of energy carried per unit time by a wave is called its power and power per unit area held perpendicular to the direction of energy flow is called intensity.

$$I = \frac{1}{2}\rho\omega^2 A^2 V$$

For Sound,

$$V = \sqrt{\frac{B}{\rho}}$$

$$\text{So } = \frac{P_0}{BK}$$

$$I = \frac{1}{2}\frac{B}{V^2}.\omega^2.\frac{P_0^2}{B^2K^2}.V$$

$$I = \frac{P_0^2.V}{2K}$$

$$I = \frac{P_0^2}{2\rho V}$$

NOTE:

If The If the source is a point source, then $I \propto \frac{1}{r^2}$ and

$s_0 \propto \frac{1}{r}$ and $s = \frac{a}{r}\sin(\omega t - kr + \theta)$

If a sound source is a line source, then $I \propto \frac{1}{r}$ and

$s_0 \propto \frac{1}{\sqrt{r}}$ and $s = \frac{a}{\sqrt{r}}\sin(\omega t - kr + \theta)$

Loudness: Audible intensity range for humans:

The ability of human to perceive intensity at different frequency is different. The perception of intensity is maximum at 1000 Hz and perception of intensity decreases as the frequency decreases or increases from 1000Hz.

For a 1000Hz tone, the smallest sound intensity that a human ear can detect is 10^{-12} watt. / m^2. On the other hand, continuous exposure to intensities above $1 W/m^2$ can result in permanent hearing loss.

The overall perception of intensity of sound to human ear is called loudness.

Human ear do not perceives loudness on a linear intensity scale rather it perceives loudness on organismic intensity scale.

For a example:

If intensity is increased 10 times human ear does not perceive 10 times increase in loudness. It roughly perceived that loudness is doubled where intensity increased by 10 times. Hence it is prudent to define a logarithmic scale for intensity.

Decibel Scale: The logarithmic scale which is used for comparing two sound intensity is called decibel scale.

The intensity level β described in terms of decibels is

defined as $\beta = 10\log\left(\frac{I}{I_0}\right)(dB)$

Here I_0 is the threshold intensity of hearing for human ear

i.e $I = 10^{-12}$ watt / m^2.

In terms of decibel threshold of human hearing is 1dB

Note that intensity level β is a dimensionless quantity and is not same as intensity expressed in W/m^2.

7. Superposition of Waves

7.1 Superposition of Waves

Two or more waves can traverse the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any is instant is equal to the vector sum of the displacement produced by individual by wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore, according to superposition principle.

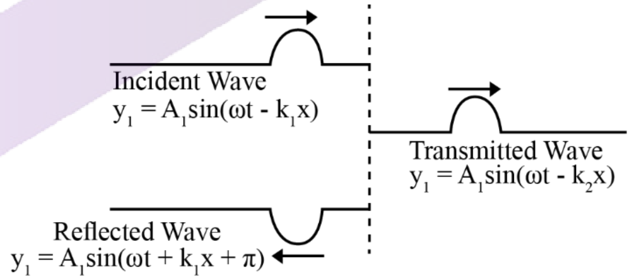
The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacement caused to the particle by the individual waves.

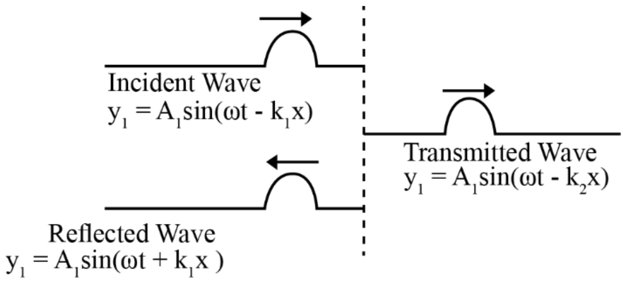
If $\vec{y}_1, \vec{y}_2, \vec{y}_3$..... are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

7.2 Reflection of Waves

Reflection of String Waves: A travelling wave, at a rigid or denser boundary, is reflected with a phase reversal but the reflection at an open boundary (rarer medium) takes place without any phase change. The transmitted wave is never inverted, but propagation constant k is changed.





Amplitude of reflected and transmitted waves:

v_1 and v_2 are speeds of the incident wave and reflected wave in mediums respectively then

$$A_r = \frac{v_2 - v_1}{v_1 + v_2} A_i, \quad A_t = \frac{2v_2}{v_1 + v_2} . A_i$$

A_r is positive if $v_2 > v_1$, i.e., wave is reflected from a rarer medium.

Reflection of Sound Waves: Reflection of sound waves from a rigid boundary (e.g. closed end of an organ pipe) is analogous to reflection of a string wave from rigid boundary; reflection accompanied by an inversion i.e. an abrupt phase change of π. This is consistent with the requirement of displacement amplitude to remain zero at the rigid end, since a medium particle at the rigid end can not vibrate. As the excess pressure and displacement corresponding to the same sound wave vary by $\pi/2$ in term of phase, a displacement minimum at the rigid end will be a point of pressure maxima. This implies that the reflected pressure wave from the rigid boundary will have same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse.

On the other hand, reflection of sound wave from a low-pressure region (like open end of an organ pipe) is analogous to reflection of string wave from a free end. This point corresponds to a displacement maximum, so that the incident & reflected displacement wave at this point must be in phase. This would imply that this point would be a minimum for pressure wave (i.e. pressure at this point remains at its average value), and hence the reflected pressure wave would be out of phase by π with respect to the incident wave. i.e. a compression pulse is reflected as a rarefaction pulse and vice-versa.

7.2 Interference of Waves

Interference of String Waves: Suppose two identical sources send sinusoidal waves of same angular frequency ω in positive x-direction. Also, the wave velocity and hence, the wave number k is same for the two waves. One source may be situated at different points. The two waves arriving at a point then differ in phase. Let the amplitudes of the two waves be A_1 and A_2 and the two waves differ in phase by an angle ϕ. Their equations may be written as

$y_1 = A_1 \sin(kx - \omega t)$

And $y_2 = A_2 \sin(kx - \omega t + \phi)$

According to the principle of superposition, the resultant wave is represented by

$y = y_1 + y_2 = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \phi)$

We get $y = A \sin(kx - \omega t + \alpha)$

Where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$ (A is amplitude of the resultant wave)

Also, $\tan\alpha = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$ (α is phase difference of the resultant wave with the first wave)

Constructive and Destructive Interference

Constructive Interference:

When resultant amplitude A is maximum $A = A_1 + A_2$

When $\cos\phi = +1$ or $\phi = \pm 2n\pi$

Where n is an integer.

Destructive Interference:

When resultant amplitude A is minimum

Or $A = |A_1 - A_2|$

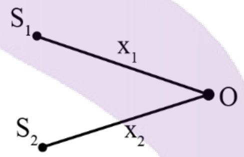
When $\cos\phi = -1$ or $\phi = \pm(2n+1)\pi$

When n is an integer.

Interference of Sound Waves:

If $p_1 = p_{m1} \sin(\omega t - kx_1 + \theta_1)$

and $p_2 = p_{m2} \sin(\omega t - kx_2 + \theta_2)$



Resultant excess pressure at point O is

$p = p_1 + p_2$

$\Rightarrow p = p_0 \sin(\omega t - kx + \theta)$

Where,

$$p_0 = \sqrt{p_{m1}^2 + p_{m2}^2 + 2p_{m1}p_{m2}\cos\phi}, \phi = |k(x_1 - x_2) + (\theta_2 - \theta_1)| \ldots(1)$$

(i) For constructive interference

$\phi = 2n\pi \Rightarrow p_0 = p_{m1} + p_{m2}$

(ii) For destructive interference

$\phi = (2n+1)\pi \Rightarrow p_0 = |p_{m1} - p_{m2}|$

If ϕ is only due to path difference, then $\phi = \frac{2\pi}{\lambda}\Delta x$, and condition for constructive interference:

$$\Delta x = \pm n\lambda, \quad n = 0, \pm\frac{1}{1}, \pm\frac{2}{2}$$

Condition for destructive interference:

$$\Delta x = \pm(2n+1)\frac{\lambda}{2}, \quad n = 0, \pm\frac{1}{1}, \pm\frac{2}{2}$$

From equation (1)

$P_0^2 = P_{m1}^2 + P_{m2}^2 + 2P_{m1} + P_{m2} \cos\phi$

Since intensity, $I \propto$ (Pressure amplitude)^2,

We have, for resultant intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad .....(2)$$

$$I_1 = I_2 = I_0$$

$$I = 2I_0(1+\cos\phi) \Rightarrow I = 4I_0\cos^2\frac{\phi}{2} \quad ......(3)$$

Hence in this case,

For constructive interference: $\phi = 0, \pm 2\pi, \pm 4\pi$ and $I_{max} = 4I_0$

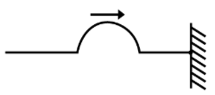
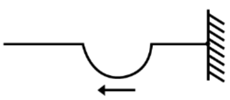
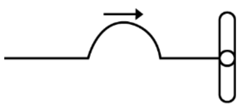
And for destructive interference: $\phi = \pi, 3\pi....$ and $I_{min} = 0$

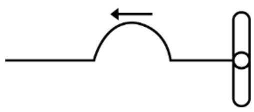
Coherence: Two sources are said to be coherent if the phase difference between them at a given point does not change with time. In this case their resultant intensity at any point in space remains constant with time. Two independent sources of sound are generally incoherent in nature, i.e. phase difference
between them changes with time and hence the resultant intensity due to them at any point in space changes with time.

8. Standing Waves

8.1 Introduction to Standing Waves

- When two identical progressive waves (transverse or longitudinal) propagating in opposite direction superimpose in a bounded medium (having boundaries) the resultant wave is called stationary wave or standing wave.
- Stationary wave pattern may be obtained only and only in limited region.
- We can obtain two same type of progressive waves, only & only by method of reflection.
- According to the nature of reflected surface, reflection are of two types –

(a) Rigid End	(b) Free End
In such type of reflection incident and reflected waves have phase difference of π and direction of propagation are opposite.   	In such type of reflection incident and reflected waves are in phase and direction of propagation are opposite 

Incident wave $y_1 = a\sin(\omega t - kx)$ Reflected wave $y_2 = a\sin(\omega t + kx + \pi)$ Or, $y_2 = -a\sin(\omega t + kx)$ $y = y_1 + y_2$ $y = a\begin{Bmatrix}\sin(\omega t - kx) \\ -\sin(\omega t + kx)\end{Bmatrix}$ after solving $y = -2a\sin kx\cos\omega t$ $y = -A\cos\omega t$ where $A = 2a\sin kx$ at $x = 0$ $A = 0$	 Incident wave $y_1 = a\sin(\omega t - kx)$ Reflected wave $y_2 = a\sin(\omega t + kx)$ From superposition of wave $y = y_1 + y_2$ $y = a\begin{Bmatrix}\sin(\omega t - kx) \\ +\sin(\omega x + kx)\end{Bmatrix}$ after solving $y = 2a\cos kx\sin\omega t$ $y = A\sin\omega t$ where $A = 2a\cos kx$ So $x = 0$ and $A = 2a$

Special Properties of Stationary Wave Pattern

- Zero wave velocity: No transfer of energy between two points, particle velocity is non zero but wave velocity is zero.
- Position of antinode-4 & nodes in this pattern remains fix.
- The particles between two consecutive nodes vibrate in same phase while medium particles nearby of any node on both sides always vibrate in opposite phase.
- All medium particles doing simple harmonic vibrations have identical time period but different vibration Amplitude and because of this their maximum velocity at mean position is different
- All medium particles pass through their mean position simultaneously but with different maximum velocity.
- All medium particles pass their mean position in their one complete vibration two times hence stationary wave pattern is obtained as straight line twice in its one complete cycle.
- In this pattern, at antinode position, displacement and velocity is maximum, but wave strain is minimum.

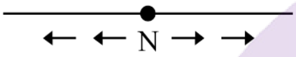
  Strain = slope of stationary wave pattern $\left(\frac{dy}{dx}\right)$

  At node position displacement and velocity is minimum but wave strain is maximum.
- Amplitude of incident wave > Amplitude of reflected wave

For node $a_1 - a_2 \Rightarrow$ minima

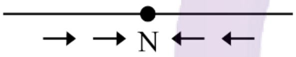
For antinode $a_1 + a_2 \Rightarrow$ maxima

- For any wave each and every reflecting surface have some absorptive power and due to this the energy, intensity & amplitude of reflected wave is always less compared to that of incident wave. Two waves difference in their amplitude having same frequency and wavelength and propagate in reverse or opposite direction always give stationary wave pattern by their superposition.
- According to nature of superposing waves stationary waves are of two types-
  Transverse stationary waves $\rightarrow$ Musical instruments based on wire (sonometer).
  Longitudinal stationary waves $\rightarrow$ Musical instruments based on air (resonance tube).
- Only applied to longitudinal stationary wave



Pressure↓ density↓
(compression)

If medium particle move in this way



Pressure↑ density↑
(Rarefaction)

At antinode $\rightarrow$ pressure & density constant so variations min.

At node $\rightarrow$ Pressure & density variations maximum.

- $E_{gas} = \dfrac{\text{change in pressure}}{\text{volumetric strain}} = \dfrac{dP}{\left(\dfrac{dy}{dx}\right)} = \text{const.}$

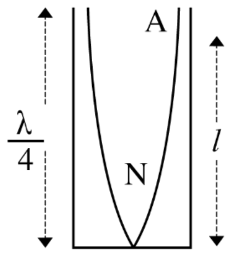
Then $dP \propto \text{strain}\left(\dfrac{dy}{dx}\right)$

So strain is maximum at node positions and minimum at antinode positions.

Types of Stationary Waves

- Longitudinal Stationary Waves [Organ pipe(ex : Resonance Tube)]
- Transverse Stationary Waves [Vibration in streched string (ex : Sonom

Closed Organ Pipe

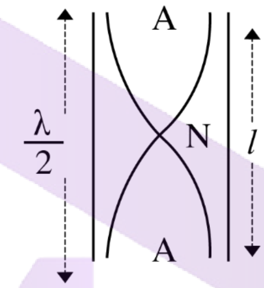


l = Length of air coloumn

$n = \dfrac{v}{4l}$ Fundamental Freqency

$n_1 : n_2 : n_3 : n_4 :: 1 : 3 : 5 : 7$

Open Organ Pipe



$n = \dfrac{v}{2l}$ Fundamental Freqency

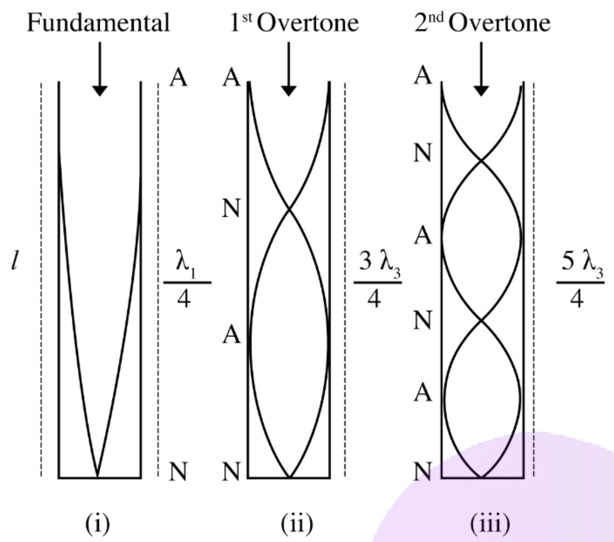
$n_1 : n_2 : n_3 : n_4 :: 1 : 2 : 3 : 4$

Vibration of Air columns:

When two longitudinal waves of same frequency and amplitude travel in a medium in opposite direction then by superposition, standing waves are produced. These waves are produced in air columns in cylindrical tube of uniform diameter. These sound producing tubes are called organ pipes.

Vibration of air column in Closed Organ Pipe:

The tube which is closed at one end and open at the other end is called closed organ pipe. On blowing air at the open end, a wave travels towards dosed end from where it is reflected towards open end. As the wave reaches open end, it is reflected again. So two longitudinal waves travel in opposite directions to superpose and produce stationary waves. At the closed end there is a node since particles does not have freedom to vibrate whereas at open end there is an antinode because particles have greatest freedom to vibrate.



Hence on blowing air at the end, the column vibrates formatting antinode at free end and node at closed end. If l is length of pipe and λ be the wavelength and v be the velocity of of sound in organ pipe then

Case (i) $l = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4l$

$\Rightarrow n_1 = \frac{v}{\lambda_1} = \frac{v}{4l}$ Fundamental frequency.

Case (ii) $l = \frac{3\lambda_2}{4} \Rightarrow \lambda_2 = \frac{4l}{3}$

$\Rightarrow n_2 = \frac{v}{\lambda_2} = \frac{3v}{4l} = 3n_1$ First overtone frequency.

Case (iii) $l = \frac{5\lambda_3}{4} \Rightarrow \lambda_3 = \frac{4l}{5}$

$\Rightarrow n_3 = \frac{v}{\lambda_3} = \frac{5v}{4l} = 5n_1$ Second overtone frequency.

When closed organ pipe vibrate in mth overtone then

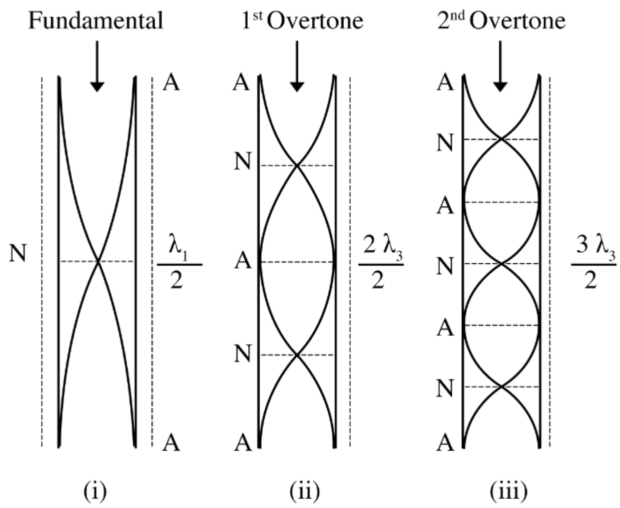
$$l = (2m+1)\frac{\lambda}{4} \text{ so } \lambda = \frac{4l}{(2m+1)} \Rightarrow n = (2m+1)\frac{v}{4l}$$

Hence frequency of overtone is given by

$n_1 : n_2 : n_3 ..... = 1 : 3 : 5 ....$

Vibration of air columns in open organ pipe:

The tube which is open at both ends is called an open organ pipe. On blowing air at the open end, a wave trawl towards the other end from which waves travel in opposite direction to superpose and produce stationary wave. Now the pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open-end antinodes are formed at each end and nodes in the middle. If f is length of the pipe and X be the wavelength and v is velocity of sound in organ pipe.



Case (i) $l = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2l$

$\Rightarrow n_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$ Fundamental frequency.

Case (ii) $l = \frac{2\lambda_2}{2} \Rightarrow \lambda_2 = \frac{2l}{2}$

$\Rightarrow n_2 = \frac{v}{\lambda_2} = \frac{2v}{2l} = 2n_1$ First overtone frequency.

Case (iii) $l = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2l}{3}$

$\Rightarrow n_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3n_1$ Second overtone frequency.

Hence frequency of overtones are given by the relation

$n_1 : n_2 : n_3 ..... = 1 : 2 : 3$

If an open pipe and a closed pipe have same length

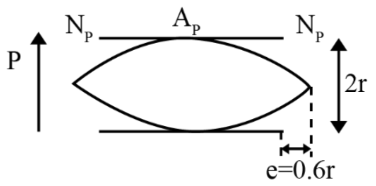
$n_{open} = n_{closed}$

When open organ pipe vibrate in nth overtone then

$$l = (m+1)\frac{\lambda}{2} \text{ so } \lambda = \frac{2l}{(m+1)} \Rightarrow n = (m+1)\frac{v}{2l}$$

8.2 End Correction

As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open end. The distance of the antinode from the open end is called end correction and its value is given by $e = 0.6r$



where r = radius of the organ pipe. with end correction, the fundamental frequency of a closed pipe (f_c) and an open organ pipe (f_0) will be given by

$$f_c = \frac{v}{4(L+0.6r)} \text{ and } f_0 = \frac{v}{2(L+1.2r)} \quad ......(5)$$

9. Experimental study of Standing Waves

9.1 Sonometer

- If a vibrating Tuning fork is pressed against a sonometer wire then forced vibrations are produced in table of hollow box & these vibrations are transferred to air coloum filled in hollow box which results into increase in vibration amplitude of sound & intensity of sound increases. Air filled hollow box is called sound box.
- During contact with table some energy is transferred to table so TF can not do vibrations for longer duration
- At resonance maximum energy is transferred to table so TF can do vibrations not for longer duration.
- At resonance maximum energy is transferred from TF to vibrating wire and sound intensity is maximum.

Laws of Transverse Vibrations of a String: Sonometer

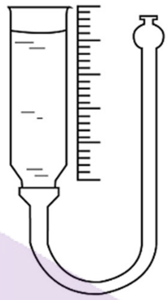
The fundamental frequency of vibration of a string fixed at both ends is given by equation. From this equation, one can immediately write the following statements known as "Laws of transverse vibrations of a string".

- **Law of Length** - The fundamental frequency of vibration of a string (fixed at both ends) is inversely proportional to the length of the string provided its tension and its mass per unit length remain the same.

  $v \propto \frac{1}{L}$ If F and μ are constants.
- **Law of Tension -** The fundamental frequency of a string is proportional to the square root of its tension provided its length and the mass per unit length remain the same.
- $v \propto \sqrt{F}$ if L and μ are constants.
- **Law of Mass** - The fundamental frequency of a string is inversely proportional to the square root of the linear mass density, i.e., mass per unit length, provided the length and the tension remain the same.
- $v \propto \frac{1}{\sqrt{\mu}}$ if L and F are constants.

9.2 Resonance Tube

Figure shows schematically the diagram of a simple apparatus used in laboratories to measure the speed of sound in air. Along cylindrical glass tube (say about 1 in) is fixed on a vertical wooden frame. It is also called a resonance tube. A rubber tube connects the lower end of this glass tube to a vessel which can slide vertically on the same wooden frame. A meter scale is fitted parallel to and close to the glass tube.



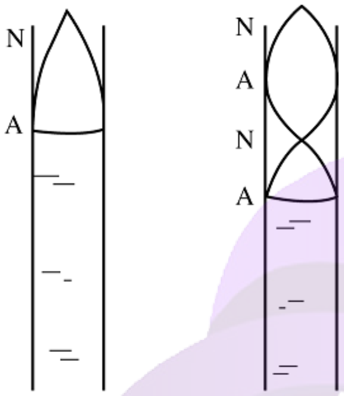
The vessel contains water which also goes in the resonance tube through the rubber tube. The level of water in the resonance tube is same as that in the vessel. Thus, by sliding the vessel up and down, one can change the water level in the resonance tube.

A tuning fork (frequency 256 Hz if the tube is 1 in long) is vibrated by hitting it on a rubber pad and is held near the open end of the tube in such a way that the prongs vibrate parallel to the length of the tube. Longitudinal waves are then sent in the tube.

The water level in the tube is initially kept high. The tuning fork is vibrated and kept dose to the open end, and the loudness of sound coming from the tube is estimated. The vessel is brought down a little to decrease the water level in the resonance tube. The tuning fork is again vibrated, kept close to the open end and the loudness of the sound coming from the tube is estimated. The process is repeated until the water level corresponding to the maximum loudness is located. Fine adjustments of water level are made to locate accurately the level corresponding to the maximum loudness. The length of the air column is read on the scale attached. In this case, the air column vibrates in resonance with the tuning fork. The minimum length of the air column for which the resonance takes place corresponds to the fundamental mode of vibration. A pressure antinode is formed at the water surface (which is the closed end of the air column) and a pressure node is formed near the open end. In fact, the node is formed slightly above the open end (end correction) because of the air-pressure from outside. Thus, for the first resonance the length l_1 of the air column in the resonance tube is given by

$$l_1 + e = \frac{\lambda}{4}, ...(i)$$

Where d is the end correction.



The length of the air column is increase to a little less than three times of l. The water level is adjusted so that the loudness of the sound coming from the tube becomes maximum again. The length of the air column is noted on the scale. In this second resonance the air column vibrates in the first overtone. There is one node and one antinode in between the ends of the column. The length l_2 of the column is given by

$$l_2 + e = \frac{3\lambda}{4}. ....(ii)$$

By (i) and (ii),

$$(l_2 - l_1) = \frac{\lambda}{2}, \text{ or } \lambda = 2(l_2 - l_1).$$

10. Beats

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are,

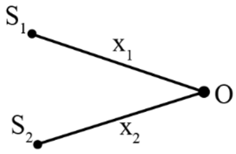
$$p_1 = p_0 \sin(2\pi f_1 t - k_1 x_1 + \theta_1)$$

$$p_2 = p_0 \sin(2\pi f_2 t - k_2 x_2 + \theta_2)$$

Let $-k_1 x_1 + \theta_1 = \phi_1$ and $-k_2 x_2 + \theta_2 = \phi_2$

By principle of superposition

$$= 2p_0 \sin\left(\pi(f_1 + f_2)t + \frac{\phi_1 + \phi_2}{2}\right)\cos\left(\pi(f_1 - f_2)t + \frac{\phi_1 - \phi_2}{2}\right)$$



i.e., the resultant sound at point O has frequency $\left(\frac{f_1 + f_2}{2}\right)$

while pressure amplitude $p_0'(t)$ varies with time as

$$p_0'(t) = 2p_0 \cos\left\{\pi(f_1 - f_2)t + \frac{\phi_1 - \phi_2}{2}\right\}$$

Hence pressure amplitude at point O varies with time with a frequency of $\left(\frac{f_1 - f_2}{2}\right)$

Hence sound intensity will vary with a frequency $f_1 - f_2$

This frequency is called beat frequency (f_B) and the time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$$f_B = |f_1 - f_2| \qquad T_B = \frac{1}{|f_1 - f_2|}$$

IMPORTANT POINTS:

1. The two superimposed sound wave producing beats must have a frequency in the audible range i.e., 20Hz to 20Khz.
2. Beat phenomenon can be used for determining an unknown frequency by sounding it together with a source of known frequency.
3. If the arm of a tuning fork is waxed or loaded, then its frequency decreases.
4. If arm of tuning fork is filed, then its frequency increases.
5. The two waves should have a very small difference in there frequencies. This difference should not be more than 10Hz
6. The amplitude of the waves should preferably b equal so that the maximum and the minimum intensity of sound produced in the beats can be heard distinctly and clearly.

11. Doppler's Effect

The apparent change in frequency or pitch due to relative motion of source and observer along the line of sight is called Doppler Effect. While deriving these expressions, we make the following assumptions:

1. The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.
2. The velocity of the source and the observer is less than velocity of sound.

Doppler effect takes place both in sound and light. In sound it depends on whether the source or observer or both are in motion while in light it depends on whether the distance between source and observer is increasing or decreasing.

Notations:

$n \rightarrow$ Actual frequency

$n' \rightarrow$ observed frequency (apparent frequency)

$\lambda \rightarrow$ actual wavelength

$\lambda' \rightarrow$ observed (apparent) wavelength

$v \rightarrow$ velocity of sound

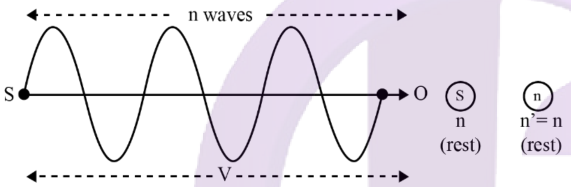
$v_s \rightarrow$ velocity of source

$v_0 \rightarrow$ velocity of observer

$v_w \rightarrow$ wind velocity

Case I:

Source in motion, Observer at rest, Medium at rest:



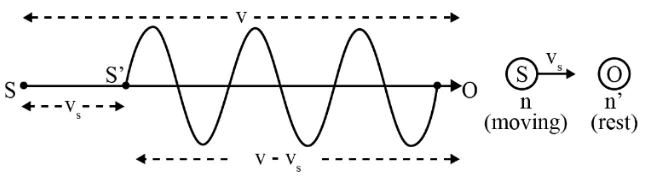
both source and observer at rest

Suppose the source S and observer O are separated by distance v. Where v is the velocity of sound. Let n be the frequency of sound emitted by the source. Then n waves will be emitted by the source in one second. These n waves will be accommodated in distance v.

So, wavelength $\lambda = \dfrac{\text{total distance}}{\text{total number of waves}} = \dfrac{v}{n}$

1. Source moving towards stationary observer:

Let the source start moving towards the observer with velocity v_s, after one second, the n waves will be crowded in distance $(v - v_s)$. Now the observer shall feel that he is listening to sound of wave length λ' and frequency n.



Now apparent wavelength

$$\lambda' = \frac{\text{total distance}}{\text{total number of waves}} = \frac{v - v_s}{n}$$

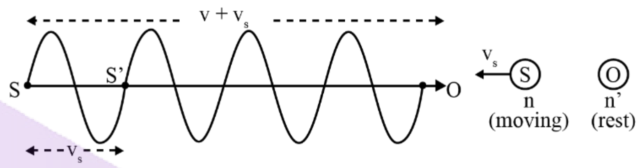
And changed frequency,

$$f' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v - v_s}{n}\right)} = n\left(\frac{v}{v - v_s}\right) = \frac{fv}{v - v_S}$$

So, as the source of sound approaches the observer the apparent frequency n' becomes greater than be true frequency n,

2. When source move away from stationary observer:

For this situation n waves will be crowded in distance $v + v_s$.



So, apparent wavelength $\lambda' = \dfrac{v + v_s}{n}$

And apparent frequency

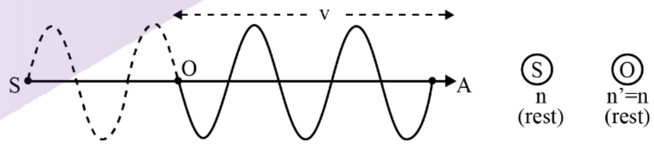
$$f' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v + v_s}{n}\right)} = n\left(\frac{v}{v + v_s}\right) = f\left(\frac{v}{v + v_S}\right)$$

So, n' becomes less thatn $n.(f' < f)$

Case II

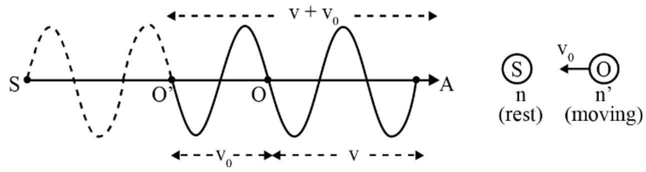
Observer in motion, source at rest, medium at rest:

Let the source (S) and observer (O) are in rest at their respective places. Then n waves given by source 'S' would be crossing observer ' O' in one second and fill the space $OA(= v)$



both source S and O at rest

1. Observer move towards Stationary Source:



When observer 'O' moves towards 'S' with velocity v_0, it will cover v_0 displace in one second. So the observer has received not only the f waves occupying OA but also received additional number of Δn waves occupying the distance $OO'(=v_o)$.

So, total waves received by observer in one second

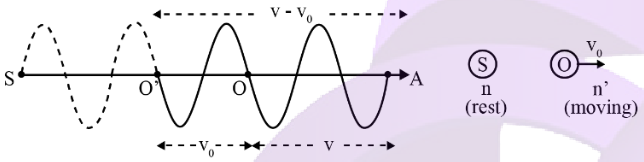
i.e., apparent frequency (n') = Actual waves (n) + Additional waves (Δn)

$$f' = \frac{v}{\lambda} + \frac{v_o}{\lambda} = \frac{v+v_o}{(v/f)} = f\left(\frac{v+v_o}{v}\right) \left(\because \lambda = \frac{v}{n}\right)$$

(so, $f' > f$)

2. Observer Move Away from Stationary Source: -

For this situation n waves will be crowded in distance $v - v_o$.



When observer move away from source with v_0 velocity then he will get Δn waves less than real number of waves. So, total number of waves received by observer i.e.,

Apparent frequency (f') = Actual waves (v) – reduction in number of waves (Δv)

$$v' = \frac{v}{\lambda} - \frac{v_o}{\lambda} = \frac{v-v_o}{\lambda} = \left(\frac{v-v_o}{v}\right)v \left(\because \lambda = \frac{v}{v'}\right)$$

(so $v' < v$)

Case III:

Effect of Motion of Medium:

General formula for a Doppler effect

$$= n' = n\left[\frac{v \pm v_0}{v \mp v_s}\right] \quad ......(i)$$

If medium (air) is also moving with v_m velocity in direction of source and observer. Then velocity of sound relative to observer will be $v \pm v_m$ (-ve sign, if v_m is opposite to sound velocity). So,

$$n' = n\left(\frac{v \pm v_m \pm v_o}{v \pm v_m \mp v_s}\right)$$ [on replacing v by $v \pm v_m$ is equal (i)]

NOTE:

When both 'S' and 'O' are in rest (i.e., $v_s = v_o = 0$) then there is no effect on frequency due to motion of air.

Case-I

If medium moves in a direction opposite to the direction of propagation of sound, then $v' = \left(\frac{v - v_m \pm v_o}{v - v_m \pm v_s}\right)v$

Case-II

Source in motion towards the observer. Both medium and observer are at rest.

$$v' = \left(\frac{v}{v - v_s}\right)v; \text{clearly } v' > v$$

So, when a source of sound approaches a stationary observer, the apparent frequency is more than the actual frequency.

Case-III:

Source in motion away from the observer. Both medium and observer are at rest.

$$v' = \left(\frac{v}{v + v_s}\right)v; \text{clearly } v' < v$$

So, when a source of sound moves away from a stationary observer, the apparent frequency is less than actual frequency.

Case-IV:

Observer in motion towards the source. Both medium and source are at rest.

$$v' = \left(\frac{v + v_o}{v}\right)v; \text{clearly } v' > v$$

So, when observer is in motion towards the source, the apparent frequency is more than the actual frequency.

Case-V:

Observer in motion away from the source. Both medium and source are at rest.

$$v' = \left(\frac{v - v_0}{v}\right)v; \text{clearly } v' < v$$

So, when observer is in motion away from the source, the apparent frequency is less than the actual frequency.

Case-VI:

Both source and observer are moving away from each other. Medium at rest.

$$v' = \left(\frac{v - v_0}{v + v_s}\right)v; \text{clearly } v' > v$$

NCERT Corner

1. A **wave** is a disturbance that propagates in space, transport energy and momentum from one point to another without the transport of matter.
2. Mechanical transverse waves are produced in such type of medium which have shearing property, so they are known as shear wave or S-wave
3. A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium, when a transverse wave passes.
4. A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium, when a transverse wave passes.
5. **Longitudinal Wave Motion:** Longitudinal wave have oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).
6. The regions of high particle density are called compressions and regions of low particle density are called rarefactions.
7. **Wavelength (λ) [length of one wave]:** Distance travelled by the wave during the time interval in which any one particle of the medium completes one cycle about its mean position. We may also define wavelength as the distance between any two nearest particle of the medium, vibrating in the same phase
9. **Phase:** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation $y = A\sin(\omega t - kx);(\omega t - kx) = \text{phase}$.
10. **Wave number $(\bar{v})$:** it is defined as $\bar{v} = \frac{1}{\lambda} = \frac{k}{2\pi} =$ number of waves in unit length of the wave pattern.
11. **Differential equation of Harmonic Progressive Waves:**

Differential equation of Harmonic Progressive Waves is given by:

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx)$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx)$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2}\frac{\partial y}{\partial t}$$

12. **Wave velocity:** The velocity with which the disturbance, or planes of equal (wave front), travel through the medium is called wave (or phase) velocity
13. **Transverse wave:** A transverse wave is a moving wave whose oscillations are perpendicular to the direction of the wave

The speed of a wave on a string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where T is tension in the string (in Newtons) and μ is mass per unit length of the string (kg/m).

14. When a travelling wave s established on a string, energy is transmitted along the direction of propagation of the wave, in form of potential energy and kinetic energy
15. **Intensity of Sound Waves:** The amount of energy carried per unit time by a wave is called its power and power per unit area held perpendicular to the direction of energy flow is called intensity.
16. **Loudness:** Audible intensity range for humans: The ability of human to perceive intensity at different frequency is different. The perception of intensity is maximum at 1000 Hz and perception of intensity decreases as the frequency decreases or increases from 1000Hz.
17. **Decibel Scale:** The logarithmic scale which is used for comparing two sound intensity is called decibel scale. The intensity level β described in terms of decibels is defined as $\beta = 10\log\left(\frac{I}{I_0}\right)(\text{dB})$
18. **Superposition of Waves**: The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore, according to superposition principle.
19. The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacement caused to the particle by the individual waves.

20. **Coherence:** Two sources are said to be coherent if the phase difference between them does not change with time. In this case their resultant intensity at any point in space remains constant with time. Two independent sources of sound are generally incoherent in nature, i.e. phase difference between them changes with time and hence the resultant intensity due to them at any point in space changes with time.

21. **Standing Waves:** Standing waves can be transverse or longitudinal, e.g., in strings (under tension) if reflected wave exists, the waves are transverse-stationary, while in organ pipes waves are longitudinal-stationary.

22. **Beats:** When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats. Beat phenomenon can be used for determining an unknown frequency by sounding it together with a source of known frequency.

23. **Doppler's Effect:** The apparent change in frequency or pitch due to relative motion of source and observer along the line of sight is called Doppler Effect.

    Assumptions: (i) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

    (ii) The velocity of the source and the observer is less than velocity of sound.

14

THERMAL PHYSICS

Thermal Physics

1. Temperature & Heat

- **Temperature :** Temperature is a relative measure of hotness or coldness of a body.
  SI Unit : Kelvin (K)
  Commonly Used Unit : °C or °F
- **Heat :** Heat is a form of energy flow
  (i) between two bodies or
  (ii) between a body and its surroundings
  by virtue of temperature difference between them
  SI Unit : Joule (J)
  Commonly Used Unit : Calorie (Cal)
  Conversion : 1 cal = 4.186 J

NOTE:

Heat always flows from a higher temperature system to a lower temperature system.

1.1 Zeroth Law of Thermodynamics

The zeroth law of thermodynamics states that if two thermodynamic systems are each in thermal equilibrium with a third one, then they are in thermal equilibrium with each other.

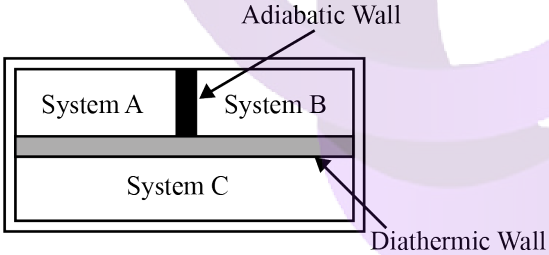


Fig. 14.1

- Zeroth law of thermodynamics takes into account that temperature is something worth measuring because it predicts whether the heat transfer between objects or not. This is true regardless of how the objects interact.
- Even if two objects are not in physical contact, heat still can flow between them, by means of radiation mode of heat transfer & zeroth law of thermodynamics states that, if the system are in thermal equilibrium, no heat flow will take place.

1.2 Temperature Scale

Measurement of Temperature

The measurement of temperature is done by some specified as given below:

Different Scales to Measure the Temperature			
Name of Scale	**Measuring Unit**	**Frezzing or ice point** (Lower fixed point)	**Boiling or steam point** (Upper fixed point)
Celsius Point	Degree Centigrade (°C)	0°C	100°C
Fahrenheit Point	Degree Fahrenheit (°F)	32°F	212°F
Reaumur scale	Degree Reaumur (°R)	0°R	80°R
Kelvin Scale	Kelvin (K)	273.15 K	373.15 K

Principle: Observation of Thermometric property with the change in temperature and comparing it with certain reference situations.
Reference situation is generally ice point or steam point.

1.2.1 Celsius and Fahrenheit Temperature Scales

In Celsius Scale	In Fahrenheit Scale
Ice Point $\rightarrow$ 0°C .	Ice Point $\rightarrow$ 32°F .
Steam Point $\rightarrow$ 100°C .	Steam Point $\rightarrow$ 212°F .

It implies that 100 division in Celsius scales is equivalent to 180 divisions in Fahrenheit scale.

Hence $\Rightarrow \frac{t_f - 32}{180} = \frac{t_c}{100}$

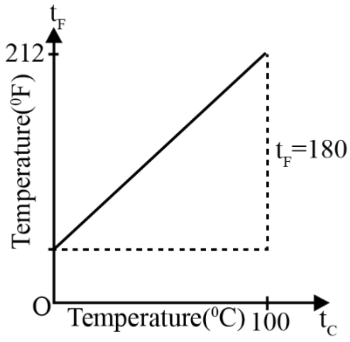


Fig. 14.2

1.2.2 Absolute Temperature Scale

It is kelvin scale

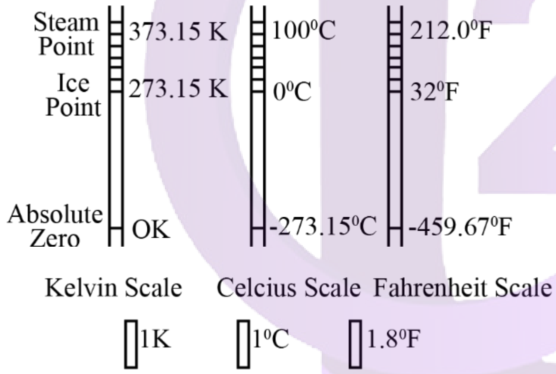
Ice point → 273.15 K

Steam point → 373.15 K

Comparing it with the Celsius scale, number of scale division in both the scales is same.

$$\frac{t_c - 0°C}{100} = \frac{t_k - 273.15}{100}$$

Kelvin scale is called as absolute scale, because it is practically impossible to go beyond 0 K in the negative side.



Comparison of Temperature Scales

Fig. 14.3

To convert the temperature to one scale to another, the following relation is used

$$\frac{\text{Temperature on one scale} - \text{LFP(ice point)}}{\text{UFP(Steam point)} - \text{LFP(ice point)}}$$

$$= \frac{\text{Temperature on other scale} - \text{LFP(ice point)}}{\text{UFP(Steam point)} - \text{LFP(ice point)}}$$

Relation between C, F and K temperature scales is given below:

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5}.$$

1.2.3 Thermometers

Instrument used to measure temperature of any system is called as **thermometer**.

Examples : Liquid in Glass thermometer, Platinum Resistance Thermometer, Constant Volume Gas Thermometers.

a) Liquid in Glass thermometer and Platinum Resistance thermometer give uniform readings for ice point & steam point but go non uniform for different liquids and different materials.

b) Constant volume gas thermometer gives same readings irrespective of which gas. It is based on the fact that at low pressures and constant volume, P × T for a gas is constant.

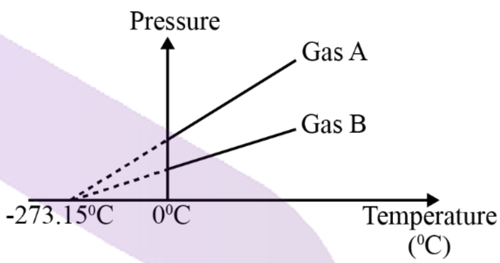


Fig. 14.4

c) All gases converge to absolute zero at zero pressure.

2. Thermal Expansion

When matter is heated without any change in its state, it usually expands. This phenomena of expansion of matter on heating is called thermal expansion of matter.

There are three types of thermal expansions.

2.1 Expansion of Solids

Three types of expansion takes place in solid as given below:

a) **Linear Expansion** The expansion in length of a body due to increase in its temperature is called the **linear expansion.**

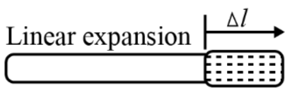


Fig. 14.5

Increase in length, $\ell_2 = \ell_1(1+\alpha\Delta t)$

where, ℓ_1 and ℓ_2 are initial and final lengths,

Δt = change in temperature and

α = coefficient of linear expansion.

Coefficient of linear expansion,

$$\alpha = \frac{\Delta \ell}{\ell \times \Delta t}$$

where, ℓ = real length

and $\Delta \ell$ = change in length

and Δt = change in temperature.

The coefficient of linear expansion of a material of a solid rod is defined as increase in length per unit original length per unit rise in temperature. Its unit is $°C^{-1}$ or K^{-1}.

b) Superficial Expansion: The expansion in the area of a surface due to increase in its temperature is called **area expansion.**

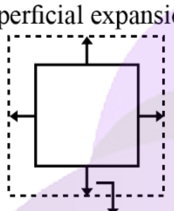


Fig. 14.6

Increase in area, $A_2 = A_1(1+\beta\Delta t)$

where, A_1 and A_2 are initial and final area and β is a coefficient of superficial expansion,

Coefficient of superficial expansion, $\beta = \frac{\Delta A}{A \times \Delta t}$

where, A = area,

ΔA = change in area

and Δt = change in temperature.

The coefficient of area expansion of metal sheet is defined as the increase in its surface area per unit original surface area per unit rise in temperature. Its unit is $°C^{-1}$ or K^{-1}.

c) Cubical Expansion: The expansion in the volume of an object due to increase in its temperature is known as cubical or **volume expansion.**

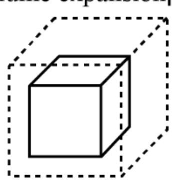


Fig. 14.7

Increase in volume, $V_2 = V_1(1+\gamma\Delta t)$

where V_1 and V_2 are initial and final volumes and γ is a coefficient of cubical expansion.

The coefficient of cubical expansion, $\gamma = \frac{\Delta V}{V \times \Delta t}$

where, V = real volume,

ΔV = change in volume and Δt = change in temperature.

The coefficient of volume (cubical) expansion of a substance is defined as the increase in volume per unit original volume per unit rise in its temperature. Its unit is $°C^{-1}$ or K^{-1}.

2.2 Expansion of Liquids

- Liquids do not have linear and superficial expansion but these only have volumetric expansion.
- Since, liquids are always heated in a vessel, so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid. Thus, liquids have two coefficients of volume expansion.

a) Apparent Expansion of Liquids: When expansion of the container containing liquid, on heating is not taken into account, the observed expansion is called apparent expansion of liquids.

Coefficient of apparent expansion of a liquid

$$(\gamma_a) = \frac{\text{apparent (or observed) incerease in volume}}{\text{original volume} \times \text{change in temperature}}.$$

b) Real Expansion of Liquids: When expansion of the container, containing liquid, on heating is also taken into account, then observed expansion is called real expansion of liquids.

Coefficient of real expansion of liquid

$$(\gamma_r) = \frac{\text{real incerease in volume}}{\text{original volume} \times \text{change in temperature}}$$

Both γ_r and γ_a are measured in $°C^{-1}$.

We can show that $\gamma_r = \gamma_a + \gamma_g$

where, γ_g is the coefficient of cubical expansion of the container (vessel).

2.2.1 Anomalous Expansion of Water

Generally, with increasing temperature, the volume expansion coefficient of liquids is about ten times greater than that of solids. Water is an exception to this rule. From $0°C$ to $4°C$ water contracts and beyond $4°C$, it expands. Thus, density of water reaches a maximum value of $1000 kgm^{-3}$ at $4°C$.

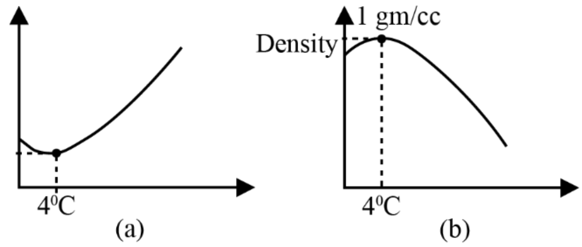


Fig. 14.8

2.3 Thermal Expansion of Gases

On heating, gases expand more than solids or liquids and equal volume of different gases expands equally, when heated by the same amount.

All gases have coefficient of volume expansion γ_v with volume variation given by $V = V_0(1+\gamma_v \Delta T)$ and pressure variation is given by $p = p_0(1+\gamma_p \Delta T)$.

2.3.1 Variation of Density with Temperature

Most substances expand when they are heated, i.e., volume of a given mass of a substance increases on heating, so density decreases.

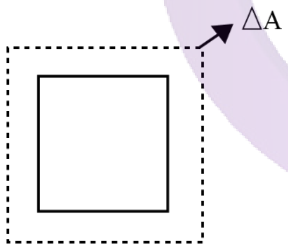
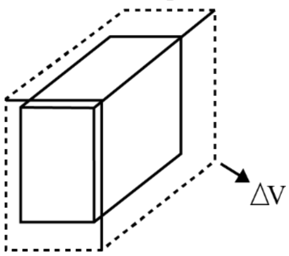
Hence $\rho \propto \frac{1}{V'}$

$\rho' = \rho(1+\gamma\Delta T)^{-1}$

As γ is small

$(1+\gamma\Delta T)^{-1} = 1-\gamma\Delta T$

$$\rho' = \rho(1-\gamma\Delta T)$$

Types of Expansion	Fractional Change	Coefficient of Expansion
Linear Expansion 	$\frac{\Delta L}{L} = \alpha\Delta T$	Coefficient of Linear Expansion (α): Increase in length per unit length per degree rise in temperature.
Area Expansion 	$\frac{\Delta A}{A} = \beta\Delta T$	Coefficient of Area Expansion (β): Increase in area per unit area per degree rise in temperature
Volume Expansion 	$\frac{\Delta V}{V} = \gamma\Delta T$	Coefficient of Volume Expansion (γ): Increase in area per unit volume per degree rise in temperature

NOTE:

- α for metals generally higher than α for non-metals.
- γ is nearly constant at high temperatures but for all low temperature it depends on temperature.

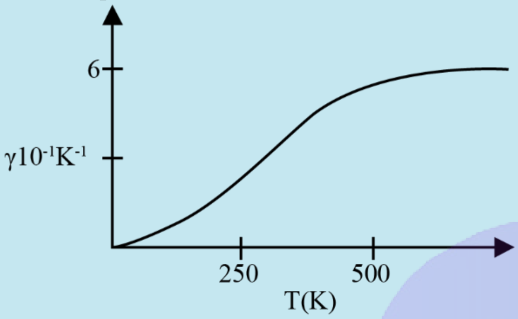


Fig. 14.9
Coefficient of volume expansion of Cu as a function of temperature.

- In general

$$\gamma = 3\alpha = \frac{3}{2}\beta$$

Proof: Imagine a cube of length ℓ that expands equally in all directions, when its temperature increases by small ΔT;

We have

$$\Delta\ell = \alpha\ell\Delta T$$

Also,

$$\Delta V = (\ell + \Delta\ell)^3 - \ell^3 = \ell^3 + 3\ell^2\Delta\ell + 3\ell\Delta\ell^2 + \Delta\ell^3 - \ell^3$$

In Equation (1) we ignore $3\ell\Delta\ell^2$ & $\Delta\ell^3$ as $\Delta\ell$ is very small as compared to ℓ.

So,

$$\Delta V = 3\ell^2\Delta\ell \quad \ldots(1)$$

$$\Delta V = \frac{3V}{\ell}\Delta\ell = 3V\alpha\Delta T \quad \ldots(2)$$

$$\therefore \frac{\Delta V}{V} = 3\alpha\Delta T$$

$$\Rightarrow \gamma = 3\alpha$$

Similarly, we can prove for areal expansion coefficient.

NOTE:

Relation between coefficients of linear, superficial and cubical expansion : $\beta = 2\alpha$ and $\gamma = 3\alpha$ or $\alpha : \beta : \gamma = 1 : 2 : 3$.

2.4 Application of Thermal Expansion

a) Thermometers: In thermometers, thermal expansion is used in temperature measurements.

b) Removing tight lids: To open the cap of a bottle that is tight enough, immerse in hot water for a minute. So, Metal cap expands and becomes loose. It would now be easy to turn it open.

c) Riveting: To join steel plates tightly together, red hot rivets are forced through holes in the plates. The ends of hot rivets is then hammered. On cooling, the rivets contract and bring the plates tightly gripped.

d) Fixing metal tires on wooden wheels: Iron rims are fixed on wooden wheels of carts. Iron rims are heated. The thermal expansion allows them to slip over the wooden wheel. Water is poured on it to cool. The rim contracts and becomes tight over the wheel.

e) Bimetallic Strip: A bimetal strip consists of two thin strips of different metals such as brass and iron joined together. On heating the strip, brass expands more than iron. This unequal expansion causes the bending of the strip.

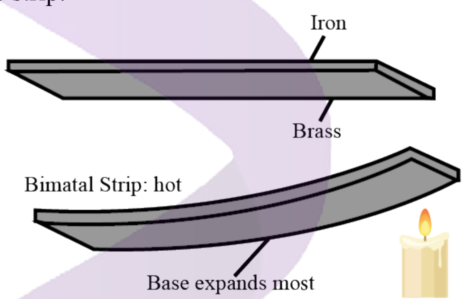


Fig. 14.10

Bimetallic strips are used for various purposes. Bimetal thermometers are used to measure temperature, especially in furnaces and ovens. Bimetal strips are used in thermostats. A bimetal thermostat is used to control the temperature of the heater coil in an electric iron

f) Thermostats: The thermostat is a heat-regulating device which works on the principle of thermal expansion.

2.5 Thermal Strain & Thermal Stress

When a metal rod whose ends are rigidly fixed so as to prevent the rod from expansion or contraction, undergoes a change in temperature, thermal strains and thermal stresses are developed in the rod.

- If a rod of length ℓ is heated by a temperature ΔT, then increase in length of rod should have been $\Delta\ell = \ell\alpha\Delta T$.

But due to being fixed at ends rod does not expand and a compressive thermal strain is developed in it whose value is

Thermal (compressive) strain $= \frac{\Delta \ell}{\ell} = \alpha \Delta T$

Here, $\alpha =$ linear expansion coefficient of the material of rod.

- Due to this strain, a thermal stress is developed in the rod.

We know,

$\frac{\Delta V}{V} = \alpha \Delta T =$ compressive strain

Also, $\frac{Y\Delta L}{L} = \sigma =$ Thermal stress

$\sigma_T = Y\alpha \Delta T$

where, Y = young's modulus of elasticity.

- Practical applications: Railway tracks, metal tyres of cart wheels, bridges and so many other applications.

3. Calorimetry

When two systems at different temperatures are connected together then heat flows from higher temperature to lower temperature till the time their temperatures do not become same.

Note:

Whenever heat is given to any body, either its temperature changes or its state changes.

Change in Temperature

When the temp changes on heating,
Then,
Heat supplied $\propto$ change in temperature (ΔT)

$\propto$ amount of substance (m/n)

$\propto$ nature of substance (s/C)

$\Rightarrow \Delta H = ms\Delta T$

m = mass of body

s = specific heat capicity per kg

ΔT = change in temperature

or $\Delta H = nC\Delta T$

n = Number of moles
C = Specific/Molar heat Capacity per mole

3.1 Mechanical Equivalent of Heat

The mechanical equivalent of heat implies that motion and heat are interchangeable, and that a defined amount of work generates the same amount of heat in all situations if the work is completely converted to heat energy.
The mathematical expression for the mechanical equivalent of heat is,

$$J = \frac{W}{q}$$

Where
W = The amount of heat required to generate heat
q = Amount of heat
The SI unit of mechanical equivalent of heat is **Joule/calorie.**

3.2 Thermal Heat Capacity

Amount of heat required to raise the temperature of a system through one degree

$$\Rightarrow \Delta Q = S\Delta T$$

where S = Heat Capacity

Units
SI : J / K
Common : Cal / °C

Note:

Materials with higher specific heat capacity require a lot of heat for same one degree rise in temperature.

3.2.1 Molar Heat Capacity

The amount of heat required to change the temperature of unit mole of substance by 1°C is termed as its molar heat capacity,

$$C = \frac{Q}{\mu \Delta T}$$

where, $\mu =$ number of moles = m/M

- Types of molar specific heat capacity are as follows

a) Molar specific heat capacity at constant pressure (C_p) is expressed as $C_P = \left[\frac{\Delta Q}{\Delta T}\right]_{p=constant}$

b) Molar specific heat capacity at constant volume (C_V)

$$C_V = \left(\frac{\Delta Q}{\Delta T}\right)_{V=constant}$$

- Relation between specific heat and molar heat capacity can be expressed as C = Ms
  where, C = molar heat capacity,
  M = molecular mass of the substance
  and s = specific heat capacity
  Thermal stress = $Y \times \text{Thermal strain} = Y\alpha\Delta T$
  Thermal stress $= Y\alpha\Delta T$
  Here, Y = Young's modulus of the material of given rod.
  If A be the cross-section area of the rod, then force exerted by the rod on the supports will be
  $F = Y\alpha\Delta T A$.

3.2.2 Specific Heat Capacity

- The quantity of heat Q required to change the temperature of a mass m of certain material by ΔT, is approximately proportional to the product of m and ΔT, i.e.,
  $Q \propto m\Delta T$
  or $Q = ms\Delta T$
  where, s = specific heat capacity of the material.
- Specific heat of ice is $500\,\text{cal/kg}°\text{C}$ and that of water is $1000\,\text{cal/kg}°\text{C}$.

Note:

Specific heat capacity can have any value from 0 to ∞. For some substances under particular situations, it can have negative values also.

3.2.3 Water Equivalent

Water equivalent is the amount of water that would absorb the same amount of heat as the calorimeter per degree temperature increase.

3.4 Latent Heat

The amount of heat transferred per unit mass during the change of state of a substance without any change in its temperature is called latent heat of the substance for particular change.

$Q \propto m$

$\Rightarrow Q = mL$

where, L = latent heat of the material.

- There are two types of latent heat of materials

a) Latent Heat of Fusion or Melting It is the quantity of heat required to change the state of a substance from solid to liquid state at its melting point. It is denoted by L_f.

Latent heat of fusion,

$$L_f = \frac{Q}{m}$$

Its SI unit is Jkg^{-1}.

b) Latent Heat of Vaporization It is the quantity of heat required to change the state of unit mass of a substance from liquid to vapour state at its boiling point. It is denoted by L_V.

Latent heat of vaporization, $L_V = \frac{Q}{m}$ and its SI unit is Jkg^{-1}.

3.5 Calorimeter

It is the branch of science which deals with the measurement of heat.

Principle of calorimetry states that, neglecting heat loss to surroundings, heat lost by a body at higher temperature is equal to heat gained by a body at lower temperature.

It is expressed as

Heat lost by hotter body = Heat gained by colder body

$$m_1 s_1 \Delta T_1 = m_2 s_2 \Delta T_2$$

where, m_1 = mass of hot body,
m_2 = mass of cold body,
s_1 = specific heat of hot body,
s_2 = specific heat of cold body
ΔT_1 = change in temperature of hotter body
and ΔT_2 = change in temperature of colder body.

3.6 Change of State

- The process of converting one state of a substance into another state is known as change of state of a substance or matter.
- Matter generally exists in three states
  (i) Solid (ii) Liquid (iii) Gas
- These states can be changed into one another by absorbing heat or rejecting heat. The process is so called the change of state. The temperature of a substance remains constant during change of state.

3.6.1 Triple Point of Water

The values of pressure and temperature at which water coexists in equilibrium in all three states of matter, i.e., ice, water and vapour is called triple point of water.

- Triple point of water is 273 K temperature and 0.46 cm of mercury pressure.
- **For example**
  Solid $\rightleftharpoons$ Liq L_f = Latent heat of fusion
  Liq $\rightleftharpoons$ Gas L_v = Latent heat of vaporization

NOTE:

In case any material is not at its B.P or M.P, then on heating the temperature will change till the time a particular state change temperature reaches.

For Example: If water is initially at –50°C at 1 Atm pressure in its solid state.

On heating

Step - 1 : Temperature changes to 0°C first

Step - 2 : Ice melts to $H_2O(\ell)$ keeping the temperature constant

Step - 3 : Temperature increase to 100°C

Step - 4 : $H_2O(\ell)$ boils to steam keeping the temp constant

Step - 5 : Further temperature increases.

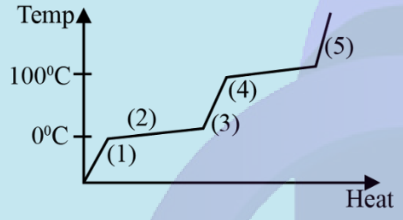


Fig. 14.11

The slope is inversely proportional to heat capacity.

Length of horizontal line depends upon mL for the process.

3.6.2 Pressure Dependence on Melting Point and Boiling Point

- For some substance melting point decreases with increase in pressure and for other melting point increases
- Melting point increases with increase in pressure.

We can observe the above results through phaser diagrams.

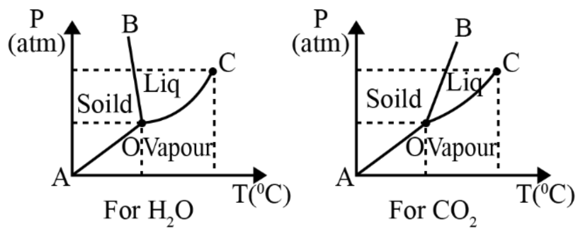


Fig. 14.12

Line AO → Sublimation curve

Line OB → Fusion curve

Line OC → Vaporization curve

Point O → Triple Point

Point C → Critical temperature

- **Triple Point**: The combination pressure and temperature at which all three states of matter (i.e. solids, liquids and gases co-exist.
  For H_2O it is at 273.16 K and 0.006 Atm.
- **Critical Point**: The combination of pressure & temp beyond which a vapour cannot be liquified is called as critical point.
  Corresponding temperature, pressure are called as critical temperature & critical pressure.
- From the phasor diagram, we can see that melting point decreases with increases in pressure for H_2O.
- **Regelation:** The phenomena of refreezing of water melted below the normal melting point due to increase in pressure.
- It is due to this pressure effect on melting point that cooking is tough on mountains and easier in pressure cooker.

4. Heat Transfer

There are three modes of heat transfer

- Conduction
- Convection
- Radiation

5. Conduction & Convection

5.1 Conduction

5.1.1 Thermal

Thermal conduction is the process in which thermal energy is transferred from the hotter part of a body to the colder one or from hot body to a cold body in contact with it without any transfer of material particles

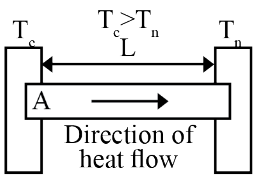


Fig. 14.13

At steady state, the rate of heat energy flowing through the rod becomes constant.

Then its rate (for uniform cross-sectional rod)

$$Q = KA\frac{(T_C - T_D)}{L} \quad ...(i)$$

where Q = Rate of heat energy flow (J/s or W)

A = Area of cross-section (m^2)

T_C, T_D = Temperature of hot end and cold end respectively (°C or K)
L = Length of the rod (m)
K = coefficient of thermal conductivity
Coefficient of Thermal Conductivity: It is defined as amount of heat conducted during steady state in unit time through unit area of any cross-section of the substance under unit temperature gradient, the heat flow being normal to the area.
Units
SI → J/msK or W/mK.

Note:

Larger the thermal conductivity, the greater will be rate of heat energy flow for a given temperature difference.
$K_{metals} > K_{non\ metals}$
Thermal conductivity of insulators is very low. Therefore, air does not let the heat energy to be conducted very easily

- **Temperature Gradient**: The fall in temperature per unit length in the direction of flow of heat energy is called as Temperature Gradient i.e., $\frac{T_C - T_D}{L}$

  Units
  SI → K/m
  The term Q, (i.e.) rate of flow of heat energy can also be named as heat current.
  The term (L/KA) is called as thermal resistance of any conducting rod.
- **Thermal Resistance:** Obstruction offered to the flow of heat current by the medium
  Units → K/W

5.1.2 Slabs in Parallel & Series

- **Slabs in Series (in steady state)**
  Consider a composite slab consisting of two materials different thickness L_1 and L_2 different cross-sectional area A_1 and A_2 and different thermal conductivities K_1 and K_2. The temperature at the outer surface of the slabs are maintained at T_H and T_C, and all lateral surfaces are covered by an adiabatic coating.

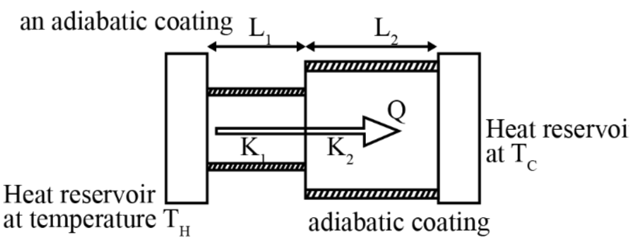


Fig. 14.14

Let temperature at the junction be T, since steady state has been achieved thermal current through each slab will be equal. Then thermal current through the first slab.

$$i = \frac{Q}{t} = \frac{T_H - T}{R_1} \text{ or } T_H - T = iR_1$$

and that through the second slab.

$$i = \frac{Q}{t} = \frac{T - T_C}{R_2} \text{ or } T - T_C = iR_2$$

adding eqn. 5.1 and eqn 5.2

$$T_H - T_C = (R_1 + R_2)i \quad \text{or } i = \frac{T_H - T_C}{R_1 + R_2}$$

Thus these two slabs are equivalent to a single slab of thermal resistance $R_1 + R_2$.
If more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$$R = R_1 + R_2 + R_3 + ...$$

- **Slabs in Parallel**

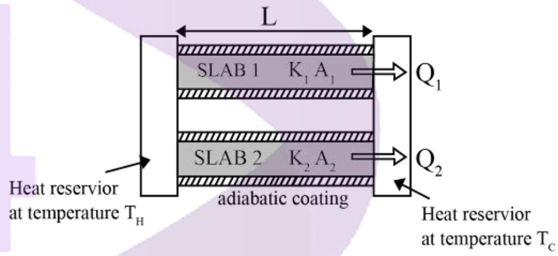


Fig. 14.15

Consider two slabs held between the same heat reservoirs, their thermal conductivities K_1 and K_2 and cross-sectional area A_1 and A_2

then $R_1 = \frac{L}{K_1 A_1}, R_2 = \frac{L}{K_2 A_2}$

thermal current through slab 1

$$i_1 = \frac{T_H - T_C}{R_1}$$

and that through slab 2

$$i_2 = \frac{T_H - T_C}{R_2}$$

Net heat current from the hot to cold reservoir

$$i = i_1 + i_2 = (T_H - T_C)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Comparing with $i = \frac{T_H - T_C}{R_{eq}}$,we get ,

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}$$

If more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}.....$$

5.1.3 Growth of Ice on Lake

- Water in a lake starts freezing if the atmospheric temperature drops below $0°C$. Let y be the thickness of ice layer in the lake at any instant and atmospheric temperature is $-\theta°C$.
- The temperature of water in contact with lower surface of ice will be zero.
- If A is the area of lake, heat escaping through ice in time dt is $dQ_1=\frac{KA\left[0-(-\theta)dt\right]}{y}$.
- Suppose the thickness of ice layer increase by dy in time dt, due to escaping of above heat. Then $dQ_2=mL-\rho(dyA)L$.

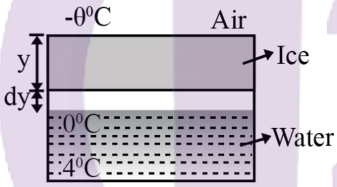


Fig. 14.16

- As $dQ_1=dQ_2$ hence, rate of growth of ice will be $(dy/dt)=(K\theta/\rho Ly)$. So, the time taken by ice to grow to a thickness y is $t=\frac{\rho L}{K\theta}\int_0^y ydy=\frac{\rho L}{2K\theta}y^2$.
- If the thickness is increased from y_1 to y_2 then time taken $t=\frac{\rho L}{2K\theta}\left(y_2^2-y_1^2\right)$.
- Do not apply negative sign for putting values of temperature in formula and also do not convert it to absolute scale.
- Ice is a poor conductor of heat, therefore the rate of increase of thickness of ice on ponds decreases with time.
- It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of $t_1:t_2:t_3::1^2:2^2:3^2$ i.e., $t_1:t_2:t_3::1:4:9$.

5.2 Convection

- The process in which heat is transferred from one point to another by the actual movement of the heated material particles from a place at higher temperature to another place of lower temperature is called as thermal convection.
- If the medium is forced to move with the help of a fan or a pump, it is called as forced convection.
- If the material moves because of the differences in density of the medium, the process is called natural or free convection.

Examples of forced convection:
Circulatory system, cooling system of an automobile heat connector

Examples of natural convection:
Trade winds, Sea Breeze/Land Breeze, Monsoons, Burning of Tea.

6. Radiation

Radiation is **energy that comes from a source and travels through space at the speed of light**. This energy has an electric field and a magnetic field associated with it, and has wave-like properties. You could also call radiation "electromagnetic waves".
Also, this mode of heat transfer, does not need medium to travel.

6.1 Theory of Exchange

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it.
If a body radiates more than what is absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings it radiates at the same rate as it absorbs.

6.2 Ideal Black Body & Black Body Radiation

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.

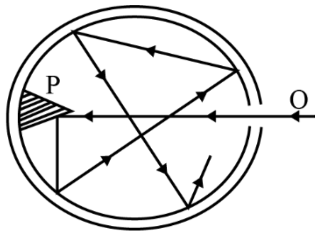


Fig. 14.17

In actual practice, no natural object possesses strictly the properties of a perfectly black body. But the lamp-black and platinum black are good approximation of black body. They absorb about 99% of the incident radiation. The most simple and commonly used black body was designated by Ferry. It consists of an enclosure with a small opening which is painted black from inside. The opening acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little change of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.

6.2.1 Absorption, Reflection & Emission of Radiations

$$Q = Q_r + Q_t + Q_a$$

$$\Rightarrow 1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$$\Rightarrow 1 = r + t + a$$

where r = reflecting power, a = absorptive power\

amd t = transmission power.

(i) r = 0, t = 0, a = 1, perfect black body

(ii) r = 1, t = 0, a = 0, perfect reflector

(iii) r = 0, t = 1, a = 0, perfect transmitter

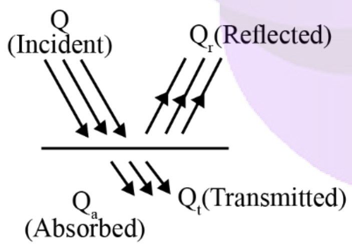


Fig. 14.18

- **Absorptive Power:** In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

  $$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

  As all the radiations incident on a black body are absorbed, a = 1 for a black body.

- **Emissive power:** Energy radiated per unit time per unit area along the normal to the area is knonw as emissive power.
- **Spectral Emissive power (E_λ):** Emissive power per unit wavelength range at wavelength λ is spectral emissive power, they are related as follows,

  $$E = \int_0^\infty E_\lambda d\lambda \text{ and } \frac{dE}{d\lambda} = E\lambda .$$

- **Emissivity:**

  $$e = \frac{\text{Emissive power of a body at temperature T}}{\text{Emissive power of a black abody at same temperature T}}$$

  $$= \frac{E}{E_0}$$

6.3 Kirchhoff's Law of Thermal Radiation

The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of perfectly black body

$$\frac{E(\text{body})}{a(\text{body})} = E(\text{black body})$$

Note:

Hence, we can conclude that good emitters are also good absorbers.

6.4 Stefan's Law

According to this law, the emissive power of a perfectly black body (energy emitted by a black body per unit surface area per unit time) is directly proportional to the fourth power of its absolute temperature.

Mathematically, $E \propto T^4$

or $E = \sigma T^4$

where, σ is a constant known as the Stefan's constant. Value of σ is $5.67 \times 10^{-8} Wm^{-2}K^{-4}$.

- The total radiant energy Q emitted by a body of surface area A in time t is given by

  $Q = Ate = At\varepsilon\sigma T^4$

- The radiant power (P), i.e. energy radiated by a body per unit time is given by

  $$P = \frac{Q}{t} = A\varepsilon\sigma T^4 .$$

- If a body at temperature T is surrounded by another body at temperature T_0 (where, $T_0 < T$), then Stefan's law is modified as

$$E = \sigma\left(T^4 - T_0^4\right)$$

and $e = \varepsilon\sigma\left(T^4 - T_0^4\right)$.

6.5 Newton's Law of Cooling

Newton's Law of cooling states that, the rate of loss of heat $\left(\frac{-dQ}{dt}\right)$ of the body is directly proportional to the difference of temp between body and surrounding.

Now, $\frac{-dQ}{dt} = k\left(T_2 - T_1\right)$

where k is a positive constant depending upon area and nature of the surface of the body.

Suppose a body of mass m, specific heat capacity s is at temperature T_2 & T_1 be the temperature of surroundings, if dT the fall of temperature in time dt.

Amount of heat lost is

dQ = msdT

∴ Rate of loss of heat is given by

$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

From above equations

$$-ms\frac{dT}{dt} = k\left(T_2 - T_1\right)$$

$$\Rightarrow \frac{dT}{T_2 - T_1} = \frac{-k}{ms}dt = -Kdt$$

where $K = \frac{k}{ms}$

On integrating

$$\log\left(T_2 - T_1\right) = -Kt + C$$

or $T_2 = T_1 + C_1e^{-Kt}$ where $C_1 = e^C$

Thus, above equation enables you to calculate the time of cooling of a body through a particular range of temperature.

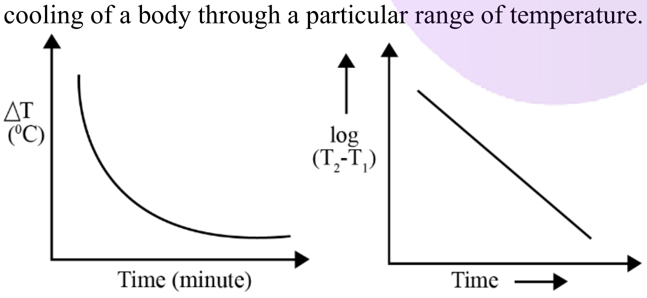


Fig. 14.19

NOTE:

> For small temperature differentiation, the rate of cooling, due to conduction, convection & radiation combined is proportional to difference in temperature

Approximation: If a body cools from T_a to T_b in t times in medium where surrounding temp is T_0, then

$$\frac{T_a - T_b}{t} = K\left(\frac{T_a + T_b}{2} - T_0\right).$$

Newton's Law of cooling can be verified experimentally

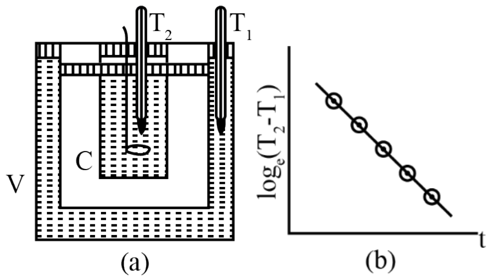


Fig. 14.20

Set Up: A double walled vessel (v) containng water in between two walls.

A copper calorimeter (C) containing hot water placed inside the double walled vessel. Two thermometers through the lids are used to note the temperature T_2 of H_2O in calorimeter and T_1 of water in between the double walls respectively.

Experiment: The temperature of hot water in the calorimeter after equal intervals of time.

Result: A graph is plotted between log $(T_2 - T_1)$ and time (t). The nature of the graph is observed to be a straight line as it should be from Newton's law of cooling.

6.6 Wien's Displacement Law

From the energy distribution curve of black body radiation, the following conclusions can be drawn:

a) The higher the temperature of a body, the higher is the area under the curve i.e., more amount of energy is emitted by the body at higher temperature.

b) The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.

c) For a given temperature, there is a particular wavelength $\left(\lambda_m\right)$ for which the energy emitted $\left(E_\lambda\right)$ is maximum.

d) With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.

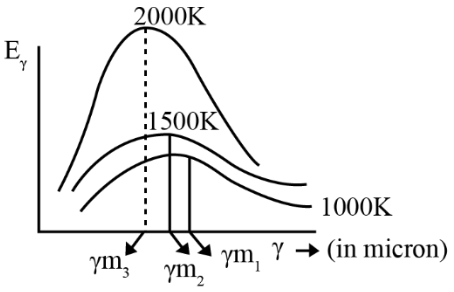


Fig. 14.21

From the study of energy distribution of black body radiation discussed as above, it was established experimentally that the wavelength (λ_m) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body. i.e.,

$$\lambda_m \propto \frac{1}{T} \text{ or } \lambda_m T = b$$

This is called Wien's displacement law.
Here b = 0.282 cm-K, is the Wien's constant.

6.7 Solar Constant

- A solar constant is a measurement of the solar electromagnetic radiation available in a meter squared at Earth's distance from the sun. The solar constant is used to quantify the rate at which energy is received upon a unit surface such as a solar panel. In this context, the solar constant provides a total measurement of the sun's radiant energy as it is absorbed at a given point.
- Solar constants are used in various atmospheric and geological sciences. Though called a constant, the solar constant is merely relatively constant. The relative constant does vary by 0.2% in a cycle that peaks once every eleven years. The constant is rated at a solar minimum of 1.361 kW/m^2 and a solar maximum of 1.362.

NCERT Corner

(Some Important Points to Remember)

1. Heat is a form of energy that flows between a body and its surrounding medium by virtue of temperature difference between them. The degree of hotness of the body is quantitatively represented by temperature.

2. A temperature-measuring device (thermometer) makes use of some measurable property (called thermometric property) that changes with temperature. Different thermometers lead to different scales. To construct a temperature scale, two fixed points are chosen and assigned some arbitrary values of temperature. The two numbers fix the origin of the scale and the size of its unit.

3. The Celsius temperature (t_C) and the Fahrenheit temperature (t_V) are related by $t_V = (9/5)t_C + 32$.

4. The ideal gas equation connecting pressure (P), volume (V) and absolute temperature (T) is: $PV = \mu RT$ where μ is the number of moles and R is the universal gas constant.

5. In the absolute temperature scale, the zero of the scale is the absolute zero of temperature the temperature where every substance in nature has the least possible molecular activity. The Kelvin absolute temperature scale (T) has the same unit size as the Celsius scale (T_C), but differs in the origin: $T_C = T - 273.15$.

6. The coefficient of linear expansion (α_t) and volume expansion (α_v) are defined by the relations:

$$\frac{\Delta \ell}{\ell} = \alpha_t \Delta T$$

$$\frac{\Delta V}{V} = \alpha_V \Delta T$$

where $\Delta \ell$ and ΔV denote the change in length ℓ and volume V for a change of temperature ΔT. The relation between them is : $\alpha_V = 3\alpha_t$.

7. The specific heat capacity of a substance is defined by

$$s = \frac{1}{m}\frac{\Delta Q}{\Delta T}.$$

where m is the mass of the substance and ΔQ is the heat required to change its temperature by ΔT. The molar specific heat capacity of a substance is defined by

$$C = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$$

where μ is the number of moles of the substance.

8. The latent heat of fusion (L_f) is the heat per unit mass required to change a substance from solid into liquid at the same temperature and pressure. The latent heat of vaporization (L_v) is the heat per unit mass required to change a substance from liquid to the vapour state without change in the temperature and pressure.

9. The three modes of heat transfer are conduction, convection and radiation.

10. In conduction, heat is transferred between neighbouring parts of a body through molecular collisions, without any flow of matter. For a bar of length L and uniform cross section A with its ends maintained at temperatures T_C and T_D, the rate of flow of heat H is:

$$H = KA\frac{T_C - T_D}{L}.$$

where K is the thermal conductivity of the material of the bar.

11. Newton's Law of Cooling says that the rate of cooling of body is proportional to the excess temperature of the body over the surroundings:
where T_1 is the temperature of the surrounding medium and T_2 is the temperature of the body.

12. Emissivity of a body at a given temperature is equal to the ratio of the total emissive power of the body (e_λ) to the total emissive power of perfectly black body (E_λ) at that temperature. Emissivity, $E = \frac{e_\lambda}{E_\lambda}$

13. Perfectly Black Body: A body which absorbs completely the radiations of all wavelengths incident on it, is called a perfectly black body. For a perfectly black body, emissive power $(E_\lambda) = 1$.

Lamp black is 96% black and platinum black is about 98% black.

14. Kirchhoff's Law of Radiation: This law states that, the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature. Mathematically, $\frac{e_1}{a_1} = \frac{e_2}{a_2} = ... = E$.

15. Stefan's Law: According to this law, the emissive power of perfectly black body (energy emitted by a black body per unit surface area per unit time) is directly proportional to the fourth power of its absolute temperature.

Mathematically, $E = \sigma T^4$

where σ is a constant known as the Stefan's constant.

Value of σ is $5.67 \times 10^{-8}\, Wm^{-2}K^{-4}$.

16. Wien's Displacement Law: This law states that as temperature of black body T increase, the wavelength λ_m corresponding to the maximum emission decreases such that $\lambda_m \propto \frac{1}{T}$ or $\lambda_m T = b$.

where, b is known as Wien's constant and its value is $2.89 \times 10^{-3}\, m - K$.

15

KINETIC THEORY OF GASES

Kinetic Theory of Gases and Thermodynamics

1. Kinetic Theory of Gases

1.1 Introduction to Kinetic Theory of Gases:

In this topic, we discuss the behaviour of gases and how are the various state variable like P, V, T, moles, U etc are interrelated with each other

1.1.1 Postulates of Kinetic Theory of Gases

- A gas consists of a very large number of molecules (of the order of Avogadro's number, 10^{23}), which are perfect elastic spheres. For a given gas they are identical in all respects, but for different gases, they are different.
- The molecules of a gas are in a state of incessant random motion. They move in all directions with different speeds., (of the order of 500 m/s) and obey Newton's laws of motion.
- The size of the gas molecules is very small as compared to the distance between them. If typical size of molecule is 2Å, average distance between the molecules is 320Å. Hence volume occupied by the molecules is negligible in comparison to the volume of the gas.
- The molecules do not exert any force of attraction or repulsion on each other, except during collision.
- The collisions of the molecules with themselves and with the walls of the vessel are perfectly elastic. As such, that momentum and the kinetic energy of the molecules are conserved during collisions, though their individual velocities change.
- There is no concentration of the molecules at any point inside the container i.e. molecular density is uniform throughout the gas.
- A molecule moves along a straight line between two successive collisions and the average straight distance covered between two successive collisions is called the **mean free path** of the molecules.
- The collisions are almost instantaneous, i.e., the time of collision of two molecules is negligible as compared to time interval between two successive collisions.

1.2 Pressure Calculation

1.2.1 Pressure of an Ideal Gas and Its Expression

Pressure exerted by the gas is due to continuous bombardment of gas molecules against the walls of the container.

Expression:

Consider a gas enclosed in a cube of side *l*. Take the axes to be parallel to the sides of the cube, as shown in figure. A molecule with velocity (v_x, v_y, v_z) hits the planar wall parallel to yz-plane of area A (= l^2). Since the collision is elastic, the molecule rebounds with the same velocity; its y and z components of velocity do not change in the collision but the x-component reverses sign. That is, the velocity after collision is $(-v_x, v_y, v_z)$. The change in momentum of the molecule is: $-mv_x - (mv_x) = -2mv_x$. By the principle of conservation of momentum, the magnitude of momentum imparted to the wall in the collision = $2mv_x$.

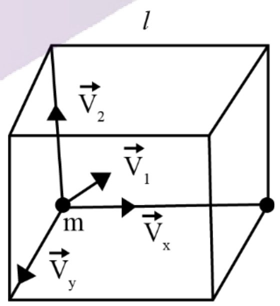


Fig 15.1

To calculate the force (and pressure) on the wall, we need to calculate momentum imparted to the wall per unit time, if it is within the distance v_x V_x Δt from the wall, that is, all molecules within the volume. $Av_x\Delta t$ only can hit the wall in time ΔT is $\frac{1}{2}A\ v_x \Delta t\ n$, where n is the member of molecules per unit volume. The total momentum transferred to the wall by these molecules in time Δt is: $Q=(2mv_x)\left(\frac{1}{2}nAv_x\Delta t\right)$. The force on the wall is the rate of momentum transfer $Q/\Delta t$ and pressure is force per unit area:

$$P=\frac{Q}{(A\ \Delta t)}=nmv_x^2$$

Actually, all molecules in a gas do not have the same velocity; there is a distribution in velocities. The above equation therefore, stands for pressure due to the group of molecules with speed v_x in the x-direction and n stands for the number density of that group of molecules. The total pressure is obtained by summing over the contribution due to all groups:

$$P=nm\vec{v}_x^2$$

where $\vec{v}_x^2$ is the average of v_x^2. Now the gas is isotropic, i.e. there is no preferred direction of velocity of the molecules in the vessel. Therefore by symmetry,

$$\vec{v}_x^2=\vec{v}_y^2=\vec{v}_z^2$$

$$\vec{v}_x^2=\left(\frac{1}{3}\right)\left(\vec{v}_x^2+\vec{v}_y^2+\vec{v}_z^2\right)=\left(\frac{1}{3}\right)\vec{v}^2$$

where v is the speed and $\vec{v}^2$ denotes the mean of the squared speed. Thus

$$P=\left(\frac{1}{3}\right)nm\vec{v}^2$$

$$P=\frac{1}{3}mn\vec{v}^2=\frac{1}{3}\frac{M}{V}=\frac{1}{3}\rho\vec{v}^2$$

M = Total mass of gas molecules

V = Total volume of gas molecules

1.2.2 Relation Between Pressure and KE of Gas Molecules

From equation $P=\frac{1}{3}\rho\vec{v}^2$

$$\Rightarrow P=\frac{2}{3V}\left(\frac{1}{2}Mv^{-2}\right)$$

$$\Rightarrow P=\frac{2}{3}\frac{K.E}{V}$$

$$\Rightarrow P=\frac{2}{3}E$$

Pressure exerted by an ideal gas is numerically equal to two third of mean kinetic energy

1.3 Ideal Gas Law's:

- **Ideal Gas**

That gas which strictly obeys the gas laws, (such as Boyle's Law, Charles', Gay Lussac's Law etc.)

Characteristics

1. The size of the molecule of an ideal gas is zero.

2. There is no force of attraction or repulsion amongst the molecules of an ideal gas.

- **Real Gas**

  All gases are referred to as real Gases. All real gas near the ideal gas behavior at low pressures and temperatures high enough, where they cannot be liquified.

Gay Lussac's Law:

We know that $PV=\frac{2}{3}N\vec{K}$

where $\vec{K}$ is the average kinetic energy of translation per gas molecule. At constant temperature. $\vec{K}$ is constant and for a given mass of the gas. N is constant.

Thus, PV = constant for given mass of gas at constant temperature, which is also called Boyle's Law.

Charle's Law:

We know that $PV=\frac{2}{3}N\vec{K}$

For a given mass of gas, N is constant.

Since $\vec{K}=\frac{3}{2}k_BT,\ \vec{K}\propto T$ and as such $PV\propto T$.

If P is constant, $V\propto T$, which is the Charles' Law.

Constant Volume Law:

We know that $PV=\frac{2}{3}N\vec{K}$

For a given mass of gas, N is constant. Since

$\vec{K}=\frac{3}{2}k_BT,\ \vec{K}\propto T$

Thus, $PV\propto T$

If V is constant, $P\propto T$, which the constant volume law.

Avogadro's Law:

Consider two gases 1 and 2. We can write

$$P_1V_1=\frac{2}{3}N_1\vec{K}_1,\ P_2V_2=\frac{2}{3}N_2\vec{K}_2$$

If their pressures, volumes and temperatures are the same, then

$P_1 = P_2,\ V_1 = V_2,\ \overline{K}_1 = \overline{K}_2.$

Clearly, $N_1 = N_2$ Thus:

Equal volumes of all ideal gases existing under the same conditions of temperature and pressure contain equal number of molecules which is Avogadro's Law or hypothesis**.**

This law is named after the Italian physicist and chemist, Amedeo Avogadro (1776 – 1856).

Alliter: As $PV = Nk_BT,\ N = \frac{PV}{k_BT}$

If P, V and T are constants, N is also constant.

1.4 Ideal Gas Equation

As $PV = \frac{2}{3}N\overline{K}$ and $\overline{K} = \frac{3}{2}k_BT$

$$PV = \frac{2}{3}N\left(\frac{3}{2}k_BT\right) \text{ or } PV = Nk_BT$$

which is the ideal gas equation

1.5 Real Gas Equation and Related Concepts

Real Gas Definition: A real gas is defined as a gas that at all standard pressure and temperature conditions does not obey gas laws. It deviates from its ideal behavior as the gas becomes huge and voluminous. True gases have velocity, mass, and volume. They liquefy when cooled to their boiling point. The space filled by gas is not small when compared to the total volume of gas.

Ideal and Real Gas Equation An ideal gas is defined as a gas that obeys gas laws at all pressure and temperature conditions. Ideal gases have velocity as well as mass. They have no volume. The volume taken up by the gas is small as compared to the overall volume of the gas. It does not condense, and triple-point does not exist.

The ideal gas law is the equation of the state of a hypothetical ideal gas, also called the general gas equation. Under many conditions, it is a reasonable approximation of the behavior of several gases, but it has many limitations. In 1834, Benoit Paul Emile Clapeyron first described it as a variation of the empirical law of Boyle, the law of Charles, the law of Avogadro, and the law of Gay-Lussac. In an empirical form, the ideal gas law is also written:

$$PV = nRT$$

Real Gas Law

By explicitly including the effects of molecular size and intermolecular forces, the Dutch physicist Johannes van der Waals modified the ideal gas law to explain the behavior of real gases. The Vander Waal real gas equation is given below.

Real gas law equation,

$$\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$

Where a and b represent the empirical constant which is unique for each gas.

$\frac{n^2}{V^2}$ represents the concentration of gas.

P represents pressure

R represents a universal gas constant and T is the temperature.

Difference Between Ideal and Real Gases

The table below shows the properties and the behaviour of ideal and real gases.

Ideal Gas	Real Gas
No definite volume	Definite volume
Elastic collision of particles	Non-elastic collisions between particles
No intermolecular attraction force	Intermolecular attraction force
Does not really exist in the environment and is a hypothetical gas	It really exists in the environment
High pressure	The pressure is less when compared to ideal gas
Independent	Interacts with others
Obeys $PV = NRT$	Obeys $\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$

2. Speeds of Gas Molecules

Maxwell's speed Distribution Law, average, RMS and most Probable Speeds.

Molecule Nature of Matter: Same as Atomic Theory given by Dalton, according to him, atoms are the smallest constituents of elements. All atoms of one element are identical, but atoms of different element are different.

In solids: Atoms are tightly packed, interatomic spacing about 1A°. Interatomic force of attraction are strong.

In liquids: Atoms are not as rigidly fixed as in solids. Interatomic spacing is about the same 2Å. Interatomic force a attraction are relative weaker.

In Gases: Atoms are very free. Inter atomic spacing is about tens of Angstroms. Interatomic forces are much weaker in gases than both in solids and liquids.

In this chapter, we mainly focus on gases

2.1 Maxwell's Law of Distribution of Molecular Velocities

Assumptions of Maxwell Distribution

- Molecules of all velocities between 0 to ∞ are present.
- Velocity of one molecule, continuously changes, though fraction of molecules in one range of velocities is constant.

Result

$$N_v = 4\pi N\left(\frac{M}{2\pi k_B T}\right)^{3/2} V^2 e^{-\frac{mv^2}{2k_BT}}$$

where $N_v = \frac{dN_v}{dV}$

where dN_v = Total number of molecules with speeds between V and V + dV

N = Total number of molecules.

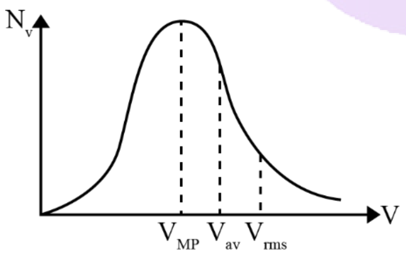


Fig 15.2

Based on this we define three types of speed for molecules of gas

$$V_{rms} = \left(\overline{\vec{V}^2}\right)^{1/2} = \left[\frac{1}{N}\int V^2 dNv\right]^{1/2}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Where M = Molecular Mass of Gas

Similarly, $V_{av} = \overline{V} = \frac{1}{N}\int V dN_v$

$$= \sqrt{\frac{8RT}{\pi M}}$$

But V_{MP} is velocity at which $\frac{dN_v}{dv} = 0$

$$\Rightarrow V_{MP} = \sqrt{\frac{3RT}{M}}$$

Physically V_{MP} is velocity possessed by Maximum number of molecules.

NOTE:

$V_{rms} > V_{av} > V_{MP}$

3. Energy of Gas Molecules

3.1 Degrees of Freedom

The number of degrees of freedom of a dynamical system is defined as the total number of co-ordinates or independent quantities required to describe completely the position and configuration of the system.

Example:

- A particle moving in straight line, say along X-axis need only x coordinate to define itself. It has only one degree of freedom.
- A particle in a plane, needs 2 co-ordinates, hence has 2 degrees of freedom.
  In general if
  A = number of particles in the system
  R = number of independent relations among the particles
  N = Number of degrees of freedom of the system
  N = 3A – R

Monoatomic Gases:

The molecules of a monoatomic gas (like neon, argon, helium etc) consists only of one atom.

∴ A = 1

R = 0

∴ N = 3

Here 3 degrees of freedom are for translational motion

Diatomic Gases

A = 2

Assuming the distance between the two molecules is fixed then R = 1

$\Rightarrow N = 3 \times 2 - 1 = 5$

Here 5 degrees of freedom implies combination of 3 translational energies and 2 rotational energies.

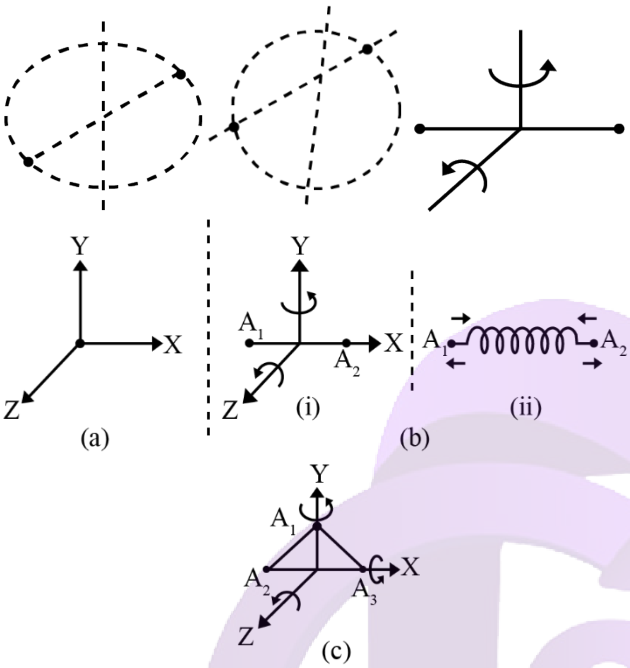


Fig 15.3

If vibrational motion is also considered then [only at very high temperatures]

N = 7

where 3 for translational

2 for rotational

2 for vibrational

Triatomic Gas

Fig 15.4

Linear

A = 3

R = 2

$\Rightarrow N = 3 \times 3 - 2 = 7$

Non-Linear

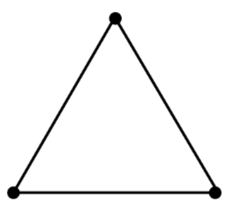


Fig 15.5

A = 3

R = 3 $\Rightarrow N = 3 \times 3 - 3 = 6$

- Here again vibrational energy is ignored.

Polyatomic Gas

A polyatomic gas has 3 translational, 3 rotational degrees of freedom. Apart from them if there V vibrational modes then there will be additional 2V vibrational degrees of freedom.

$\therefore$ Total degree of freedom

$n = 3 + 3 + 2V = 6 + 2V$

3.2 Internal Energy and Kinetic Energy

Internal Energy: As studied in thermodynamics, Internal Energy of any substance is the combination of Potential Energies and Kinetic Energies of all molecules inside a given gas.

- In real gas
  Internal Energy = P.E of molecules + K.E of Molecules
- In real gas: Internal Energy = K.E of Molecules
  Here PE of molecules is zero as assumed in Kinetic theory postulates; There is no interaction between the molecules hence its interactional energy is zero.

Average KE per Molecule of the Gas:

We know, $P = \frac{1}{3}\frac{M}{V}\vec{v}^2$

$\Rightarrow PV = \frac{1}{3}M\vec{v}^2$

Hence, $nRT = \frac{1}{3}M\vec{v}^2$

$\Rightarrow nRT = \frac{1}{3}Nm\vec{v}^2$

$\Rightarrow \frac{n}{N}\frac{3RT}{2} = \frac{1}{2}m\vec{v}^2$

Also $N = nN_A$

$\Rightarrow \frac{3}{2}\frac{R}{N_A}T = \frac{1}{2}m\vec{v}^2 \Rightarrow \frac{3}{2}K_BT = (K.E)_{avg}$

Average KE of translation per molecule of the gas $\frac{3}{2}K_BT$

Kinetic Interpretation of Temperature

From above equations, we can easily see that KE of one molecule is only dependent upon its temperature.

$\Rightarrow$ KE of molecule will cease if, the temperature of the gas molecules become absolute zero.

$\therefore$ Absolute zero of a temperature may be defined as that temperature at which the root mean square velocity of the gas molecule reduces to zero.

All the Ideal gas laws can be derived from Kinetic Theory of gases.

3.3 Law of Equipartition of Energy

Statement: According to this law, for any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom, and the energy associated with each molecule per degree of freedom is $\frac{1}{2}k_BT$, where k_B is Boltzman constant and T is temperature of the system.

Application: $U = f\frac{k_BT}{2}$ where f = Total degrees of freedom. This law is very helpful in determining the total internal energy of any system be it monatomic, diatomic or any polyatomic. Once the internal energy is know we can very easily predict C_v and C_p for such systems.

Remark: In case vibrational motion is also there in any system, say for diatomic molecule, then there should be energy due to vibrational as well given by

$$E_v = \frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}ky^2$$

where $\frac{dy}{dt}$ = vibrational velocity and $\frac{ky^2}{2}$ = Energy due to configuration.

According to Law of Equipartition

Energy per degree of freedom $= \frac{1}{2}k_BT$

$\Rightarrow$ Total energy $= \frac{1}{2}k_BT + \frac{1}{2}k_BT = k_BT$ is energy for complete one vibrational mode

3.4 Specific Heat of Gases

Specific Heat Capacity:

As we know the law of equipartition, we can predict the heat capacity of various gases.

Monoatomic Gas

Degree of freedom = 3.

$\therefore$ Average Energy of a molecule at temperature T

$$\Rightarrow E = 3\left(\frac{1}{2}k_BT\right)$$

Energy for one mole $\Rightarrow E \times N_A$

$$\Rightarrow U = \frac{3}{2}(k_BN_A)T$$

$$\Rightarrow U = \frac{3}{2}RT$$

In thermodynamics, we studied

$$C_V = \left.\frac{\Delta Q}{\Delta T}\right]_V = \frac{\Delta U}{\Delta T} \qquad [\because W = 0 \text{ for constant V}]$$

$$\Rightarrow C_v = \frac{3R}{2}$$

$$\therefore C_p = \frac{5R}{2} \text{ and } \gamma = \frac{C_p}{C_v} = \frac{5}{3}.$$

Diatomic Gases

When no vibration

Degree of freedom = 5

Average energy for one mole $= \frac{5}{2}RT$

$$\therefore C_v = \frac{\Delta U}{\Delta T} = \frac{5}{2}R$$

$$C_p = \frac{7R}{2}$$

$$\gamma = \frac{C_p}{C_v} = \frac{7}{5}$$

When vibration is present.

There is only one mode of vibration between 2 molecules.

$\therefore$ Degree of freedom = 7

$$\therefore U = \frac{7}{2}RT$$

$$\Rightarrow C_v = \frac{7}{2}R \text{ and } C_p = \frac{9}{2}R$$

and $\gamma = \frac{9}{7}$

Polyatomic Gases

Degree of freedom

= 3 for translational

+ 3 for rotational

+ 2V for vibrational

= 6 + 2V

If v = Number of vibrational modes

$$\therefore U = (6 + 2V)K\frac{RT}{2}$$

$$\Rightarrow C_v = (3 + V)R$$

$$C_p = (4 + V)R$$

and $\gamma = \frac{4+V}{3+V}$

Specific Heat Capacity of Water

Water is treated like solid.

Water has three atoms, 2 of hydrogen and one of oxygen

$\therefore$ Total degree of freedom for every atom

$= 3 \times 2 = 6$

$\therefore$ Total degree of freedom for every molecule of water

$= 3 \times 6 = 18$

$$\therefore\ C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = \frac{\left(18 \times \frac{1}{2} R\Delta T\right)}{\Delta T}$$

$C = 9R$

Specific Heat Capacity of Solids

- In solids, there is very less difference between heat capacity at constant pressure and at that constant volume. Therefore we do not differentiate between C_p and C_v for solids.

  $$\therefore\ C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T}$$

  {As solids hardly expand or expansion is negligible}

  Now in solid the atoms are arranged in an array structure and they are not free to move independently like in gases.

  Therefore the atoms do not possess any translational or rotational degree of freedom.

  On the other hand, the molecules do possess vibrational motion along 3 mutually perpendicular directions.

  Hence for 1 mole of a solid, there are N_A number of atoms. The energy associated with every molecule

  $$= 3\left[2 \times \frac{1}{2} k_B T\right] = 3K_B T$$

  $\therefore$ U = 3 RT for one mole

  $$\therefore\ C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 3R$$

- The above equation is called as Dulong and Petit's Law.
- At low temperatures the vibrational mode may not be that active hence, heat capacity is low at low temperatures for solids.

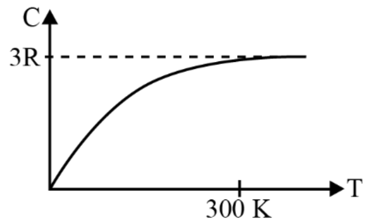


Fig 15.6

3.5 Mean Free Path

The path traversed by a molecule between two successive collisions with other molecule is called the mean free path

$$\vec{l} = \frac{\text{Total distance travelled by a molecule}}{\text{No. of collisions it makes with other molecules}}$$

Expression:

Mean Free Path

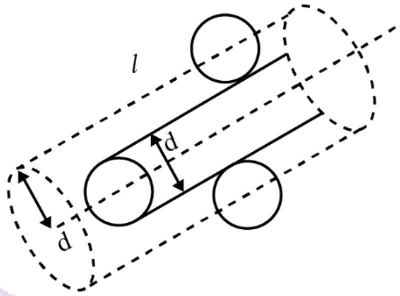


Fig 15.7

Suppose the molecules of a gas are spheres of diameter d. Focus on a single molecule with the average speed $\vec{V}$. It will suffer collision with any molecule that comes within a distance d between the centres. In time Δt, it sweeps a volume $\pi d^2\ \vec{V}\ \Delta t$ wherein any other molecule will collide with it (as shown in figure). If n is the number of molecules per unit volume, the molecule suffers $n\pi d^2\ \vec{V}\ \Delta t$ collisions in time Δt. thus the rate of collisions is $n\pi d^2\ \vec{V}$ or the time between two successive collisions is on the average.

$$\tau = \frac{1}{\left(n\pi\ \vec{V}\ d^2\right)}$$

The average distance between two successive collisions, called the mean free path l, is:

$$l = \vec{V}\ \tau = \frac{1}{\left(n\pi d^2\right)}$$

In this derivation, we imagined the other molecules to be at rest. But actually all molecules are moving and the collision rate is determined by the average relative velocity of the molecules. Thus we need to replace $\vec{V}$ by $\sqrt{2}\vec{V}$ in equation. A more exact treatment.

$$\vec{\ell} = \frac{1}{\left(\sqrt{2}n\pi d^2\right)}$$

Result

$$\vec{l} = \frac{1}{\left(\sqrt{2}n\pi d^2\right)}$$

for N molecules $PV = NK_BT$

$$\Rightarrow n = \frac{N}{V} = \frac{P}{K_BT}$$

$$\vec{\ell} = \frac{K_BT}{\sqrt{2}\pi d^2 P}$$

NOTE:

Mean free path depends inversely on the number density and size of the molecule.

4. Thermodynamics

4.1 Introduction to Thermodynamics

It is the study of interrelations between heat and other forms of energy

Thermodynamic System: A collection of large number of molecules of matter (solid, liquid or gas) which are so arranged that these possess certain values of pressure, volume and temperature forms a thermodynamic system.

- The parameters pressure, volume, temperature, internal energy etc which determine the state or condition of system are called thermodynamic state variables.
- In thermodynamics we deal with the thermodynamic systems as a whole and study the interaction of heat and energy during the change of one thermodynamic state to another.

Thermal Equilibrium

The term 'equilibrium' in thermodynamics implies the state when all the macroscopic variables characterising the system (P, V, T, mass etc) do not change with time.

- Two systems when in contact with each other come to thermal equilibrium when their temperatures become same.
- Based on this is zeroth law of thermodynamics. According to zeroth law, when the thermodynamic systems A and B are separately in thermal equilibrium with a third thermodynamic system C, then the systems A and B are in thermal equilibrium with each other also.

4.2 Basic Terms of Thermodynamics

State Variables: P, V, T, no. of moles and internal energy They can be extensive or intestive.

Equation of State: The equation which connects the pressure (P), the volume (V) and absolute temperature (T) of a gas is called the equation of state.

$PV = \text{constant}$ (Boyle's law)

$\frac{V}{T} = \text{constant}$ (Charle's law)

$\Rightarrow PV = nRT$

Thermodynamic Process: A thermodynamic process is said to take place when some changes occur in the state of a thermodynamic system, i.e., the thermodynamic parameters of the system change with time. Types of these thermodynamic process are Isothermal, Adiabatic, Isobaric and Isochoric.

Quasi Static Process: A thermodynamic process which is infinitely slow is called as quasi-static process.

- In quasi static process, system undergoes change so slowly, that at every instant, system is in equilibrium, both thermal and mechanical, with the surroundings.
- Quasi-static process is an idealised process. We generally assume all the processes to be quasistatic unless stated.

Indicator or P-V, Diagram: A graph between pressure and volume of a gas under thermodynamic operation is called P-V. diagram.

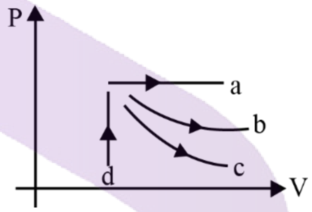


Fig 15.8

a → Isobaric

b → Isothermal

c → Adiabatic

d → Isochoric

Area under P – V diagram gives us work done by a gas.

4.3 Heat, Work Done and Internal Energy of Gas

Internal Energy is the energy possessed by any system due to its molecular K.E. and molecular P.E. Here K.E and PE are with respect to centre of mass frame. This internal energy depends entirely on state and hence it is a state variable. For a real gases internal energy is only by virtue of its molecular motion.

$U = \frac{nfRT}{2}$ for ideal gases where

n = number of moles

f = Degree of freedom

R = Universal Gas Constant

T = Temperature in Kelvin

Internal Energy can be change either by giving heat energy or by performing some work.

Heat Energy is the energy transformed to or from the system because of the difference in temperatures by conduction, convection or radiation.
The energy that is transferred from one system to another by force moving its point of application in its own direction is called work.

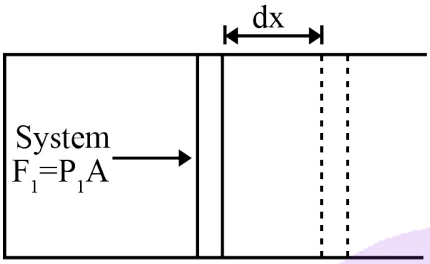


Fig 15.9

Work done by the system $= \int F\,dx$

$$= \int P_s\,Adx$$

$$= \int P_s\,dV$$

Where P_s is the Pressure of system on the piston. This work done by system is positive if the system expands and it is negative if the system contracts.

- Work and Heat are path functions whereas internal energy is a state function.
- Heat and work are two different terms through they might look same.

5. First Law of Thermodynamics

The first law of thermodynamics is a thermodynamics-adapted version of the law of conservation of energy. In principle, the conservation law asserts that an isolated system's total energy remains constant; energy can be transferred from one form to another, but it cannot be created or destroyed.

The first law states that the change in internal energy of the system (ΔU system) is equal to the difference between the heat provided to the system (Q) and the work (W) done by the system on its surroundings in a closed system (i.e., there is no transfer of matter into or out of the system).

$$\Delta U_{system} = Q - W$$

5.1 Relation of Heat and Internal Energy

Let ΔQ = Heat supplied to the system by the surroundings

ΔW = Work done by the system on the surroundings

ΔU = Change in internal energy of the system.

First law of thermodynamics states that energy can neither be created nor be destroyed. It can be only transformed from one form to another.

Mathematically: $\Delta Q = \Delta U + \Delta W$

Sign Conventions:

- When heat is supplied to the system, then ΔQ is positive and when heat is withdrawn from the system, ΔQ is negative.
- When a gas expands, work done by the gas is positive and when a gas contracts then work is negative
- ΔU is positive, when temperature rises and ΔU is negative, when temperature falls.
  Remember here we always take work done by the system.

5.2 Mayer's Formula

$$C_P = \left(\frac{dQ}{dT}\right)_P$$

Or, $dQ = C_P dT$

From equation

$$dQ = C_P dT = dU + PdV$$

Again, from equation (2) substituting

$$dU = C_V dT$$

$$C_P dT = C_V dT + PdV \ldots(4)$$

For one mole of gas $(\mu = 1)$, from ideal gas equation,

$$PV = RdT$$

$$PdV = RdT$$

From equations

$$(C_P - C_V)\,dT = RdT$$

Or, $C_P - C_V = R$

Where C_P is the Specific heat at constant pressure and C_V is the specific heat at constant volume.

6. Cyclic and Non-Cyclic Processes

6.1 Introduction to Cyclic and Non-Cyclic Processes

Cyclic Process: A cyclic process is one in which the system returns to its initial stage after undergoing a series of changes.

Indicator Diagram

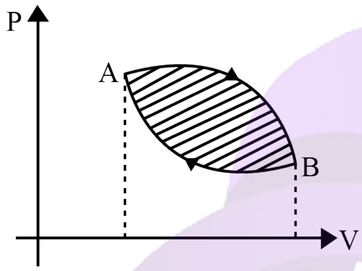


Fig 15.10

$\Delta U = 0$

W = Area enclosed by the loop.

Q = W as per First Law of thermodynamics

Here W is positive if the cycle is clockwise and it is negative if the cyclic is anti clockwise.

Non-Cyclic Process: In Non-cyclic process the series of changes involved do not return the system back to its initial state.

i.e., change in internal energy for cyclic process is zero and also $\Delta U \propto \Delta T \Rightarrow \Delta T = 0$

i.e., temperature of system remains constant. i.e, heat supplied is equal to the work done by the system.

6.2 Work done and Heat exchange in Cyclic Processes

In cyclic process, initial and final points are same.

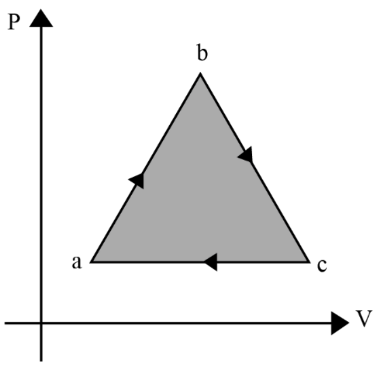


Fig 15.11

Therefore, $(p_i V_i, T_i) = (p_f, V_f, T_f)$

Internal energy is a state function which only depends on temperature (in case of an ideal gas).

$T_i = T_f$

$\Rightarrow U_i = U_f$

Or, $\Delta U_{net} = 0$

If there are three process in a cyclic abc, then

$\Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0$

From first law of thermodynamics,

$Q = W + \Delta U, \text{if } \Delta U_{net} = 0, \text{then}$

$Q_{net} = W_{net}$

Or, $Q_{ab} + Q_{bc} + Q_{ca} = W_{ab} + W_{bc} + W_{ca}$

Further, W_{net} = area under P-V diagram. For example,

W_{net} = + area of triangle 'abc' in the shown diagram. Cycle is clockwise. So, work done will be positive.

6.3 Reversible and Irreversible Processes

Reversible Process: A reversible process is the process where it never occurs; on the contrary the irreversible process is the one which can be said to be the natural process and cannot be reversed.

Thermodynamics is the example of the reversible process. Here the system and the surroundings return to the same stage at the end of the process.

NOTE:

A Reversible process takes two processes into account while in the first process participants convert into another form, tin the case of this second process the reverse reaction takes space where the resultants get back to the initial state.

Types of reversible processes: There are two types of reversible processes. The internally reversible process and the external reversible process. Internal reversible process involves no irreversibility within the system boundaries. This states that the system undergoes the stage of equilibrium but when it returns it again passes through the same stage.

- In the externally reversible process there are no irreversibility's

Irreversible Process: an irreversible process is a naturally occurring phenomenon, which does not go back to its original state.

Factors behind Irreversibility of process:

An irreversible process can be said to be the thermodynamics process that departs equilibrium. When we talk in terms of pressure, we can say that it occurs when the pressure of the system changes and the volume does not have time to reach equilibrium.

The system and the surrounding does not come back to the original state even after the completion of the process in the spontaneous process.

Hence, The Reversible Nature of a Process is Dependent on Multiple Factors Such as non-elasticity, friction, viscosity, electrical resistance etc.

7. Thermodynamic Processes

7.1 Isothermal Process

Description: A thermodynamic process in which temperature remains constant

Condition: The walls of the container must be perfectly conducting to allow free exchange of heat between gas and its surroundings.

The process of compression or expansion should be slow so as to provide time for exchange of heat.

These both conditions are perfectly ideal.

Equation of State: T = Constant or PV = Constant

Indicator Diagram:

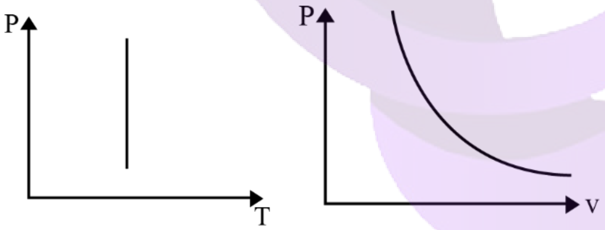


Fig 15.12

Slope of P – V curve is $\frac{dP}{dV}$ at any point.

$PV = nRT$

$\Rightarrow (dP)V + P(dV) = 0$

$\Rightarrow \frac{dP}{dV} = -\frac{P}{V}$

$\Delta U = 0$ (Temperature remains constant)

$W = \int_{v_2}^{v_2} P_g dV$

$= \int_{v_2}^{v_2} \frac{nRT}{V} dV$ [Using PV = nRT]

$= nRT \ln \frac{V_2}{V_1}$

Since $P_1V_1 = P_2V_2$

Therefore, $W = nRT \ln\left(\frac{P_1}{P_2}\right)$

First Law of Thermodynamics

$Q = \Delta U + W$

$\Rightarrow Q = nRT \ln \frac{V_2}{V_1}$

NOTE:

All the heat supplied is used entirely to do work against external surroundings. If heat is supplied then the gas expands and if heat is withdrawn then the gas contracts.

Practical Examples:

Melting of ice at 0°C

Boiling of water at 100°C

7.2 Adiabatic Process

Description: When there is no heat exchange with surroundings.

Conditions: The walls of the container must be perfectly non-conducting in order to prevent any exchange of heat between the gas and its surroundings.

The process of compression or expansion should be rapid, and so, there is no time for the exchange of heat.

These conditions are again ideal condition and are hard to obtain

Equation of State:

$PV^{\gamma} = \text{constant}$

or $TV^{\gamma-1} = \text{constant}$

or $PT^{\frac{\gamma}{1-\gamma}} = \text{constant}$

Indicator Diagram

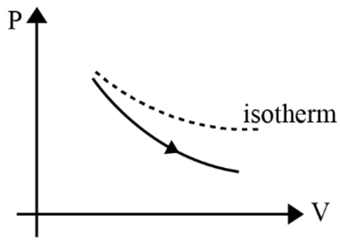


Fig 15.13

Slope of adiabatic curve $=\frac{dP}{dV}$

$PV^{\gamma}=\text{const}$

$\Rightarrow P\gamma V^{\gamma-1}(dv)+(dP)V\gamma=0$

$\Rightarrow \frac{dP}{dV}=\frac{-\gamma P}{V}$

As shown in graph adiabatic curve is steeper than isothermal curve.

$\Delta U=\frac{nfRdT}{2}=\frac{nR(T_2-T_1)}{\gamma-1}=\frac{P_2V_2-P_1V_1}{\gamma-1}$

Work Done by Gas: If a gas adiabatically expands from V_1 to V_2

$W=\int_{v_2}^{v_2} P(dv)$

$=\text{cons tan t}\int_{v_2}^{v_2}\frac{dV}{V^{\gamma}}$

$\left[\begin{array}{l}\because\ PV^{\gamma}=\text{cons tan t}\\ \Rightarrow P=\frac{\text{cons tan t}}{V^{\gamma}}\end{array}\right]$

$=\text{cons tan t}\times\left[\frac{V^{-\gamma+1}}{1-\gamma}\right]_{V_1}^{V_2}=\frac{\text{cons tan t}}{1-\gamma}\left[\frac{1}{V_2^{\gamma-1}}-\frac{1}{V_1^{\gamma-1}}\right]$

Also we know

$P_1V_1^{\gamma}=P_2V_2^{\gamma}=\text{cons tan t}$

$\Rightarrow \frac{1}{1-\gamma}\left[\frac{P_2V_2^{\gamma}}{V_2^{\gamma-1}}-\frac{P_1V_1^{\gamma}}{V_1^{\gamma-1}}\right]$

$W=\frac{P_2V_2-P_1V_1}{\gamma-1}=\frac{nR(T_1-T_2)}{\gamma-1}$

First Law of Thermodynamics

$Q=\Delta U+W$

Substituting the values

We get $Q=0$

NOTE:

> If gas expands adiabatically then its temperature decreases and vice versa.

Practical Example

- Propagation of sound waves in the form of compression and rarefaction.
- Sudden bursting of a cycle tube.

7.3 Isochoric Process

Description: Volume remains constant

Condition: A gas being heated or cooled inside a rigid container.

Equation of State: V = constant or $\frac{P}{T}$ = constant

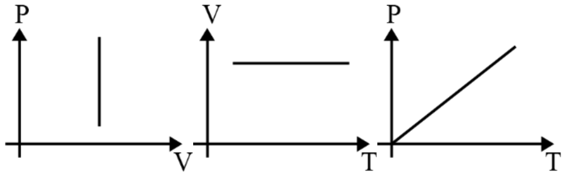


Fig 15.14

$\Delta U=\frac{nfR\Delta T}{2}$

Work

W = 0 as gas does not expands

First Law of thermodynamics

$Q=\Delta U+W$

$\Rightarrow Q=\frac{nfR\Delta T}{2}$

NOTE:

> Since we have studied earlier, that when heat is supplied to any process. Its temperature increases according to relation.

$Q=nC\Delta T$

$\Rightarrow C=\frac{Q}{n\Delta T}$ ....(1)

Now this C depends upon external conditions for gases.

Here it is referred as $\left.\frac{\Delta Q}{n\Delta T}\right]_v$ (2)

i.e. Molar heat capacity at constant volume Comparing equation 1 and 2

We get $C_v=\frac{fR}{2}$ ...(3)

7.4 Isobaric Process

Description: When pressure remains constant

Condition: When in one container, the piston is free to move and is not connected by any agent.

Equation of State: P = constant

$\frac{V}{T}=\text{cons tan t}$

Indicator Diagram:

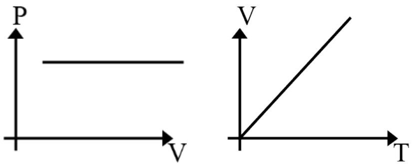


Fig 15.15

$\Delta U = \frac{nfR\Delta T}{2}$ same as always

$W = \int PdV = P\Delta V$ (as pressure is constant)

$= PV_2 - PV_1 = nR\Delta T$

First Law of Thermodynamics

$Q = \Delta U + W$

$\Rightarrow Q = \frac{nfR\Delta T}{2} + nR\Delta T$

$\Rightarrow Q = n\left[\frac{fR}{2} + R\right]\Delta T \quad ...(4)$

Similar to C_v, we can define molar heat capacity at constant pressure

$\Rightarrow C_p = \left.\frac{Q}{n\Delta T}\right]_p \quad ...(5)$

From equation 4 and 5

We get $C_p = \frac{fR}{2} + R \quad ...(6)$

From equation 3 and 6

Replacing $\frac{fR}{2}$ by C_v we get

$V_p = C_v + R$

which is also called Mayer's Relation.

Similar to molar specific heat at constant pressure and molar specific heat at constant volume, we can define molar specific heat for any process.

For example:

$C_{adiabatic} = 0$

$C_{isothermal} = \infty$

Basically gas does not possess a unique specific heat. Mainly we have C_p and C_v.

- **Specific Heat at Constant Volume:** It is defined as the amount of heat required to raise the temperature of 1g of a gas through 1C°, when its volume is kept constant. It is denoted as C_V.
- **Specific Heat at Constant Pressure:** It is defined as the amount of heat required to raise the temperature of 1g of a gas through 1°C keeping its pressure constant. It is denoted as C_P.

NOTE:

c_P, c_v means Molar heat Capacity and C_P, C_V means specific heat capacity

$C_V = Mc_V$ and $C_P = Mc_P$ where M stands for molar mass of any sample.

$c_p - c_v = \frac{R}{M}$

7.5 Melting Process

In any case first law is always applicable

$Q = mL_f$ as learned earlier.

$W = 0$

(In the change of state from solid to liquid we ignore any expansion or contraction as it is very small)

According to first law of thermodynamics

$\Delta U = Q - W$

$\Delta U = mL_f$

NOTE:

The heat given during melting is used in increasing the internal energy of any substance

7.6 Boiling Process

Here, $Q = mL_v$

$W = P[V_2 - V_1]$

(Pressure is constant during boiling and it is equal to atmosphere pressure)

$\Rightarrow \Delta U = Q - W$

$\Delta U = mL_v - P(V_2 - V_1)$

7.7 Polytropic Process

A polytropic process is a thermodynamic process that obeys the relation:

$pv^n = C$

Where p is the pressure, V is volume, n is the polytropic index, and C is a constant. The polytropic process equation can describe multiple expansion and compression processes which include heat transfer.

Particular Cases:

Some specific values of n correspond to particular cases:

$n = 0$ for an isobaric process,

$n = +\infty$ for an isochoric process

In addition, when the ideal gas law applies:

$n = 1$ for an isothermal process,

$n = \gamma$ for an isentropic process.

Where γ is the ratio of the heat capacity at constant pressure (C_P) to heat capacity at constant volume (C_v).

7.8 Free-Expansion

A process in which gas is allowed to expand in vacuum and this happens so quickly that no heat leaves or enters the system this type of process is also known as adiabatic process and because this happens so fast the gas does not cross the system boundaries, hence no work is done by the system or on the system, then the expansion is called the free expansion.

We can for the equate free expansion into

$U_f - U_i = Q - W$

Now, as know heat is exchange and no work is done

$Q = 0$ and $W = 0$ i.e., $U_f = U_i$

Further, as we see there is no change in the internal energy, Hence, the temperature remains constant.

7.9 Limitation of First Law of Thermodynamics

- The first law does not indicate the direction in which the change can occur.
- The first law gives no idea about the extent of change.
- The first law of thermodynamics gives no information about the source of heat. i.e. whether it is a hot or a cold body.

8. Heat Engine

It is a device that converts heat energy into mechanical energy.

Key Elements:

- A source of heat at higher temperature
- A working substance
- A sink of heat at lower temperature.

Working:

- The working substance goes through a cycle consisting of several processes.
- In some processes it absorbs a total amount of heat Q_1 from the source at temperature T_1.
- In some processes it rejects a total amount of heat Q_2 to the sink at some lower temperature T_2.
- The work done by the system in a cycle is transferred to the environment via some arrangement.

Schematic Diagram

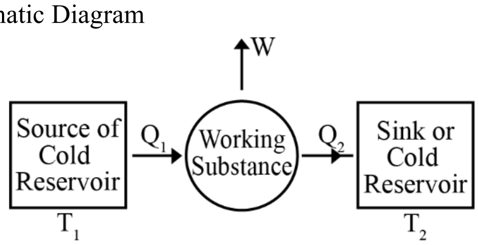


Fig 15.16

First Law of Thermodynamics

∵ Energy is always conserved

$\Rightarrow Q_1 = W + Q_2$

8.1 Thermal Efficiency

Thermal Efficiency of a heat engine is defined of the ratio of net work done per cycle by the engine to the total amount of heat absorbed per cycle by the working substance from the source.

It is denoted by $\eta = \dfrac{W}{Q_1}$ ...(1)

Using equation 1 and 2 we get

$\eta = 1 - \dfrac{Q_2}{Q_1}$ ...(2)

Ideally engines should have efficiency = 1

NOTE:

The mechanism of conversion of heat into work vanes for different heat engines.

The system heated by an external furnace, as in a steam engine. Such engines are called as external combustion engine.

The system in which heat is produced by burning the fuel inside the main body of the engine is called as Internal Combustion Engine.

9. Carnot Cycle

Sadi Carnot devised on ideal cycle of operation for a heat engine called as carnot cycle.

Engine used for realising this ideal cycle is called as carnot heat engine.

The essential parts of an Ideal heat engine are shown in figure.

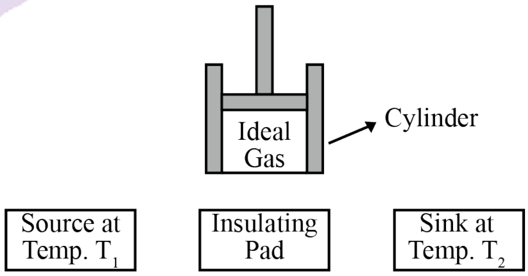


Fig 15.17

- **Source of heat:** The source is maintained at a fixed higher temperature T_1, from which the working substance draws heat. The source is supposed to possess infinite thermal capacity and as such any amount of heat can be drawn from it without changing its temperature.

- **Sink of heat:** The sink is maintained at a fixed lower temperature T_2, to which any amount of heat can be rejected by the working substance. It has also infinite thermal capacity and as such its temperature remains constant at T_2, even when any amount of heat is rejected to it.
- **Working substance:** A perfect gas acts as the working substance. It is contained in a cylinder with non-conducting sides but having a perfectly conducting-base. This cylinder is fitted with perfectly non-conducting and frictionless piston.
- Apart from these essential parts, there is a perfectly **insulating stand** or **pad** on which the cylinder can be placed. It would isolate the working substance completely from the surroundings. Hence, the gas can undergo adiabatic changes.
  The Carnot cycle consists of the following four stages:
  - Isothermal expansion
  - Adiabatic expansion
  - Isothermal compression
  - Adiabatic compression

The cycle is carried out with the help of the Carnot engine as detailed below:

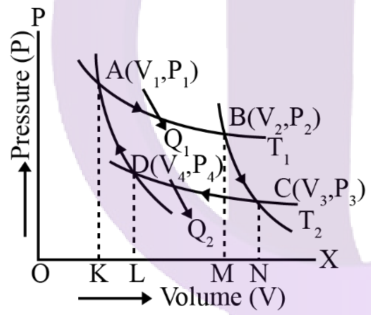


Fig 15.18

Consider one gram mole of an ideal gas enclosed in the cylinder. Let V_1, P_1, T_1 be the initial volume, pressure and temperature of the gas. The initial state of the gas is represented by the point A on P–V. diagram, We shall assume that all the four processes are quasi-static and dissipative, the two conditions for their reversibility.

Steps

- **Isothermal Expansion:** The cylinder is placed on the source and gas is allowed to expand by slow outward motion of piston. Since base is perfectly conducting therefore the process is isothermal.
  Now
  $\Delta U_1 = 0$
  $$q_1 = W_1 = RT_1 \ln.\frac{V_2}{V_1} = \text{Area ABMKA}$$
  $q_1 \to$ Heat absorbed by gas
  $w_1 \to$ Work done by gas
- **Adiabatic Expansion:** The cylinder is now removed from source and is placed on the perfectly insulating pad. The gas is allowed to expand further from B (P_2, V_2) to C (P_3, V_3). Since the gas is thermally insulated from all sides, therefore the processes is adiabatic
  $$q_2 = 0 \quad \Delta U_2 = \frac{R(T_2 - T_1)}{\gamma - 1}$$
  $$W_2 = \frac{R(T_1 - T_2)}{\gamma - 1} = \text{Area BCNMB}$$
- **Isothermal Compression:** The cylinder is now removed from the insulating pad and is placed on the sink at a temperature T_2. The piston is moved slowly so that the gas is compressed until is pressure is P_4 and volume is V_4.
  $\Delta U_3 = 0$
  $$W_3 = -RT_2 \ln\frac{V_4}{V_3} = -\text{Area CDLNC}$$
  $$q_3 = -RT_2 \ln\frac{V_4}{V_3}$$
  q_3 = Heat absorbed in this process
  W_3 = Work done by gas
- **Adiabatic Compression:** The cylinder is again placed on the insulating pad, such that the process remains adiabatic. Here the gas is further compressed to its initial P_1 and V_1.
  $$\Delta U_4 = \frac{R(T_1 - T_2)}{\gamma - 1}$$
  $$W_4 = \frac{-R(T_1 - T_2)}{\gamma - 1} = -\text{area DAKLD}$$
  $q_4 = 0$
  W_4 = work done by the gas

9.1 Analysis of Carnot Cycle

Total work done by the engine per cycle.

$= W_1 + W_2 + W_3 + W_4$

$= W_1 + W_3$

$$W = RT_1 \ln\frac{V_2}{V_1} - RT_2 \ln\frac{V_4}{V_3}$$

Q_1 = Total heat absorbed = q_1

$$= RT_1 \ln\frac{V_2}{V_1}$$

Q_2 = Total heat released = $-q_3$

[q_3 = Heat absorbed and not heat released]

$$= RT_2 \ln\frac{V_3}{V_4}$$

We can see that for heat engine

$W = Q_1 - Q_2$

= Area under ABCDA

9.2 Efficiency of Carnot Engine

$$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Now steps 2 is adiabatic and step 4 is also adiabatic

$$\Rightarrow T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

and $T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4} \quad ...(21)$$

From equation 19, 20 and 21 we get

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\therefore \; n_{Carnot} = 1 - \frac{T_2}{T_1}$$

- $\eta_{Carnot\ engine}$ – depends only upon source temperature and sink temperature.
- $\eta_{Carnot\ engine} = 1$ only when $T_2 = 0$ K or $T_1 = \infty$ which is impossible to attain.
- If $T_2 = T_1 \Rightarrow \eta = 0 \Rightarrow$ Heat cannot be converted to mechanical energy unless there is some difference between the temperature of source and sink.

9.3 Carnot Theorem

Statement: Carnot theorem states that all reversible engines working between same two temperatures have same efficiency irrespective of the nature of working substance. The source and the sink works between the same temperature.

- Working between two given temperatures, T_1 of hot reservoir (the source) and T_2 of cold reservoir (the sink), no engine can have efficiency more than that of the Carnot engine.
- The efficiency of the Carnot engine is independent of the nature of the working substance.

  Engine used for realizing this ideal cycle is called as Carnot heat engine.

Proof:

Step - 1: Imagine a reversible engine R and an irreversible engine-I working between the same source (hot reservoir T_1) and sink (cold reservoir T_2).

Step - 2: Couple two engines such that I acts like heat engine and R acts like refrigerator.

Step - 3: Let engine I absorb Q_1 heat from the source deliver work W^1 and release the balance $Q_1 - W^1$ to the sink in one cycle.

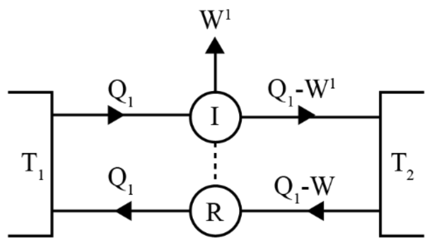


Fig 15.19

Step - 4: Arrange R, such that it returns same heat Q, to the source, taking Q_2 from the sink and requiring work $W = Q_1 - Q_2$ to be done on it.

Step - 5: Suppose $\eta_R < \eta_I$ (i.e.) If R were to act as an engine it would give less work output than that of I (i.e.) $W < W^1$ for a given Q_1 and $Q_1 - W > Q_1 - W^1$

Step - 6: In totality, the I-R system extracts heat $(Q_1 - W) - (Q_1 - W^1) = W^1 - W$ and delivers same amount of work in one cycle, without any change in source or anywhere else. This is against second Law of Thermodynamics. (Kelvin - Planck statement of second law of thermodynamics)

Hence the assertion $\eta_I > \eta_R$ is wrong.

Similar argument can be put up for the second statement of Carnot theorem, (i.e) Carnot efficiency is independent of working substance.

$\therefore$ We use ideal gas for calculating but the relation.

$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ will always hold true for any working substance used in a Carnot engine.

9.4 Second Law of Thermodynamics

The second law of thermodynamics states that the heat energy cannot transfer from a body at a lower temperature to a body at a higher temperature without the addition of energy.

There are number of ways in which this law can be stated. Though all the statements are the same in their contents, the following two are significant.

Kelvin Planck Statement: No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.

Clausius Statement: No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.

Significance: 100% efficiency in heat engines or infinite CoP in refrigerators is not possible.

10. Refrigeration

A refrigerator or heat pump is a device used for cooling things.

Key Elements:

- A cold reservoir at temperature T_2.
- A working substance.
- A hot reservoir at temperature T_1.

Working

- The working substance goes through a cycle consisting of several process.
- A sudden expansion of the gas from high to low pressure which cools it and converts it into a vapour-liquid mixture.
- Absorption by the cold fluid of heat from the region to be cooled, converting it into vapour.
- Heating up of the vapour due to external work done on the working substance.
- Release of heat by the vapour to the surroundings bringing it to the initial state and completing the cycle.

Schematic Diagram.

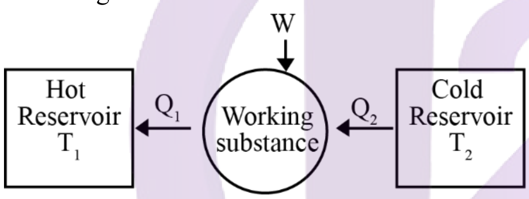


Fig 15.20

First Law of Thermodynamics

$$Q_2 + W = Q_1 \quad ...(1)$$

10.1 Coefficient of Performance

Coefficient of Performance of refrigerator (β) is defined as the ratio of quantity of heat removed per cycle from contents of the refrigerator (Q_2) to the energy spent per cycle (W) to remove this heat.

$$\beta = \frac{Q_2}{W} \quad ...(2)$$

Using equation 1 and 2 we get

$$\beta = \frac{Q_2}{Q_1 - Q_2}$$

Ideally heat pumps should have $\beta = \infty$

NCERT Corner

(Some Important Points to remember)

- The molecules of a gas are in a state of incessant random motion. They move in all directions with different speeds., (of the order of 500 m/s) and obey Newton's laws of motion.
- The molecules do not exert any force of attraction or repulsion on each other, except during collision.
- A molecule moves along a straight line between two successive collisions and the average straight distance covered between two successive collisions is called the mean free path of the molecules.
- Pressure exerted by an ideal gas is numerically equal to two third of mean kinetic energy
- **Ideal Gas:** That gas which strictly obeys the gas laws, (such as Boyle's Law, Charles', Gay Lussac's Law etc.)
- **Real Gas**: All gases are referred to as real Gases. All real gas near the ideal gas behavior at low pressures and temperatures high enough, where they cannot be liquified
- **Gay Lussac's Law/Boyle's Law:** PV = constant for given mass of gas at constant temperature, which is also called Boyle's Law.
- **Charle's Law:** $PV \propto T$, If P is constant, $V \propto T$, which is the Charles' Law
- **Constant Volume Law:** $PV \propto T$, If V is constant, $P \propto T$, which the constant volume law.
- **Avogadro's Law:** Equal volumes of all ideal gases existing under the same conditions of temperature and pressure contain equal number of molecules which is Avogadro's Law or hypothesis**.**
- **Ideal Gas Equation:** $PV = Nk_BT$ which is the ideal gas equation
- **Speed of Gas Molecules:**
- **In solids:** Atoms are tightly packed, interatomic spacing about 1A°. Interatomic force of attraction are strong.
- **In liquids:** Atoms are not as rigidly fixed as in solids. Interatomic spacing is about the same 2Å. Interatomic force a attraction are relative weaker.
- **In Gases:** Atoms are very free. Inter atomic spacing is about tens of Angstroms. Interatomic forces are much weaker in gases than both in solids and liquids.
- **Degrees of Freedom:** The number of degrees of freedom of a dynamical system is defined as the total number of co-ordinates or independent quantities required to describe completely the position and configuration of the system.
- **Absolute zero**: Absolute zero of a temperature may be defined as that temperature at which the root mean square velocity of the gas molecule reduces to zero.
- **Law of Equipartition of Energy:** According to this law, for any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom, and the energy associated with each molecule per degree of freedom is $\frac{1}{2}k_BT$, where k_B is Boltzman constant, and T is temperature of the system.
- **Mean Free Path:** The path traversed by a molecule between two successive collisions with other molecule is called the mean free path

$$\vec{l} = \frac{\text{Total distance travelled by a molecule}}{\text{No. of collisions it makes with other molecules}}$$

- Mean free path depends inversely on the number density and size of the molecule.
- **Thermodynamic System:** A collection of large number of molecules of matter (solid, liquid or gas) which are so arranged that these possess certain values of pressure, volume and temperature forms a thermodynamic system.
- The parameters pressure, volume, temperature, internal energy etc which determine the state or condition of system are called thermodynamic state variables.
- In thermodynamics we deal with the thermodynamic systems as a whole and study the interaction of heat and energy during the change of one thermodynamic state to another.
- **Thermal Equilibrium:** Two systems when in contact with each other come to thermal equilibrium when their temperatures become same.
- **Zeroth Law of Thermodynamics:** According to zeroth law, when the thermodynamic systems A and B are separately in thermal equilibrium with a third thermodynamic system C, then the systems A and B are in thermal equilibrium with each other also.
- **Thermodynamic Process:** A thermodynamic process is said to take place when some changes occur in the state of a thermodynamic system, i.e., the thermodynamic parameters of the system change with time. Types of these thermodynamic process are Isothermal, Adiabatic, Isobaric and Isochoric.
- **Quasi Static Process:** A thermodynamic process which is infinitely slow is called as quasi-static process.

- Quasi-static process is an idealised process. We generally assume all the processes to be quasistatic unless stated.
- **First Law of Thermodynamics:** The first law states that the change in internal energy of the system (ΔU system) is equal to the difference between the heat provided to the system (Q) and the work (W) done by the system on its surroundings in a closed system (i.e., there is no transfer of matter into or out of the system). $\Delta U_{system} = Q - W$
- **Cyclic Process:** A cyclic process is one in which the system returns to its initial stage after undergoing a series of changes.
- **Non-Cyclic Process:** In Non-cyclic process the series of changes involved do not return the system back to its initial state.
- **Isothermal Process**: A thermodynamic process in which temperature remains constant
- **Adiabatic Process:** There is no heat exchange with surroundings.
- **Isochoric Process**: Volume remains constant and the gas should be heated or cooled inside a rigid container.
- **Isobaric Process**: When pressure remains constant and When it is in container, the piston should be free to move and is should not be connected by any agent.
- **Specific Heat at Constant Volume:** It is defined as the amount of heat required to raise the temperature of 1g of a gas through 1C°, when its volume is kept constant. It is denoted as C_V.
- **Specific Heat at Constant Pressure:** It is defined as the amount of heat required to raise the temperature of 1g of a gas through 1°C keeping its pressure constant. It is denoted as C_p.
- **Thermal Efficiency:** Thermal Efficiency of a heat engine is defined of the ratio of net work done per cycle by the engine to the total amount of heat absorbed per cycle by the working substance from the source.

  It is denoted by $\eta = \frac{W}{Q_1}$
- **External Combustion Engine:** The system heated by an external furnace, as in a steam engine. Such engines are called as external combustion engine.
- **Internal Combustion Engine:** The system in which heat is produced by burning the fuel inside the main body of the engine is called as Internal Combustion Engine.
- **Carnot Theorem:** Carnot theorem states that all reversible engines working between same two temperatures have same efficiency irrespective of the nature of working substance. The source and the sink works between the same temperature.
- **Second Law of Thermodynamics:** The second law of thermodynamics states that the heat energy cannot transfer from a body at a lower temperature to a body at a higher temperature without the addition of energy.
- **Coefficient of Performance:** Coefficient of Performance of refrigerator (β) is defined as the ratio of quantity of heat removed per cycle from contents of the refrigerator (Q_2) to the energy spent per cycle (W) to remove this heat.